Shaoqiang YE, Kaiqing ZHOU, Azlan Mohd ZAIN, Fangling WANG, Yusliza YUSOFF, 2023. A modified harmony search algorithm and its applications in weighted fuzzy production rule extraction. *Frontiers of Information Technology & Electronic Engineering*, 24(11):1574-1590.
https://doi.org/10.1631/FITEE.2200334

A modified harmony search algorithm and its applications in weighted fuzzy production rule

Key words: Harmony search algorithm; Cuckoo search algorithm; Global convergence; Function optimization; Weighted fuzzy production rule extraction

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Motivation

1. At the optimization stage of harmony search (HS), due to the randomness of the HS pitch disturbance adjusting method, the exploitation optimization ability is insufficient because of poor solution accuracy and low convergence speed.

2. In the global exploration stage of HS, new individuals are randomly generated, which shows some defects, such as weak global search ability and poor population diversity.

3. The key parameters of HS adjustment ability are poor in the iterative process.

Main idea

1. An adaptive inertia weight is constructed based on the relationship of information transfer between individuals in HM to improve the optimization ability of HS, and then the optimal solution in HM is used to replace the bandwidth.

2. A CS operator is employed to enhance the global search ability and population diversity of HS in the random generation of new harmony (1−HMCR), and to select candidate individuals from the poor HM.

3. HMCR and PAR are adjusted adaptively in the iterative process to strengthen the adaptability of HS.

Method

1. The harmony solution vector is selected randomly to strengthen the search ability of HS by the constructed inertia weight operator in HM with the probability of HMCR.

$$X_{\text{new}}^{t+1}(i) = \text{HM}_i^{\text{worst},t} + \omega \cdot (\text{HM}_i^{\text{best},t} - \text{HM}_i^{\text{worst},t}),$$

$$\omega = \begin{cases} (\omega_2 - \omega_1)/(2\omega_2 - \omega_1), & \text{if } 2\omega_2 - \omega_1 \neq 0, \\ \text{rand}(0, 1), & \text{otherwise,} \end{cases}$$

$$\omega_1 = \min\left(\text{HM}_i^{\text{best},t}, \text{HM}_i^{\text{worst},t}\right)$$

$$\omega_2 = \max\left(\text{HM}_i^{\text{best},t}, \text{HM}_i^{\text{worst},t}\right).$$

Method (Cont’d)

2. A CS operator is used to replace the original HS population update mechanism in the random generation of new harmony (1−HMCR) to expand and update the solution vector of harmony.

$$X_{\text{new}}^{t+1} = \text{HM}^{\text{best},t} + \alpha(\text{HM}^{\text{best},t} - \text{HM}^{\text{worst},t}) \otimes \text{Levy}(\lambda)$$

3. To avoid easy-to-premature convergence and falling into a stagnant state in the final stage of the standard HS, the CS operator is used to find potential individuals from HM as candidate individuals to expand the population density.

$$(X_{\text{new}}^{t+1})' = \text{HM}^{\text{worst},t} + \alpha(\text{HM}^{\text{best},t} - \text{HM}^{\text{worst},t}) \otimes \text{Levy}(\lambda)$$

Method (Cont'd)

4. Key variables HMCR and PAR are adaptively adjusted to obtain a higher convergence efficiency of HS in the iterative process. The former increases linearly and the latter decreases linearly as the number of iterations increases.

$$\mathrm{HMCR}(t) = \mathrm{HMCR}_{\min} + (\mathrm{HMCR}_{\max} - \mathrm{HMCR}_{\min}) \cdot t/T_{\max}$$

$$\mathrm{PAR}(t) = \mathrm{PAR}_{\max} - (\mathrm{PAR}_{\max} - \mathrm{PAR}_{\min}) t/T_{\max}$$

Method (Cont'd)

- In the initial population stage, the time complexity is $O(\text{HMS} \times \text{Dim})$; the time complexity of evaluating the objective function of the individual in the population is $O(f(\text{Dim}))$.
- In the "pitch adjusting and selecting the best" stage of the improvisation process, the time complexity is $O(\text{HMS} \times \text{Dim})$

•In the population update stage, the time complexity is $O(\text{HMS} \times (\text{Dim}+O(\text{Levy})))$.

