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Causality fields in nonlinear causal effect analysis

Key words: Nonlinear causal effect; Causality field; *z*-specific causal effect; Positive causality; Negative causality; Null causality

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Motivation

1. Given the cause variable X, effect variable Y, variable Z that influences X and Y, and exogenous variable U, we can obtain the structural causal equation (SCE): $Y = f(X, Z, U)$.
2. The average causal effect (ACE) for discrete variables is:

$$
\begin{aligned}
&\mathrm{ACE}(X \rightarrow Y) \\
&\quad = \left\{E(Y \mid \mathrm{do}(X=x)) - E(Y \mid \mathrm{do}(X=x'))\right\} / (x - x') \\
&\quad = E\left\{E_Z\left[f(x, Z, U) - f(x', Z, U)\right]\right\} / (x - x').
\end{aligned}
$$

Motivation (Cont'd)

3. An Example

Table 1 The effect of a drug considering the time

Time	With or without drug	Effect
Daytime (Z=1)	With (X=1)	Recover (Y=1)
	Without (X=0)	Unchanged (Y=0)
Night (Z=0)	With (X=1)	Worsen (Y=−1)
	Without (X=0)	Unchanged (Y=0)

for Z = 0, $ACE|_{Z=1} = E(Y|do(X=1), Z=1) - E(Y|do(X=0), Z=1) = 1$

for Z = 1, $ACE|_{Z=0} = E(Y|do(X=1), Z=0) - E(Y|do(X=0), Z=0) = -1$

- **The causal effect of X on Y is fluctuant and related to Z**

4. Nonlinear causality

if X does not appear in the derivative $\frac{\partial}{\partial X} f(X, Z, U)$, the causal relation is quasi-linear.

if X appears in the derivative $\frac{\partial}{\partial X} f(X, Z, U)$, the causal relation is strong nonlinear.

Causality fields

1. Consists of the following three types of causal effect of *X* on *Y*

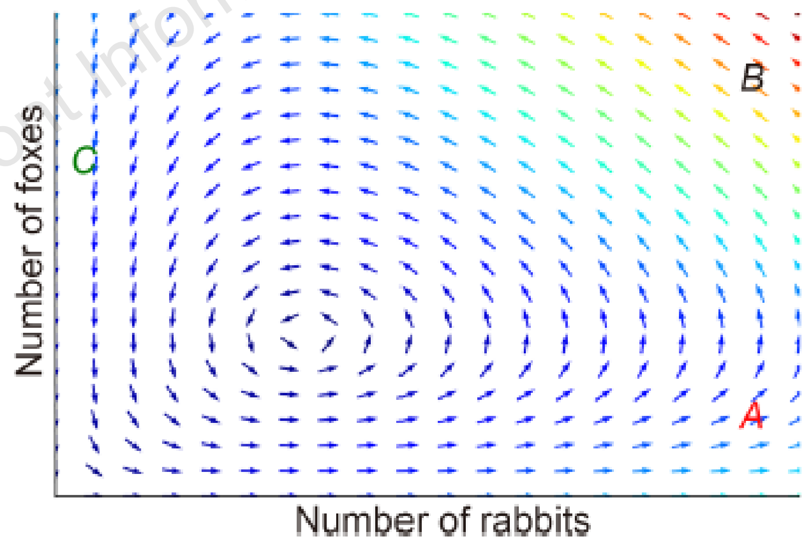
- **Positive causality**: X causes a homodromous change in *Y*, denoted as Ω^+
- **Negative causality**: X causes an antagonistic change in *Y*, denoted as Ω^-
- **Null causality**: X causes no change in *Y*, denoted as Ω^0

$$\begin{cases} \Omega^+ = \left\{ (x, z, u) \mid \frac{\partial}{\partial X} f(X, Z, U) > 0 \right\}, \\ \Omega^- = \left\{ (x, z, u) \mid \frac{\partial}{\partial X} f(X, Z, U) < 0 \right\}, \\ \Omega^0 = \left\{ (x, z, u) \mid \frac{\partial}{\partial X} f(X, Z, U) = 0 \right\}, \end{cases}$$

