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Event-triggered distributed optimization for model-free multi-agent systems

Key words: Distributed optimization; Multi-agent systems; Model-free adaptive control; Event-triggered mechanism

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Motivation

- Distributed optimization problems (DOPs) for multi-agent systems (MASs) aim to achieve a globally optimal solution by exchanging information among agents. They are of great importance in various fields.
- Distributed optimization problems rely on dynamic models of agents. Due to the increasing complexity of engineering systems, it is often difficult or even impossible to build accurate dynamic models of each agent.
- In numerous practical applications, especially for the digital-communication-based MASs, it is often the case that the communication resource is limited, which means that frequent communication is unrealistic.

Method

Dynamic linearization model

Lemma 1 (Hou and Jin, 2011a) Given that MAS (1) satisfies Assumptions 1 and 2, if $\|u_i(h)\| \neq 0$ holds, then system (1) is equivalently converted into the dynamic linearization model as follows:

$$\Delta y_i(h+1) = \mho_i(h)\Delta u_i(h), \; i = 1, 2, \ldots, N, \quad (2)$$

where

$$\mho_i(h) \triangleq \begin{bmatrix} \varpi_{i,11}(h) & \varpi_{i,12}(h) & \cdots & \varpi_{i,1n}(h) \\ \varpi_{i,21}(h) & \varpi_{i,22}(h) & \cdots & \varpi_{i,2n}(h) \\ \vdots & \vdots & & \vdots \\ \varpi_{i,n1}(h) & \varpi_{i,n2}(h) & \cdots & \varpi_{i,nn}(h) \end{bmatrix}$$

is called the pseudo-partial-derivative (PPD) matrix and satisfies $\|\mho_i(h)\| \leq m$.

ET-MFADO algorithm

Case1: $h = h_s^i$

$$\hat{\mho}_i(h) = \begin{cases} [\hat{\varpi}_{i,jl}]_{n\times n}, & \text{if } |\hat{\varpi}_{i,jl}(h_s^i)| < m_2, \; j = l \\ & \text{or } |\hat{\varpi}_{i,jl}(h_s^i)| > am_2, j = l \\ & \text{or } |\hat{\varpi}_{i,jl}(h_s^i)| > m_1, \; j \neq l \\ & \text{or } \operatorname{sign}(\hat{\varpi}_{i,jl}(h_s^i)) \neq \operatorname{sign}(\hat{\varpi}_{i,jl}), \\ \hat{\mho}_i(h_s^i - 1) & \\ \quad + \left(\Delta y_i(h_s^i) - \hat{\mho}_i(h_s^i - 1)\Delta u_i(h_s^i - 1)\right) & \\ \quad \times \dfrac{\gamma \Delta u_i^T(h_s^i - 1)}{\nu + \|\Delta u_i(h_s^i - 1)\|^2}, & \text{otherwise} \end{cases}$$

$$u_i(h) = u_i(h_s^i - 1) + \beta(h) P_i(h) \xi_i(h_s^i) - \tilde{d}_i(h-1)$$

Case2: $h \in (h_s^i, h_{s+1}^i)$

$$\hat{\mho}_i(h) = \hat{\mho}_i(h_s^i)$$

$$u_i(h) = u_i(h-1) + \beta(h) P_i(h) \xi_i(h_s^i) - \tilde{d}_i(h-1)$$

Method

Consensus analysis

Theorem 1 Given Assumptions 1 and 2, let scalars $\nu, \mu > 0$, $\tau, \lambda, \gamma \in (0,\ 1)$, and matrices $\bar{A}$, $\bar{M}$, and $\bar{Q}$ be known. MAS (1) is ultimately consensusable under the proposed algorithm in Eqs. (16)–(18) if there exist positive scalars ε_1–ε_3 and ℓ_1–ℓ_6 satisfying

$$\Pi_1 \triangleq \begin{bmatrix} \Pi_{11} & * & * & * \\ 0 & \breve{\ell}_2\bar{Q}-\varepsilon_1 I & * & * \\ 0 & 0 & (\breve{\ell}_3-\varepsilon_2)I & * \\ 0 & 0 & 0 & \breve{\ell}_4\bar{A}-\varepsilon_3 I \end{bmatrix} < 0,$$

where $\Pi_{11} \triangleq \bar{\ell}_1\bar{M} - (1-\tau)I$, $\bar{\ell}_1 \triangleq 1 + \ell_1 + \ell_2 + \ell_3$, $\breve{\ell}_2 \triangleq 1 + \ell_1^{-1} + \ell_4 + \ell_6$, $\breve{\ell}_3 \triangleq 1 + \ell_2^{-1} + \ell_4^{-1} + \ell_5$, and $\breve{\ell}_4 \triangleq 1 + \ell_3^{-1} + \ell_5^{-1} + \ell_6^{-1}$.

