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Observer-based control for fractional-order singular systems with order *α* (0<*α*<1) and input delay

Key words: Observer-based control; Singular systems; Fractional order; Input delay; Linear matrix inequality

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Motivation

Although a lot of papers have been published for input delay, it is worth mentioning that most research is mainly in the field of nonsingular fractional-order systems with input delay. For general singular systems with input delay, these were rarely studied. In short, the contributions are as follows:

1. Using the Smith predictor, observer-based control is first studied for fractional-order singular systems with order α $(0<\alpha<1)$ and input delay.
2. The necessary and sufficient condition based on nonstrict LMI is presented. Then, the condition based on strict LMI is improved.

System model

For the following fractional-order singular system (1), define the state transformation (2)

$$\begin{cases} E\mathrm{D}^{\alpha}x(t) = Ax(t) + B_1u(t) + B_2u(t-\tau), \\ y(t) = Cx(t), \end{cases} \quad (1)$$

$$Ez(t) = Ex(t) + \int_{t-\tau}^{t} e_{\alpha}^{A(t-s-\tau)}B_2u(s)\mathrm{d}s. \quad (2)$$

Using the properties of fractional calculus, we obtain

$$\bar{E}\mathrm{D}^{\alpha}\bar{z}(t) = \bar{A}\bar{z}(t), \quad (9)$$

$$\begin{cases} \bar{E} = \begin{bmatrix} E & 0 \\ 0 & E \end{bmatrix}, \ \bar{z}(t) = \begin{bmatrix} z(t) \\ z(t) - \hat{z}(t) \end{bmatrix}, \\ \bar{A} = \begin{bmatrix} A + \bar{B}K & -\bar{B}K \\ 0 & A - LC \end{bmatrix}. \end{cases} \quad (10)$$

Main lemma

Lemma 1 (Zhang XF and Chen, 2018) The fractional-order linear singular system $\boldsymbol{E}\mathrm{D}^{\alpha}\boldsymbol{x}(t)=\boldsymbol{A}\boldsymbol{x}(t)$ with rank($\boldsymbol{E}$)<n is asymptotically admissible if and only if the following inequalities hold with two matrices $\boldsymbol{X}$ and $\boldsymbol{Y}$:

$$\begin{bmatrix} EX & EY \\ -EY & EX \end{bmatrix} = \begin{bmatrix} X^{\mathrm{T}}E^{\mathrm{T}} & -Y^{\mathrm{T}}E^{\mathrm{T}} \\ Y^{\mathrm{T}}E^{\mathrm{T}} & X^{\mathrm{T}}E^{\mathrm{T}} \end{bmatrix} \geq 0,$$

$$aAX + aXA^{\mathrm{T}} + bAY - bYA^{\mathrm{T}} < 0,$$

where $a=\sin(\alpha\pi/2)$ and $b=\cos(\alpha\pi/2)$.

Main results

Theorem 1 System (9) with matrices ***K*** and ***L*** is asymptotically admissible, if and only if there exist matrices $\boldsymbol{X}_1$, $\boldsymbol{X}_2$, $\boldsymbol{Y}_1$, $\boldsymbol{Y}_2$, $\boldsymbol{Z}$, and ***R*** such that the following inequalities hold:

$$\begin{bmatrix} EX_1 & EY_1 \\ -EY_1 & EX_1 \end{bmatrix} = \begin{bmatrix} X_1^{\mathrm{T}}E^{\mathrm{T}} & -Y_1^{\mathrm{T}}E^{\mathrm{T}} \\ Y_1^{\mathrm{T}}E^{\mathrm{T}} & X_1^{\mathrm{T}}E^{\mathrm{T}} \end{bmatrix} \geq 0, \quad (11)$$

$$\begin{bmatrix} EX_2 & EY_2 \\ -EY_2 & EX_2 \end{bmatrix} = \begin{bmatrix} X_2^{\mathrm{T}}E^{\mathrm{T}} & -Y_2^{\mathrm{T}}E^{\mathrm{T}} \\ Y_2^{\mathrm{T}}E^{\mathrm{T}} & X_2^{\mathrm{T}}E^{\mathrm{T}} \end{bmatrix} \geq 0, \quad (12)$$

$$P_1 = aX_1 + bY_1, \ \mathrm{sym}(P_1A - ZC) < 0, \quad (13)$$

$$P_2 = aX_2 + bY_2, \ \mathrm{sym}(AP_2 + \bar{B}R) < 0, \quad (14)$$

where $a=\sin(\alpha\pi/2)$ and $b=\cos(\alpha\pi/2)$. Furthermore, ***L*** and ***K*** are given by

$$L = P_1^{-1}Z, \ K = RP_2^{-1}. \quad (15)$$

Main results (Cont’d)