Method (Cont'd)

- The worst-case time complexity of the HS-CS algorithm could be calculated below, while the number of the current iterations is 1.

$O(\text{HMS}\times\text{Dim})+O(\text{HMS}\times(\text{Dim}+f(\text{Dim})))+O(\text{HMS}\times(\text{Dim}+f(\text{Dim})))$

$+O(\text{HMS}\times(\text{Dim}+f(\text{Dim})))\approx O(\text{HMS}\times(\text{Dim}+f(\text{Dim})))$.

- Therefore, when the HS-CS algorithm reaches the maximum number of iterations T_{max}, the time complexity is $O(T_{max}\times\text{HMS}\times(\text{Dim}+f(\text{Dim})))$.

Method (Cont’d)

Convergence analysis of the core of HS-CS

Proof HS-CS may produce better individual conditions than the previous generation in each iteration because of the appropriate fine-tuning selection operator and greedy selection strategy that are adopted. There is a non-negative real random variable $H(\hat{\delta})$ which fulfills

$$\left|f(X_{i+1})-f(X_{i+2})\right| \le H(\hat{\delta})\left|\left(z-f(X_i)\right)-\left(z-f(X_{i+1})\right)\right|, 0 \le H(\hat{\delta}) < 1.$$

Furthermore,

$$\begin{aligned} & d\left(\varphi(\hat{\delta},X_i),\varphi(\hat{\delta},X_{i+1})\right) \\ &= d\left(X_{i+1},X_{i+2}\right) \\ &= \left|\left(z-f(X_{i+1})\right)-\left(z-f(X_{i+2})\right)\right| \\ &= \left|f(X_{i+1})-f(X_{i+2})\right| \\ &\le H(\hat{\delta})\left|\left(z-f(X_i)\right)-\left(z-f(X_{i+1})\right)\right| \\ &= H(\hat{\delta})d\left(X_i,X_{i+1}\right), \end{aligned}$$

$\Omega_0=\left\{\hat{\delta}: d\left(\varphi(\hat{\delta},X_i),\varphi(\hat{\delta},X_{i+1})\right)\le H(\hat{\delta})d\left(X_i, X_{i+1}\right)\right\} \subseteq \Omega$, and $\rho\left(\Omega_0\right)=1$.

To sum up, assuming that the Borel subset of S is $D=M_i^t$, there exist $v[D]>0$ and $\mu_t[D]=\sum_{i=1}^{N}\mu_i^t[D]=1$. It is further obtained that $\prod_{t=0}^{\infty}\left(1-\mu_t(D)\right)=0$. It is easy to conclude that HS-CS fulfills Assumption 2, so that $\lim_{k\to\infty} P\left[\mathrm{HM}^{\mathrm{best},t}\in R_\varepsilon\right]=1$.

Hence, the HS-CS algorithm converges to the global optimal solution with probability 1.