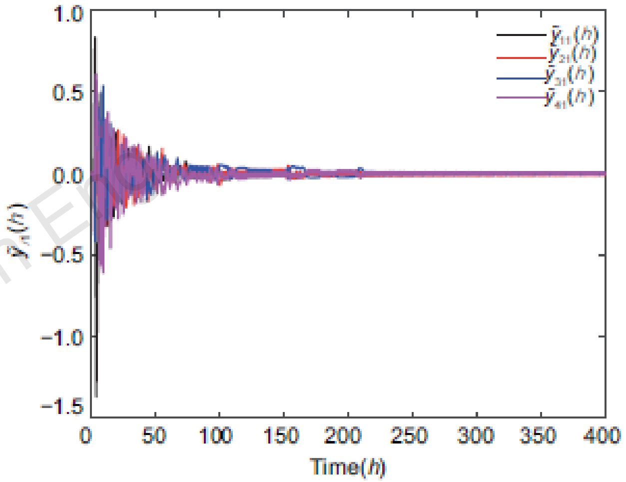

Fig. 2 The first component of the average consensus error $\tilde{y}_{i1}(h)$

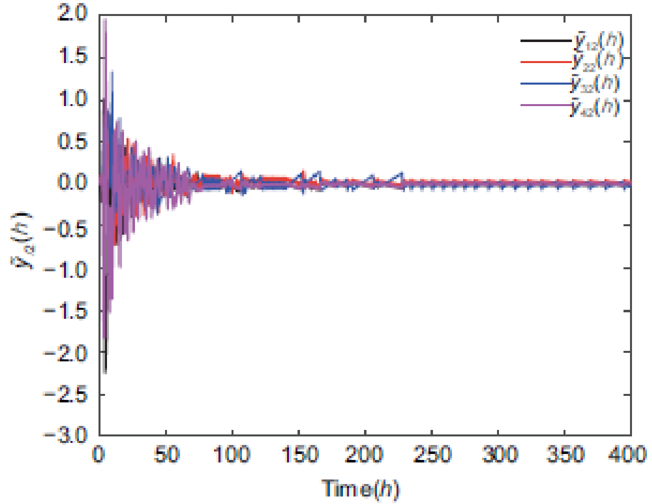

Fig. 3 The second component of the average consensus error $\tilde{y}_{i2}(h)$

Method

Optimality analysis

Theorem 2 Given Assumptions 3 and 4, let scalars $\nu, \mu > 0$ and $0 < \lambda, \gamma < 1$ be known. The output trajectories $y_i(h)$ of MAS (1) converge to the optimal solution y^* under the proposed algorithm in Eqs. (16)–(18) if there exist positive scalars ω_1–ω_3 and ℓ_7–ℓ_{12} satisfying

$$\Pi_2 \triangleq \begin{bmatrix} \breve{\ell}_5 \bar{M} - I & * & * & * \\ 0 & \breve{\ell}_6 \bar{Q} - \omega_1 I & * & * \\ 0 & 0 & \breve{\ell}_7 \bar{A} - \omega_2 I & * \\ 0 & 0 & 0 & \breve{\ell}_8 \bar{A} - \omega_3 I \end{bmatrix} < 0,$$

where $\breve{\ell}_5 \triangleq 1 + \ell_7 + \ell_8 + \ell_9$, $\breve{\ell}_6 \triangleq 1 + \ell_7^{-1} + \ell_{10} + \ell_{12}$, $\breve{\ell}_7 \triangleq 1 + \ell_8^{-1} + \ell_{10}^{-1} + \ell_{11}$, and $\breve{\ell}_8 \triangleq 1 + \ell_9^{-1} + \ell_{11}^{-1} + \ell_{12}^{-1}$. Moreover, it holds that $\lim_{h\to\infty} \bar{y}(h) = y^*$.