Remark 1 Inequalities (11) and (12) are nonstrict LMIs, which contain equality constraints. Due to round-off errors in numerical calculations, equality constraints are fragile and usually cannot be well satisfied. As a result, the strict LMI-based condition is proposed in the following theorem.

Main results (Cont'd)

Theorem 2 System (9) with matrices ***K*** and ***L*** is asymptotically admissible, if and only if there exist matrices $\boldsymbol{X}_1$, $\boldsymbol{X}_2$, $\boldsymbol{Y}_1$, $\boldsymbol{Y}_2$, ***Q***, ***Z***, and ***R*** such that the following inequalities hold:

$$\begin{bmatrix} X_1 & Y_1 \\ -Y_1 & X_1 \end{bmatrix} > 0, \tag{24}$$

$$\begin{bmatrix} X_2 & Y_2 \\ -Y_2 & X_2 \end{bmatrix} > 0, \tag{25}$$

$$P_1 = aX_1 + bY_1, \ \mathrm{sym}(P_1 E^{\mathrm{T}} A + SQA - ZC) < 0, \tag{26}$$

$$P_2 = aX_2 + bY_2, \ \mathrm{sym}(AP_2 E^{\mathrm{T}} + ASQ + \bar{B}R) < 0, \tag{27}$$

where *a*=sin(*α*π/2), *b*=cos(*α*π/2), and ***S*** satisfies ***ES***=0. Furthermore, ***L*** and ***K*** are given by

$$L = P_1^{-1} Z, \ K = RP_2^{-1}. \tag{28}$$

Simulations

Example 1

System (9) with parameters as follows is considered:

$$E = \begin{bmatrix} 0 & -1 & 0 & -1 \\ 1 & 2 & 0 & 1 \\ -2 & -2 & 1 & -2 \\ -1 & -1 & 0 & 0 \end{bmatrix},\ A = \begin{bmatrix} 1 & -1 & -1 & 1 \\ 0 & 3 & 2 & 0 \\ 2 & -5 & -1 & -5 \\ 1 & -2 & -1 & -1 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix},\ B_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 2 & 1 & -1 \end{bmatrix},\ \alpha = \frac{1}{3},\ \tau = 0.5.$$

Choose $\boldsymbol{S} = \begin{bmatrix} -1 \\ 1 \\ -2 \\ -1 \end{bmatrix}$

Then, solving inequalities (24)–(27) and Eq. (28) in Theorem 2, we can obtain the feasible solutions as follows:

$$K = [0.3225\ \ -0.6320\ \ -0.1244\ \ 0.0123],$$

$$L = [-6.3313\ \ -8.6988\ \ 1.6989\ \ -7.6680]^{\mathrm{T}}.$$

Simulations (Cont'd)

Example 1

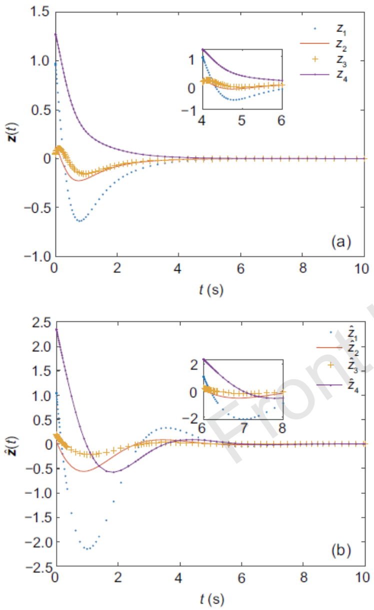

Fig. 1 System states $z(t)$ (a) and observer states $\hat{z}(t)$ (b)