Major results

Numerical optimization

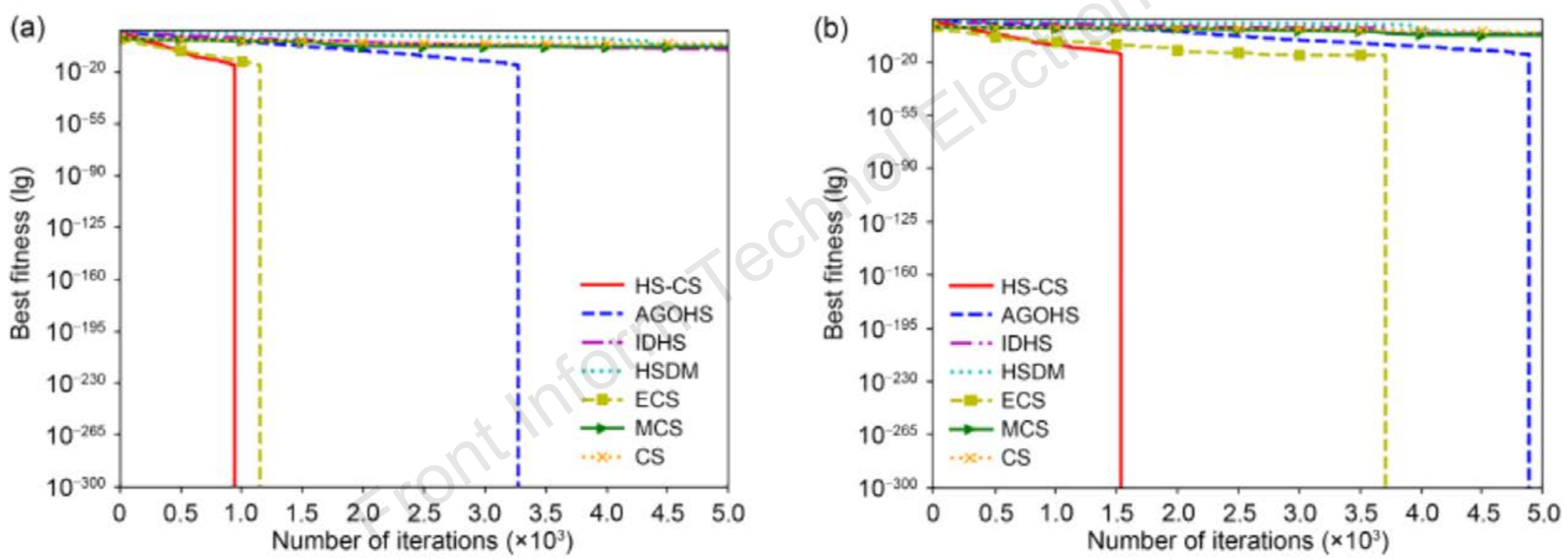

Fig. 2 Optimizing the convergence curve of the Griewank function with dimension 30 (a) and 50 (b)

Major results (Cont'd)