Causality fields (Cont'd)

2. An Example: the competition-symbiotic relationships between rabbits and foxes

- We can observe that rabbits and foxes have competitive as well as symbiotic relationships
- area *A*, ecological mutualism model
- area *B*, ecological competition model
- area *C*, ecological total-loss model



Nonlinear causality computation

1. Three key assumptions that enable causal effect estimation

- Stable unit treatment value assumption (SUTVA)
- Ignorability
- Positivity

2. With the above assumptions, we can calculate the ACE

$$ACE = E\left[E_z\left(Y^{X=x}\right) - E_z\left(Y^{X=x'}\right)\right]$$

3. Two ways for nonlinear causality computation when there are cases where the assumptions do not hold.

- SCE is more intuitive while considering estimating functions rather than probability distributions

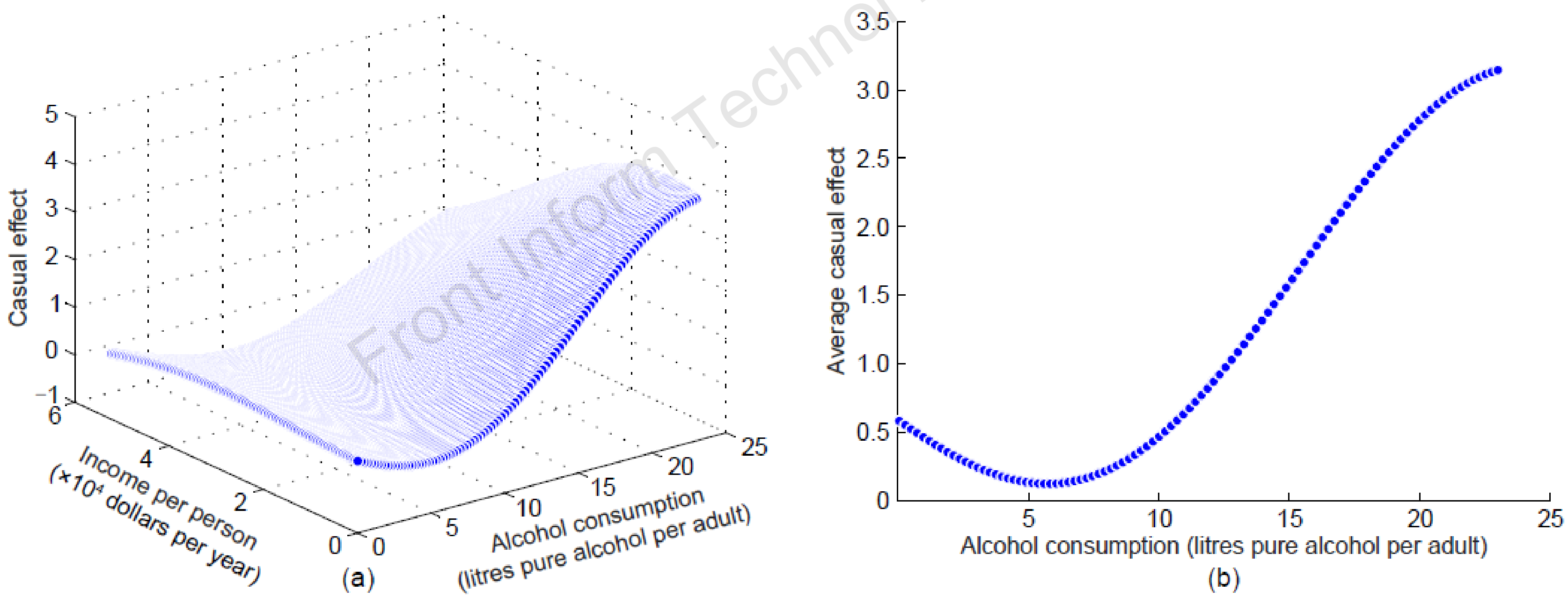
$$X_i = f_i(PA_i, U_i)$$

- Causal representation learns a latent disentangled representation

$$g: \{x_1, x_2, \dots, x_n\} \rightarrow \{w_1, w_2, \dots, w_n\}$$

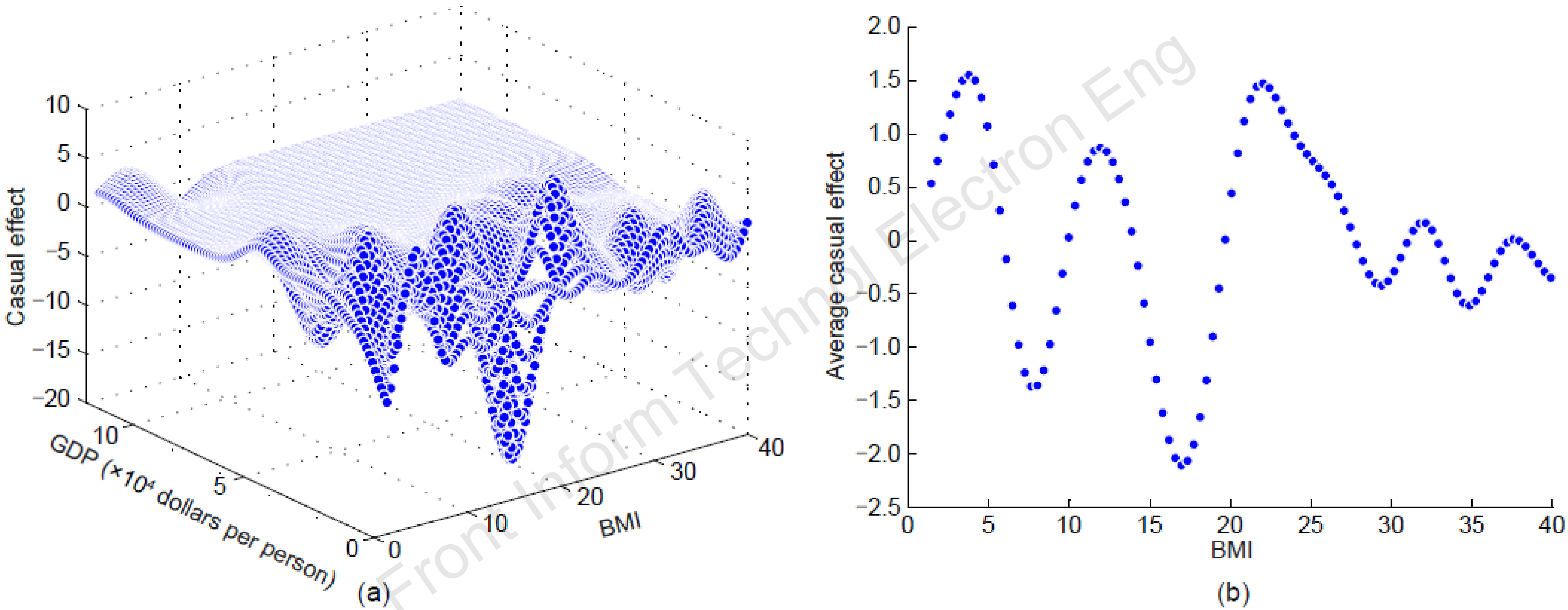
Causality field examples

1. three datasets from Kaggle (https://www.kaggle.com/)
   - The alcohol consumption dataset,
   - The life expectancy (WHO) dataset,
   - The diabetes dataset.



