Daquan LI, Weigang SUN, Hongxiang HU, 2023. Impact of distance between two hubs on the network coherence of tree networks. *Frontiers of Information Technology & Electronic Engineering*, 24(9):1349-1356.
https://doi.org/10.1631/FITEE.2200400

Impact of distance between two hubs on the network coherence of tree networks

Key words: Consensus; Coherence; Distance; Average path length

Corresponding author: Weigang SUN
E-mail: wgsun@hdu.edu.cn
ORCID: https://orcid.org/0000-0001-8699-5392

Motivation

- Propose a method of obtaining exact solutions of network coherence quantified by the Laplacian spectrum for a family of trees with two hubs.
- Study the impact of distance and difference in the degrees between two hubs on network coherence.
- Reveal the relationship between network coherence and average path length.

Main idea

- We choose a family of tree networks with two hubs controlled by two parameters.
- We obtain exact expressions of network coherence regarding the distance to investigate the role of this distance.
- We apply the distance to bridge network coherence and average path length and reveal their relationship.

Method

- ➢We use the tree's regular structure and matrix theory to obtain the analytic expressions of network coherence.
- ➢We apply theoretical analysis and Matlab simulation to discuss the influence of distance between two hubs and network parameters on coherence.
- ➢We numerically calculate network coherence and average path length to find the relationship between them.

Method

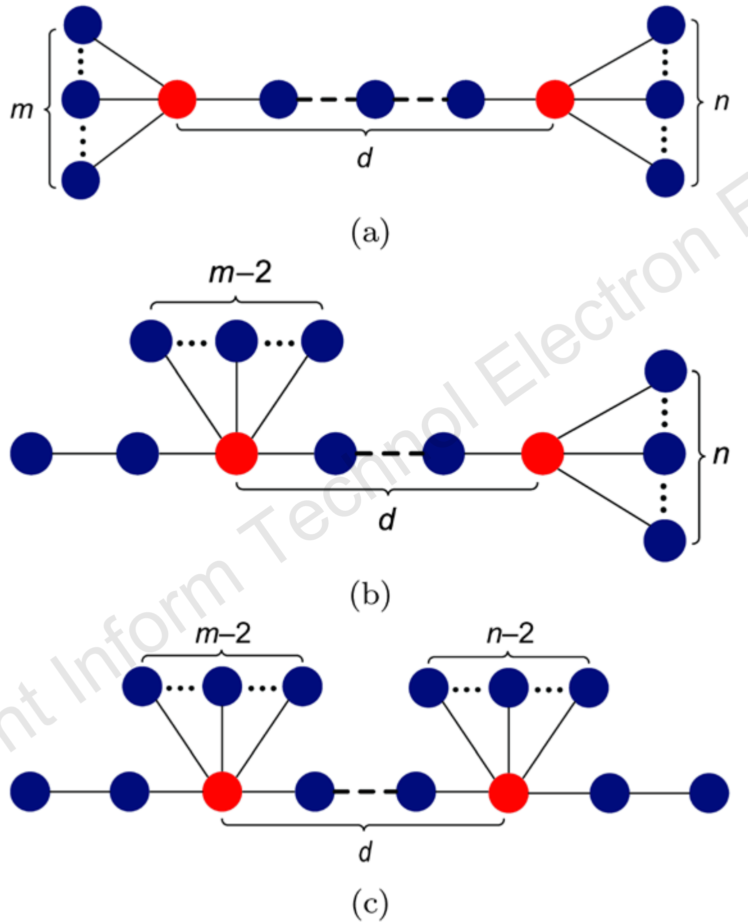

Fig. 1 A family of tree networks with the same distance d between two hubs: (a) T_1; (b) T_2; (c) T_3. The parameters m and n control the tree's asymmetry, and the network size is $N = m + n + d + 1$

Major results

Exact expression of network coherence regarding distance *d*:

$$
\begin{aligned}
H_1 &= \frac{1}{2N}\sum_{i=2}^{N}\frac{1}{\lambda_i} = -\frac{1}{2N}\frac{\overline{P}_1(1)}{\overline{P}_1(0)} \\
&= \frac{\phi_1 - d(m-\frac{N-d-1}{2})^2}{2N^2},
\end{aligned}
$$

where

$$
\begin{aligned}
\phi_1 &= \frac{1}{6}(d-2)(d-1)d + \frac{1}{2}(d-1)d(N-d+1) \\
&\quad +\frac{1}{4}d(N-d-1)^2 \\
&\quad +(N-d)(N+d-1)-(N-1).
\end{aligned}
$$