Numerical analysis

Table S1 Experimental statistics of mean and standard deviation

Function	Dim	Mean±Std						
		CS	ECS	MCS	AGOHS	IDHS	HSDM	HS-CS
F1	10	4.36E-03±1.04E-03	2.75E-164±0.00E+00	1.46E-04±3.47E-05	**0.00E+00±0.00E+00**	4.84E-15±1.45E-14	1.50E-11±9.68E-12	**0.00E+00±0.00E+00**
	30	2.96E-01±2.77E-02	6.26E-39±3.08E-39	1.00E-03±2.58E-04	**0.00E+00±0.00E+00**	1.37E-10±8.77E-11	3.38E-04±1.11E-04	**0.00E+00±0.00E+00**
	50	1.38E+00±8.42E-02	9.49E-19±2.22E-19	3.68E-02±2.72E-03	**0.00E+00±0.00E+00**	4.38E-04±3.71E-04	5.33E-02±1.50E-02	**0.00E+00±0.00E+00**
F2	10	1.10E-07±9.77E-08	1.35E-31±6.57E-47	1.67E-08±1.52E-08	**0.00E+00±0.00E+00**	**0.00E+00±0.00E+00**	2.37E-09±1.89E-09	**0.00E+00±0.00E+00**
	30	9.08E-08±6.29E-08	1.35E-31±6.57E-47	1.05E-08±1.09E-08	**0.00E+00±0.00E+00**	6.13E-08±5.87E-15	7.02E-02±2.20E-02	**0.00E+00±0.00E+00**
	50	1.31E-07±1.13E-07	1.35E-31±6.57E-47	1.14E-08±1.29E-08	**0.00E+00±0.00E+00**	6.76E-02±7.37E-02	7.68E+00±1.46E+00	**0.00E+00±0.00E+00**
F3	10	8.29E-03±9.06E-04	4.10E-04±1.20E-04	2.74E-03±3.87E-04	**0.00E+00±0.00E+00**	3.76E-02±1.13E-01	1.47E-05±1.37E-05	**0.00E+00±0.00E+00**
	30	2.39E+00±8.17E-01	3.14E+00±4.83E-01	6.44E-01±2.33E-01	5.16E-01±1.55E+00	5.37E+00±6.49E+00	6.77E-02±5.12E-02	**0.00E+00±0.00E+00**
	50	1.30E+01±2.02E+00	1.61E+01±1.41E+00	2.57E+00±5.33E-01	2.33E+00±7.00E+00	2.06E+01±1.18E+01	2.42E+00±9.29E-01	**0.00E+00±0.00E+00**
F4	10	3.13E+00±7.01E-01	1.99E+00±7.43E-01	3.82E-02±8.48E-03	**0.00E+00±0.00E+00**	**0.00E+00±0.00E+00**	1.73E-05±1.91E-05	**0.00E+00±0.00E+00**
	30	9.40E+01±9.11E+00	6.83E+01±8.57E+00	2.22E+01±3.39E+00	1.52E+01±2.49E+01	7.84E+01±3.27E+01	2.65E+01±3.32E+00	**0.00E+00±0.00E+00**
	50	3.97E+02±7.28E+01	2.39E+02±1.37E+01	1.70E+02±2.34E+01	6.46E+01±7.03E+01	2.34E+02±3.22E+01	1.14E+02±7.30E+00	**0.00E+00±0.00E+00**
F5	10	5.90E-02±2.67E-02	**0.00E+00±0.00E+00**	5.70E-02±2.30E-02	**0.00E+00±0.00E+00**	**0.00E+00±0.00E+00**	6.15E-06±2.82E-11	**0.00E+00±0.00E+00**
	30	1.72E-02±1.19E-02	**0.00E+00±0.00E+00**	8.39E-03±1.15E-02	**0.00E+00±0.00E+00**	2.93E-04±3.33E-07	3.25E-02±6.41E-03	**0.00E+00±0.00E+00**
	50	9.40E-02±1.04E-01	2.07E-16±7.07E-16	8.78E-03±1.37E-04	1.53E-13±1.24E-25	6.89E-02±1.08E-01	3.38E-02±4.89E-02	**0.00E+00±0.00E+00**
F6	10	1.06E-01±1.34E-02	2.57E-15±1.59E-15	1.63E-02±2.36E-03	**0.00E+00±0.00E+00**	**0.00E+00±0.00E+00**	7.03E-07±6.55E-07	**0.00E+00±0.00E+00**
	30	8.33E-01±6.95E-02	6.54E-14±8.40E-14	6.47E-02±3.37E-03	**0.00E+00±0.00E+00**	1.49E-06±2.29E-06	1.97E-01±2.11E-02	**0.00E+00±0.00E+00**
	50	1.71E+00±6.12E-02	2.29E-02±9.39E-03	1.13E-01±4.65E-03	1.31E-15±5.81E-15	2.41E+00±3.31E+00	1.76E+00±1.92E-01	**0.00E+00±0.00E+00**
F7	10	2.65E-03±6.29E-04	**0.00E+00±0.00E+00**	1.84E-04±4.42E-05	**0.00E+00±0.00E+00**	3.88E-08±4.22E-14	1.43E-07±1.82E-14	**0.00E+00±0.00E+00**
	30	2.65E-03±3.71E-02	**0.00E+00±0.00E+00**	7.98E-03±9.25E-07	**0.00E+00±0.00E+00**	6.09E-07±9.86E-07	5.02E+00±1.38E+00	**0.00E+00±0.00E+00**
	50	1.34E+00±1.27E-01	3.62E-16±1.13E-16	3.84E-02±2.84E-03	4.32E-15±8.19E-29	6.14E-02±2.98E-02	8.97E+01±1.53E+02	**0.00E+00±0.00E+00**

To be continued

Major results (Cont'd)