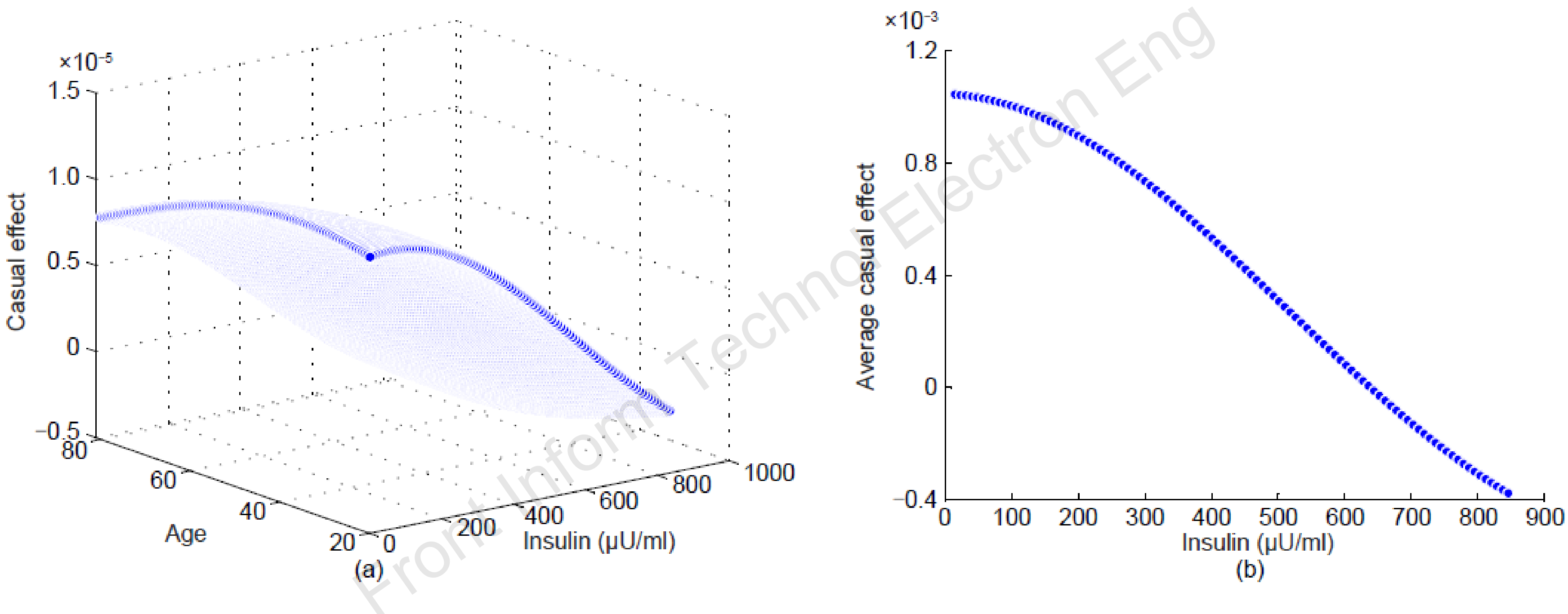
The causal effect of alcohol consumption (X) on suicide rate (Y) with income per person (Z) as the confounding variable

Causality field examples (Cont’d)



The causal effect of BMI (X) on life expectancy (Y) with GDP (Z) as the confounding variable

Causality field examples (Cont’d)



The causal effect of insulin (X) on diabetes (Y) with age (Z) as the confounding variable

Conclusions

1. The problems of nonlinear causal effect calculation and the causality field are discussed.

2. Causality fields are formulated.

3. Methods for calculating nonlinear causal effects are introduced.

4. Three examples are used to illustrate causality fields.

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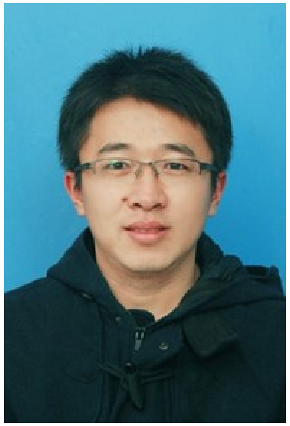

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