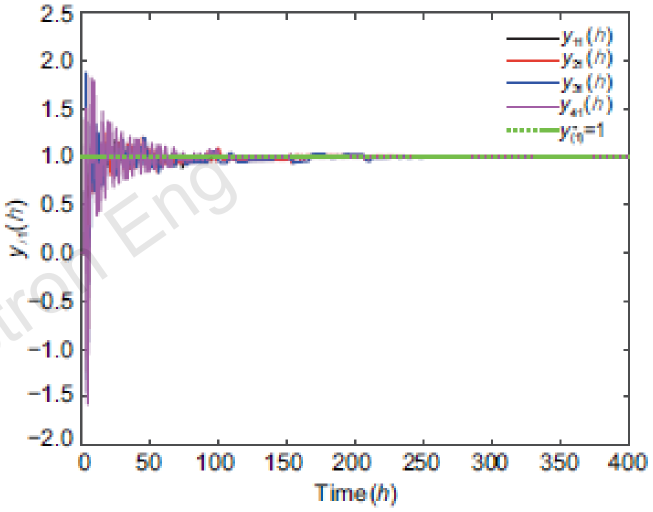

Fig. 4 The output trajectories of $y_{i1}(h)$

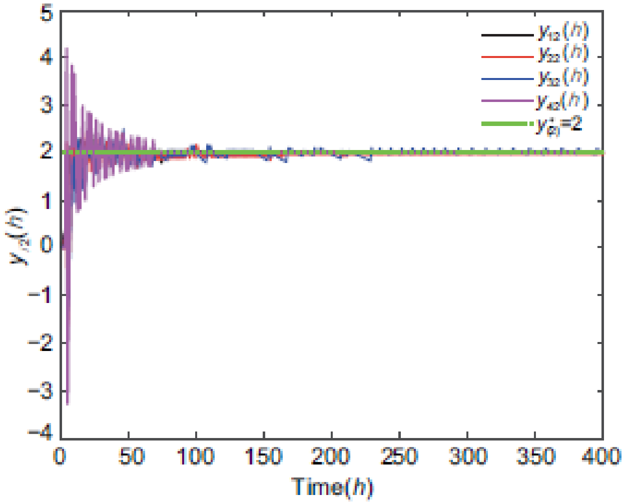

Fig. 5 The output trajectories of $y_{i2}(h)$

Method

An event-triggered machine is introduced to reduce the network communication burden and improve system performance.

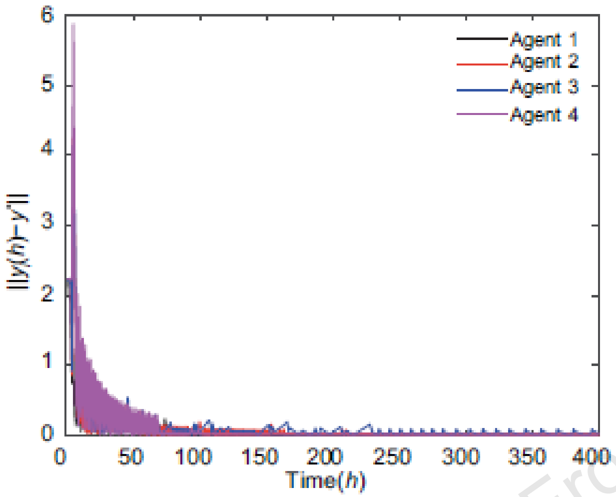

Fig. 6 The evolution of the output error $\|y_i(h) - y^*\|$

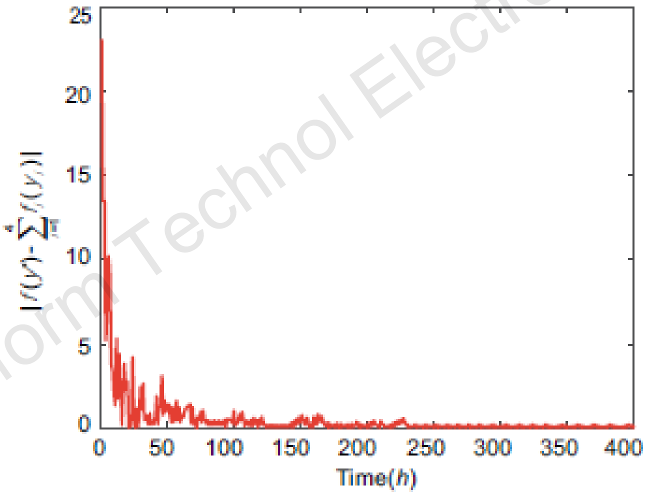

Fig. 7 The error of the optimal value $|f(y^*) - \sum_{i=1}^{4} f_i(y_i)|$

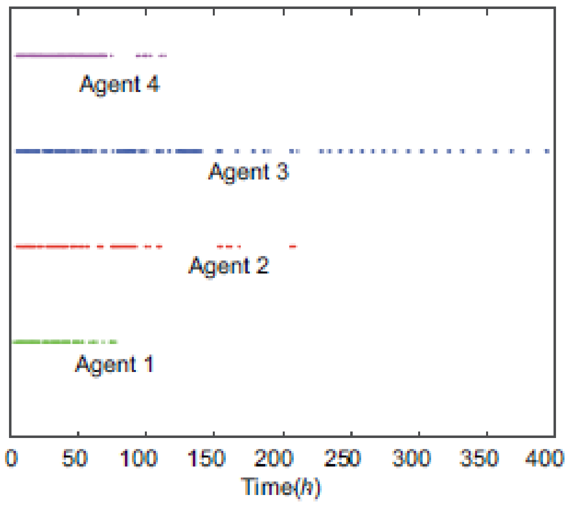

Fig. 8 Trigger time of each agent

Conclusions

- We have proposed a novel ET-MFADO algorithm to handle the distributed optimization problem for unknown nonlinear MASs.
- We have equivalently converted the original unknown nonlinear system into a dynamic linearization model.
- We have presented an event-triggered mechanism to reduce the network communication burden and improve system performance.

Shanshan ZHENG received her BS degree in statistics from Yanshan University, Hebei, China, in 2019. She is currently pursuing her MS degree in operation research and cybernetics with the University of Shanghai for Science and Technology, Shanghai, China. Her research interests include distributed optimization and consensus problems in multi-agent systems.

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