Numerical analysis

Table S1

Function	Dim	Mean±Std						
		CS	ECS	MCS	AGOHS	IDHS	HSDM	HS-CS
F8	10	1.66E-01±1.98E-02	5.56E-106±5.28E-106	3.12E-02±3.37E-03	**0.00E+00±0.00E+00**	1.93E-19±1.55E-19	1.94E-06±6.55E-07	**0.00E+00±0.00E+00**
	30	2.34E+00±1.64E-01	3.43E-27±7.18E-28	3.78E-01±1.96E-02	1.22E-22±2.30E-22	1.39E-06±2.17E-06	3.95E-01±7.72E-03	**0.00E+00±0.00E+00**
	50	6.65E+00±2.57E-01	3.04E-12±4.83E-13	1.06E+00±3.95E-02	1.32E-14±9.34E-15	1.30E+01±7.74E-03	7.02E-01±1.24E-01	**0.00E+00±0.00E+00**
F9	10	1.03E-01±1.81E-07	**0.00E+00±0.00E+00**	7.28E-03±1.84E-03	**0.00E+00±0.00E+00**	**0.00E+00±0.00E+00**	2.23E-07±1.07E-07	**0.00E+00±0.00E+00**
	30	1.03E-01±1.45E-07	**0.00E+00±0.00E+00**	3.79E-01±7.19E-04	**0.00E+00±0.00E+00**	8.90E-06±7.43E-06	4.83E+00±1.40E+00	**0.00E+00±0.00E+00**
	50	2.51E+01±2.73E+01	7.37E-01±9.10E-01	2.00E+00±2.13E-01	2.66E-16±3.95E-16	1.20E+01±6.64E+00	8.65E+01±1.17E+01	**0.00E+00±0.00E+00**
F10	10	1.46E+00±1.43E-01	1.68E-04±5.56E-05	3.78E-04±1.75E-04	**0.00E+00±0.00E+00**	2.68E-07±1.42E-06	8.09E-06±7.26E-06	**0.00E+00±0.00E+00**
	30	1.04E+01±5.16E-01	7.82E-03±2.22E-03	7.28E-03±2.25E-03	**0.00E+00±0.00E+00**	9.58E-06±2.08E-05	5.14E+00±3.11E+00	**0.00E+00±0.00E+00**
	50	2.48E+01±8.48E-01	3.84E-02±3.90E-03	3.86E-02±6.62E-03	2.78E-15±1.31E-14	7.47E+00±8.17E+00	9.49E+02±1.81E+02	**0.00E+00±0.00E+00**
F11	10	7.51E+00±5.30E+00	**0.00E+00±0.00E+00**	8.34E-12±8.34E-12	**0.00E+00±0.00E+00**	**0.00E+00±0.00E+00**	1.39E-08±9.41E-09	**0.00E+00±0.00E+00**
	30	7.83E+01±5.41E+01	**0.00E+00±0.00E+00**	1.08E-11±1.44E-11	**0.00E+00±0.00E+00**	4.31E-07±5.50E-07	5.47E-01±2.20E-01	**0.00E+00±0.00E+00**
	50	2.81E+02±1.27E+02	**0.00E+00±0.00E+00**	8.62E-12±9.48E-12	7.66E-18±4.05E-17	6.80E-01±7.09E-01	2.40E+01±2.86E+00	**0.00E+00±0.00E+00**
F12	10	7.05E-04±1.43E-04	1.27E-04±0.00E+00	1.48E-04±1.92E-05	**0.00E+00±0.00E+00**	**0.00E+00±0.00E+00**	4.26E-06±2.59E-06	**0.00E+00±0.00E+00**
	30	5.06E+02±2.34E+02	1.32E+03±1.41E+02	1.30E+01±3.59E+01	**0.00E+00±0.00E+00**	4.69E-04±9.63E-04	2.59E+01±3.87E+00	**0.00E+00±0.00E+00**
	50	4.25E+03±5.70E+02	3.30E+03±2.06E+03	1.31E+03±3.58E+02	7.10E-12±3.71E-11	4.16E+01±1.19E+01	1.44E+03±3.82E+02	**0.00E+00±0.00E+00**