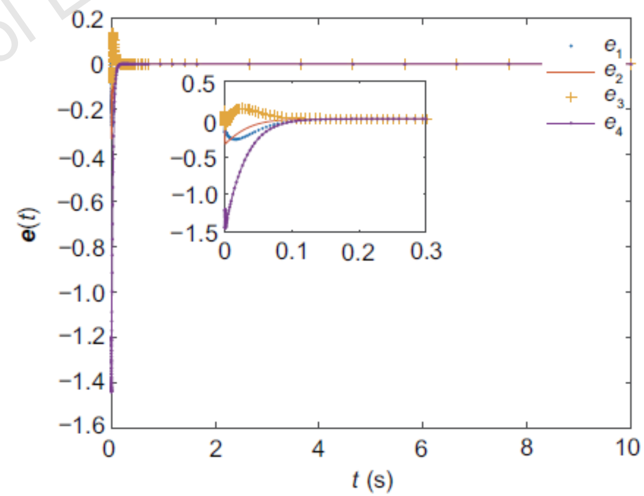

Fig. 2 Observer errors $e(t)$

Simulations (Cont'd)

Example 2

In this study, the direct current (DC) motor model is used as the example shown in Fig. 3. We can obtain Eq. (33).

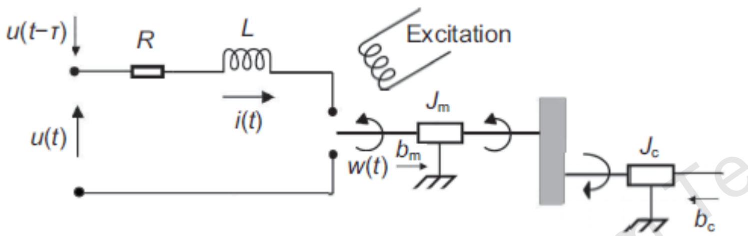

Fig. 3 **Block diagram of the direct current (DC) motor**

$$\begin{cases} \begin{bmatrix} L & 0 & 0 \\ 0 & J & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathrm{D}^{\alpha} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ K_{\mathrm{t}} & -b & 0 \\ R & -K_{\mathrm{w}} & -1 \end{bmatrix} \\ \quad \cdot \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t-\tau), \\ y(t) = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}. \end{cases} \tag{33}$$

Choose the parameter settings

Parameter	Value	Parameter	Value
L	2 mH	J_{m}	$1\ \mathrm{kg \cdot cm^2}$
R	4 Ω	b_{m}	1.5
K_{t}	$2.5\ \mathrm{kg \cdot cm^2/s^2}$	b_{c}	2
K_{w}	4 V/(rad·s)	n_0	2
J_{c}	$8\ \mathrm{kg \cdot cm^2}$		

$$\begin{cases} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathrm{D}^{\alpha} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 2.5 & -2 & 0 \\ 4 & -4 & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} \\ \quad + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t-\tau), \\ y(t) = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}. \end{cases} \tag{34}$$

Simulations (Cont'd)

Example 2

Table 5 Simulation results for K

α	τ	K
0.21	0.37	$[1.3729 \quad 2.2329 \quad -0.8073]$
	1.21	$[1.0350 \quad 2.7757 \quad -0.3342]$
	2.35	$[0.4279 \quad 3.0690 \quad -0.2465]$
0.64	0.37	$[4.1368 \quad -2.6281 \quad 0.4634]$
	1.21	$[2.5584 \quad -1.3251 \quad -0.1366]$
	2.35	$[2.0506 \quad -0.8866 \quad -0.3650]$
0.98	0.37	$[2.1249 \quad -1.2776 \quad 0.6772]$
	1.21	$[2.1000 \quad -1.2840 \quad 0.3802]$
	2.35	$[2.1017 \quad -1.2961 \quad 0.2318]$

Table 6 Simulation results for L

α	τ	L
0.21	0.37	$[12.7852 \quad 11.7590 \quad 5.2006]^{\mathrm{T}}$
	1.21	$[8.7194 \quad 12.7752 \quad 13.3779]^{\mathrm{T}}$
	2.35	$[5.0822 \quad 10.5641 \quad 16.8711]^{\mathrm{T}}$
0.64	0.37	$[-0.3521 \quad -15.2569 \quad -41.2119]^{\mathrm{T}}$
	1.21	$[-2.0668 \quad -14.5905 \quad -45.0673]^{\mathrm{T}}$
	2.35	$[-4.1104 \quad -14.1993 \quad -49.4751]^{\mathrm{T}}$
0.98	0.37	$[2.8932 \quad -7.8294 \quad -10.9265]^{\mathrm{T}}$
	1.21	$[2.1371 \quad -8.2418 \quad -14.6744]^{\mathrm{T}}$
	2.35	$[1.2399 \quad -8.6320 \quad -19.3545]^{\mathrm{T}}$