Major results

1. The network coherence is better with a smaller distance.

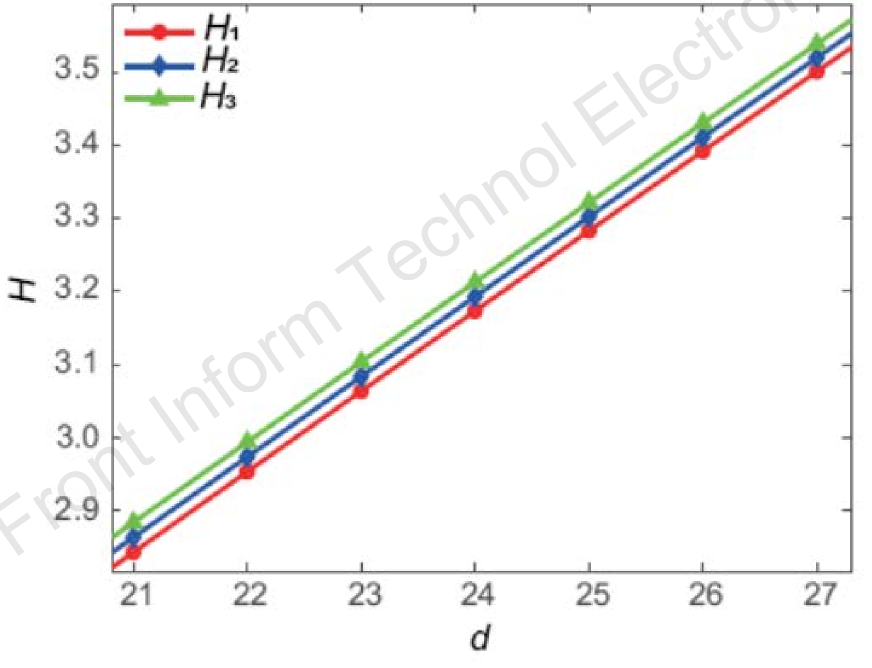

Fig. 2 Distributions of coherences H_1, H_2, and H_3 against distance d with $m = 30$ and $n = 20$

Major results

2. The larger the difference between the degrees of the two hubs, the higher the coherence.

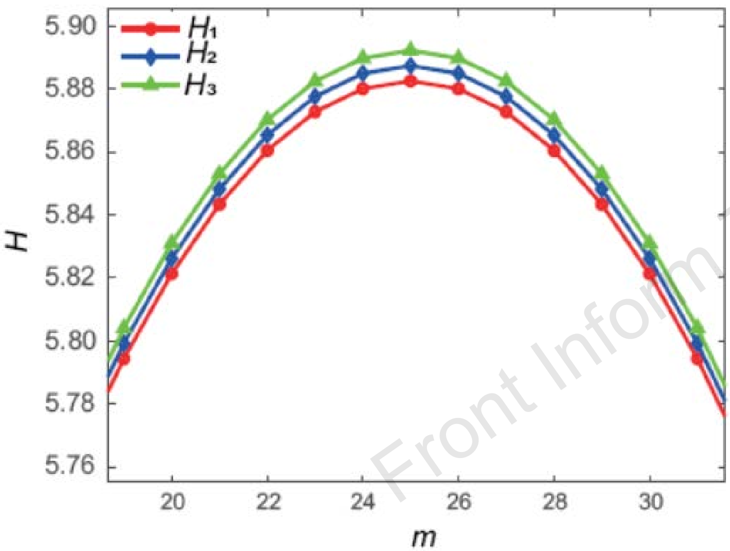

Fig. 3 Distributions of coherences H_1, H_2, and H_3 against parameter m with $N = 100$, $d = 49$, and $n = 50 - m$