Optimal values are shown in bold

Major results (Cont'd)

Numerical analysis

Table S3 Wilcoxon rank-sum test results of six algorithms referring to HS-CS in the 30-dimensional case

Function	CS		ECS		MCS		AGOHS		IDHS		HSDM		HS-CS
	Rank	Wilcoxon-test	Rank	Wilcoxon-test	Rank	Wilcoxon-test	Rank	Wilcoxon-test	Rank	Wilcoxon-test	Rank	Wilcoxon-test	Rank
F1	7	+	3	+	6	+	1	=	4	+	5	+	1
F2	7	+	3	+	4	+	1	=	5	+	6	+	1
F3	6	+	5	+	4	+	3	+	7	+	2	+	1
F4	7	+	5	+	3	+	2	+	6	+	4	+	1
F5	6	+	1	=	4	+	1	=	3	+	5	+	1
F6	7	+	3	+	5	+	1	=	4	+	6	+	1
F7	4	+	1	=	5	+	1	=	3	+	6	+	1
F8	7	+	2	+	5	+	3	+	4	+	6	+	1
F9	6	+	1	=	4	+	1	=	3	+	5	+	1
F10	7	+	5	+	4	+	1	=	3	+	6	+	1
F11	6	+	1	=	3	+	1	=	4	+	5	+	1
F12	6	+	7	+	4	+	1	=	3	+	5	+	1
Ave rank	6.33		3.08		4.25		1.42		4.08		5.08		1
Final rank	**7**		**3**		**5**		**2**		**4**		**6**		**1**
W^+/W^-		**78/0**		**36/0**		**78/0**		**6/0**		**78/0**		**78/0**	
+/−/=		12/0/0		8/0/4		12/0/0		3/0/9		12/0/0		12/0/0	

W^+ and W^- denote the rank sum of corresponding symbols; +/−/= denote the total rank number obtained by reference to the HS-CS algorithm; Ave rank and Final rank represent the average ranking and final ranking of each algorithm under the current dimension, respectively

Major results (Cont'd)

The IRIS rule extraction results by the HS-CS-BPNN framework

Table 4 Classification rules as Iris-setosa

No.	IF
1	SL is NOT MED [0.72], SW is LGR [0.44], PL is SM [21.81], and PW is SM [11.49]
2	SL is NOT MED [0.72], SW is LGR [0.44], PL is SM [21.81], and PW is NOT MED [20.79]
3	SL is NOT MED [0.72], SW is LGR [0.44], PL is SM [21.81], and PW is NOT LGR [1.4]
4	SL is NOT MED [0.72], SW is LGR [0.44], PL is NOT MED [1.25], and PW is SM [11.49]
5	SL is NOT MED [0.72], SW is LGR [0.44], PL is NOT MED [1.25], and PW is NOT MED [20.79]
6	SL is NOT MED [0.72], SW is LGR [0.44], PL is NOT MED [1.25], and PW is NOT LGR [1.4]
7	SL is NOT MED [0.72], SW is LGR [0.44], PL is NOT LGR [13.36], and PW is SM [11.49]
8	SL is NOT MED [0.72], SW is LGR [0.44], PL is NOT LGR [13.36], and PW is NOT MED [20.79]
9	SL is NOT MED [0.72], SW is LGR [0.44], PL is NOT LGR [13.36], and PW is NOT LGR [1.4]
10	SL is NOT LGR [10.92], SW is LGR [0.44], PL is SM [21.81], and PW is SM [11.49]
11	SL is NOT LGR [10.92], SW is LGR [0.44], PL is SM [21.81], and PW is NOT MED [20.79]
12	SL is NOT LGR [10.92], SW is LGR [0.44], PL is SM [21.81], and PW is NOT LGR [1.4]
13	SL is NOT LGR [10.92], SW is LGR [0.44], PL is NOT MED [1.25], and PW is SM [11.49]
14	SL is NOT LGR [10.92], SW is LGR [0.44], PL is NOT MED [1.25], and PW is NOT MED [20.79]
15	SL is NOT LGR [10.92], SW is LGR [0.44], PL is NOT MED [1.25], and PW is NOT LGR [1.4]
16	SL is NOT LGR [10.92], SW is LGR [0.44], PL is NOT LGR [13.36], and PW is SM [11.49]
17	SL is NOT LGR [10.92], SW is LGR [0.44], PL is NOT LGR [13.36], and PW is NOT MED [20.79]
18	SL is NOT LGR [10.92], SW is LGR [0.44], PL is NOT LGR [13.36], and PW is NOT LGR [1.4]

