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Phase synchronization and energy balance between neurons

Key words: Hamilton energy; Coupling synchronization; Synapse enhancement; Neural circuit

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Motivation

1. The firing modes are relative to the intrinsic energy in biophysical neurons, and it is important to clarify how energy diversity affects the synchronization stability between neurons.
2. Synaptic connection between neurons is effective to regulate the collective neural activities, as a result, energy propagation is activated to change the firing patterns. It is interesting to explore how synaptic current and coupling intensity are regulated in adaptive way.

3. Stochastic disturbance has distinct impact on the mode selection of neural activities in neurons, which is helpful to discuss the self-adaption of synaptic coupling in presence of noise.

Main idea

1. The intrinsic field energy in a neural circuit coupled by thermistor and phototube is defined, and equivalent Hamilton energy function is obtained for the temperature-sensitive neuron model driven by photocurrent.
2. Each neuron is considered as a complex charged body, and it is exposed to the superimposed electric field induced by other neurons, as a result, all neurons are under energy balance.
3. External stimuli break the energy balance, and thus synapse connection is created to couple the neurons with increasing coupling intensity exponentially until reaching energy balance and complete synchronization.

Method

1. A functional neural circuit is built to describe the response to external illumination and temperature by embedding a phototube and a thermistor into different branch circuits, and an equivalent function neuron model is obtained.

2. The field energy for neural circuit is mapped into equivalent energy function, and the energy diversity enables the creation of synapse connection to neurons with increasing coupling intensity.

3. Two criteria are proposed to control the coupling intensity between neurons by using Heaviside function, and the coupling intensity terminates its further increase under energy balance and complete synchronization.

Major results

1. A temperature-dependent neural circuit is driven by a phototube, such that the functional neuron model is sensitive to external temperature and illumination, and then Hamilton energy *H* is obtained.

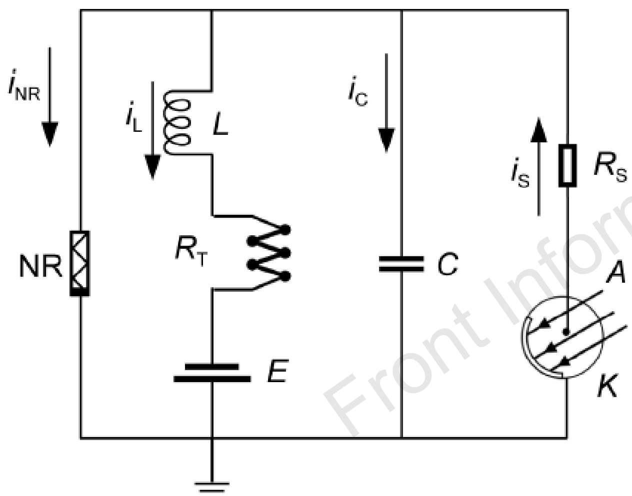

Fig.1 Schematic diagram of a neural circuit composed of a thermistor (R_T) and a phototube. *R* is the resistance of the thermistor.

$$\begin{cases} C\dfrac{dV}{dt} = i_S - i_L - i_N, \\ L\dfrac{di_L}{dt} = V + E - Ri_L, \end{cases} \quad (1)$$

$$\begin{cases} \dfrac{dx}{d\tau} = x(1-\xi) - \dfrac{1}{3}x^3 - y + I_0 \arctan(x - v_a), \\ \dfrac{dy}{d\tau} = c[x + a - b(T')y], \end{cases} \quad (4)$$

$$\begin{cases} W = \dfrac{1}{2}CV^2 + \dfrac{1}{2}Li_L^2 = CV_0^2\left(\dfrac{1}{2}x^2 + \dfrac{1}{2c}y^2\right), \\ H = \dfrac{W}{CV_0^2} = \dfrac{1}{2}x^2 + \dfrac{1}{2c}y^2. \end{cases} \quad (5)$$

Major results (Cont'd)

2. Coupling channel via synapse is controlled by energy diversity.

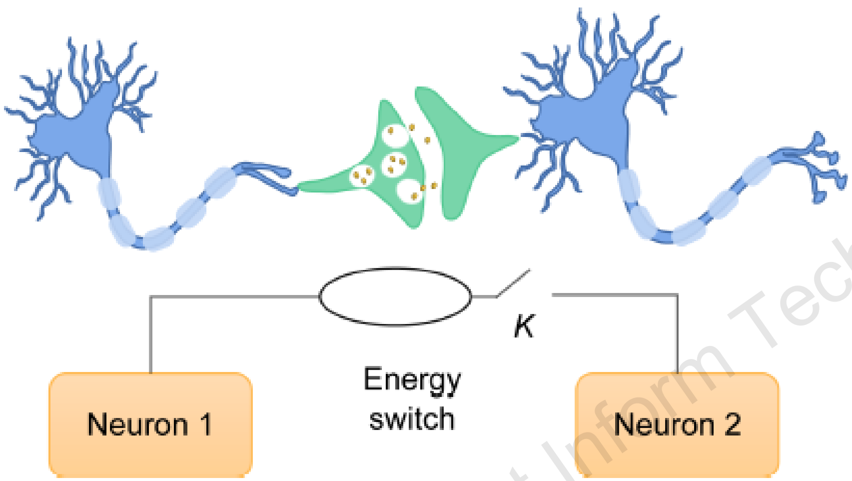

Fig. 2 Schematic for neural circuits controlled by energy diversity

The coupling channel will be switched on for enhancing a synaptic connection when field energy is propagated between two neurons

$$\begin{cases} \dot{x} = x(1-\xi) - \frac{1}{3}x^3 - y + I_0 \arctan(x - v_a) \\ \quad +k(\tau)(x' - x), \\ \dot{y} = c[x + a - b(T')y], \end{cases}$$
$$\begin{cases} \dot{x}' = x'(1-\xi) - \frac{1}{3}x'^3 - y' + I_0' \arctan(x' - v_a) \\ \quad +k(\tau)(x - x'), \\ \dot{y}' = c[x' + a - b(T')y']. \end{cases} \tag{6}$$

$$\Delta H = \left|H_1 - H_2\right| = \left|\left(\frac{1}{2}x^2 + \frac{1}{2c}y^2\right) - \left(\frac{1}{2}(x')^2 + \frac{1}{2c}(y')^2\right)\right| \sim \varepsilon. \tag{7}$$

1. Saturation gain method

$$k(\tau) = k_0 \operatorname{int}(\tau/\lambda)\vartheta(\Delta H - \varepsilon), \quad \vartheta(z) = 1, z \geq 0, \vartheta(z) = 0, z < 0, \tag{8}$$

2. Exponentially regulation and increasement

$$\frac{\mathrm{d}k(\tau)}{\mathrm{d}\tau} = \sigma \cdot k(\tau) \cdot \vartheta(\Delta H - \varepsilon). \tag{9}$$

Conclusions

1. In this work, biophysical mechanism is presented to explain the creation of synaptic connection to neurons in possible adaptive way.
2. Energy diversity between neurons can guide the synapse connections for reaching fast energy balance and complete synchronization by further increasing the coupling intensity.
3. Two criteria are proposed to predict and confirm the growth of coupling intensity for reaching effective energy balance between neurons. External stimuli will corrupt the energy balance, and synaptic coupling is regulated to reduce energy diversity and stabilizing complete synchronization.

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