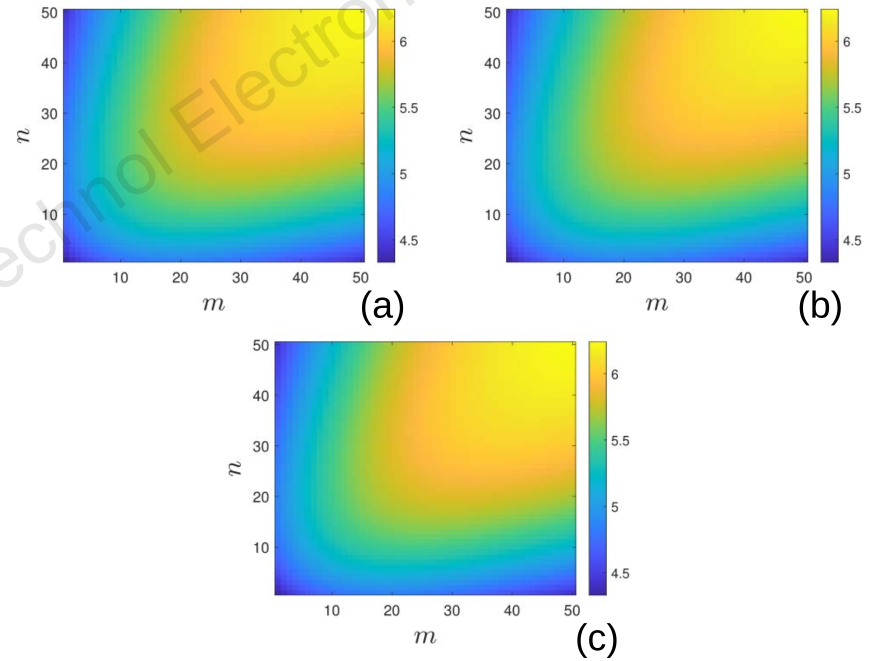

Fig. 4 Phase diagrams for coherences H_1 (a), H_2 (b), and H_3 (c) in a two-dimensional space (m, n) with $d = 49$

Major results

3. Network coherence and average path length behave linearly. When the average path length becomes smaller, the tree displays better coherence.

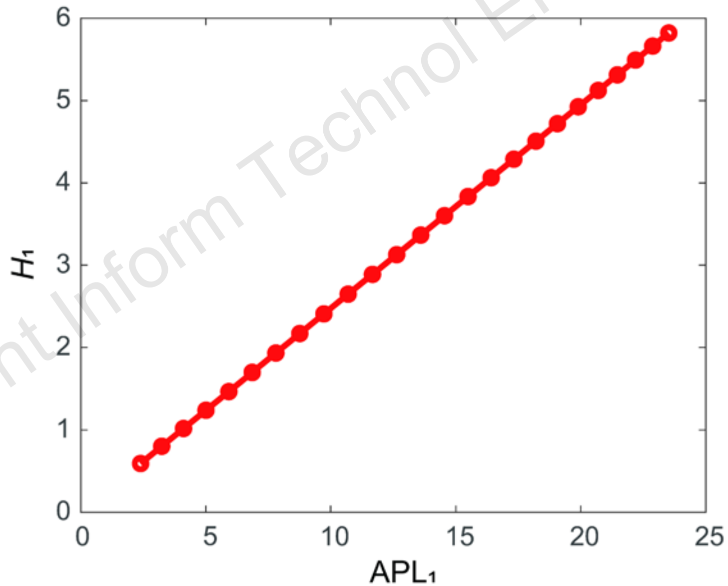

Fig. 5 Relationship between coherence H_1 and average path length APL_1 with $N = 100, m = 30$, and $n = 69 - d$

Conclusions

- The analytical solutions of network coherence are obtained based on the tree's structure.
- A smaller distance between two hubs and a large difference between the degrees will result in higher coherence.
- The tree displays better coherence when the average path length is smaller.

Weigang SUN is a full professor with the School of Sciences, Hangzhou Dianzi University, Hangzhou, China. He received his PhD degree in computational mathematics from Shanghai University in 2008. From 2009 to 2011, he was a postdoctoral fellow with the Institute of Cognitive Neurodynamics, East China University of Science and Technology. In 2011 and 2012, He was a senior research assistant with the Center of Chaos and Complex Networks, City University of Hong Kong, China. He visited University of Texas at Arlington as a visiting scholar during 2019-2020 and Arizona State University during 2013-2014. He is an author or coauthor of more than 60 refereed international journal papers, and a reviewer of many journals. His research interests include complex networks, network neuroscience, and system biology.

Hongxiang HU received his PhD degree in control theory and control engineering from Zhejiang University of Technology, Hangzhou, China, in 2013. Since September 2015, he has been a postdoctoral fellow with the School of Mathematics, Southeast University, Nanjing, China. He was a research assistant with the Department of Electronic Engineering, City University of Hong Kong, Hong Kong, China, in 2017, and a visiting scholar with the School of Electrical and Electronics Engineering, Nanyang Technological University, Singapore, in 2019. He is currently a professor with the Department of Mathematics, School of Sciences, Hangzhou Dianzi University, China. His current research interests include multiagent systems, collective dynamics, complex networks, hybrid and switched systems, and cyber-physical systems.