SL: sepal length; SW: sepal width; PL: petal length; PW: petal width; LGR: large; MED: medium; SM: small

Major results (Cont'd)

The classification accuracy of IRIS

Table 5 Comparison of IRIS classification accuracy results

Method	Accuracy (%)	
	Training set	Test set
BPNN trained by HS-CS	96.45 (95.54)	97.37 (97.37)
BPNN trained by AGOHS	96.32 (66.07)	97.36 (68.42)
BPNN trained by HS	96.39 (65.72)	97.36 (68.42)

Values in brackets indicate the corresponding accuracies of the obtained weighted fuzzy production rules

Major results (Cont'd)

Convergence results of IRIS by HS algorithms

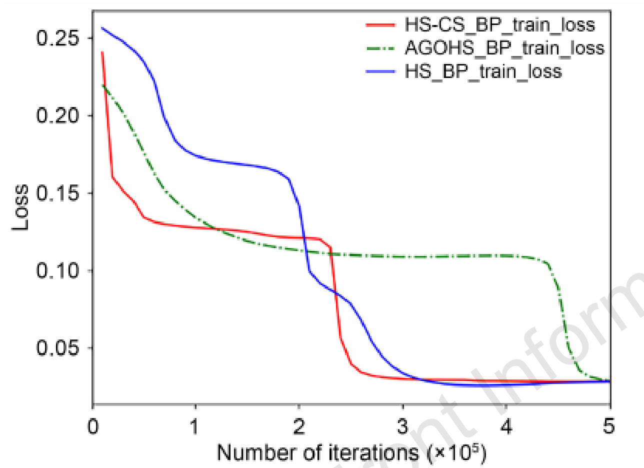

Fig. 3 Convergence curve of the loss function of BPNN with different HS algorithms (BPNN: back propagation neural network; HS: harmony search)

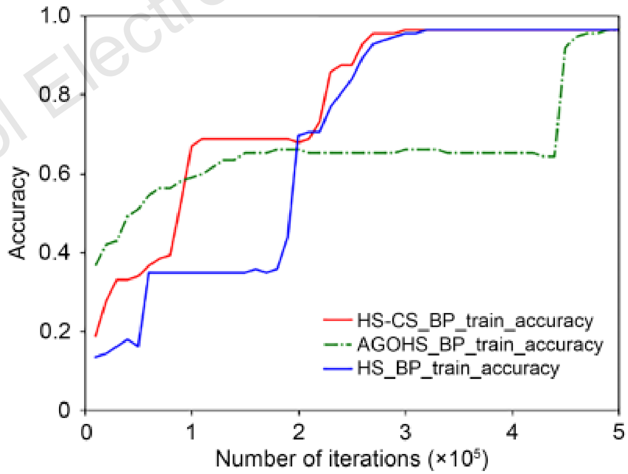

Fig. 4 Convergence precision curve of the loss function of BPNN with different HS algorithms (BPNN: back propagation neural network; HS: harmony search)

Conclusions

1. The existence of the limit of the HS-CS algorithm is proved by differential equations, and the global optimal algorithm is verified by random functional analysis and random search theory.

2. The results show that the HS-CS algorithm can still maintain the speed and precision of rapid convergence in the process of solving the optimization of high-dimensional functions and has strong robustness.

3. The HS-CS algorithm can enhance the learning and generalization abilities of BPNN effectively.