Simulations (Cont’d)

Example 2

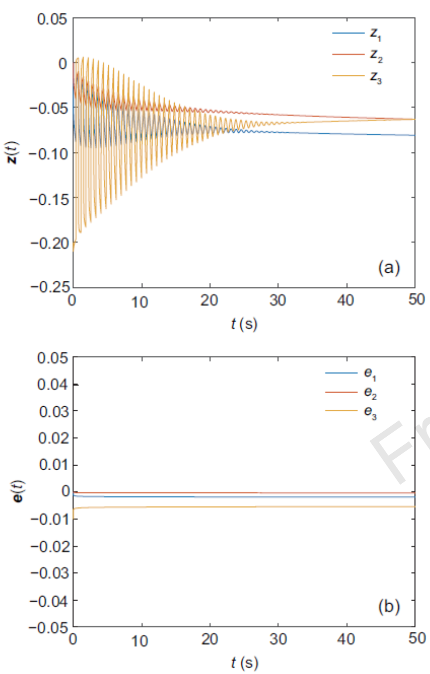

Fig. 4 States $z(t)$ (a) and errors $e(t)$ (b) with $\alpha = 0.21$ and $\tau = 0.37$

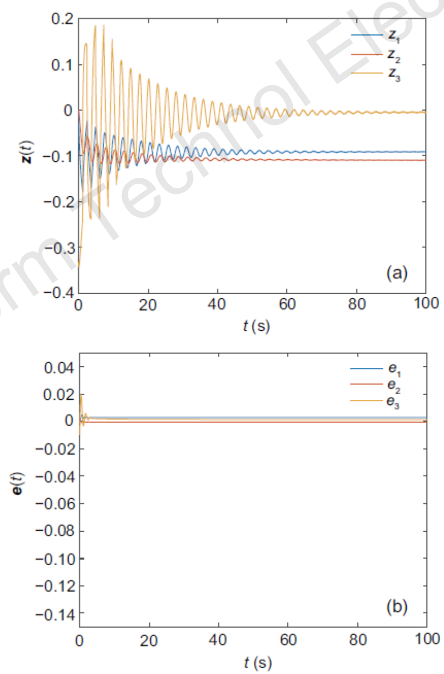

Fig. 5 States $z(t)$ (a) and errors $e(t)$ (b) with $\alpha = 0.64$ and $\tau = 1.21$

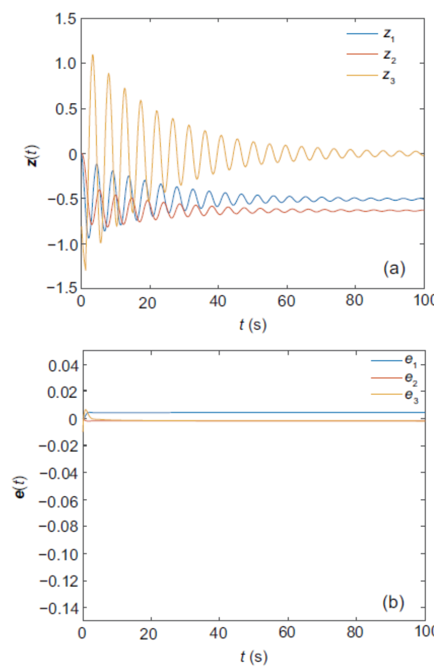

Fig. 6 States $z(t)$ (a) and errors $e(t)$ (b) with $\alpha = 0.98$ and $\tau = 2.35$

Conclusions

The paper deals with observer-based controller design for fractional-order singular systems with α ($0<\alpha<1$) and input delay. Using the linear matrix inequality (LMI) technique, the necessary and sufficient condition based on the nonstrict LMI is obtained. In the case of random error, the nonstrict LMI-based condition will cause trouble. When we improve the condition based on the strict LMI, the condition is easier to handle. Finally, the numerical example and the DC motor example are given to illustrate the effectiveness of the proposed condition.

In the future, observer-based robust control and observer-based H_∞ control for fractional-order singular systems will be studied.

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