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Semi-tensor product approach to controllability, reachability, and stabilizability of extended finite state machines

Key words: Semi-tensor product (STP); Matrix approach; Algebraic method; Finite-valued systems

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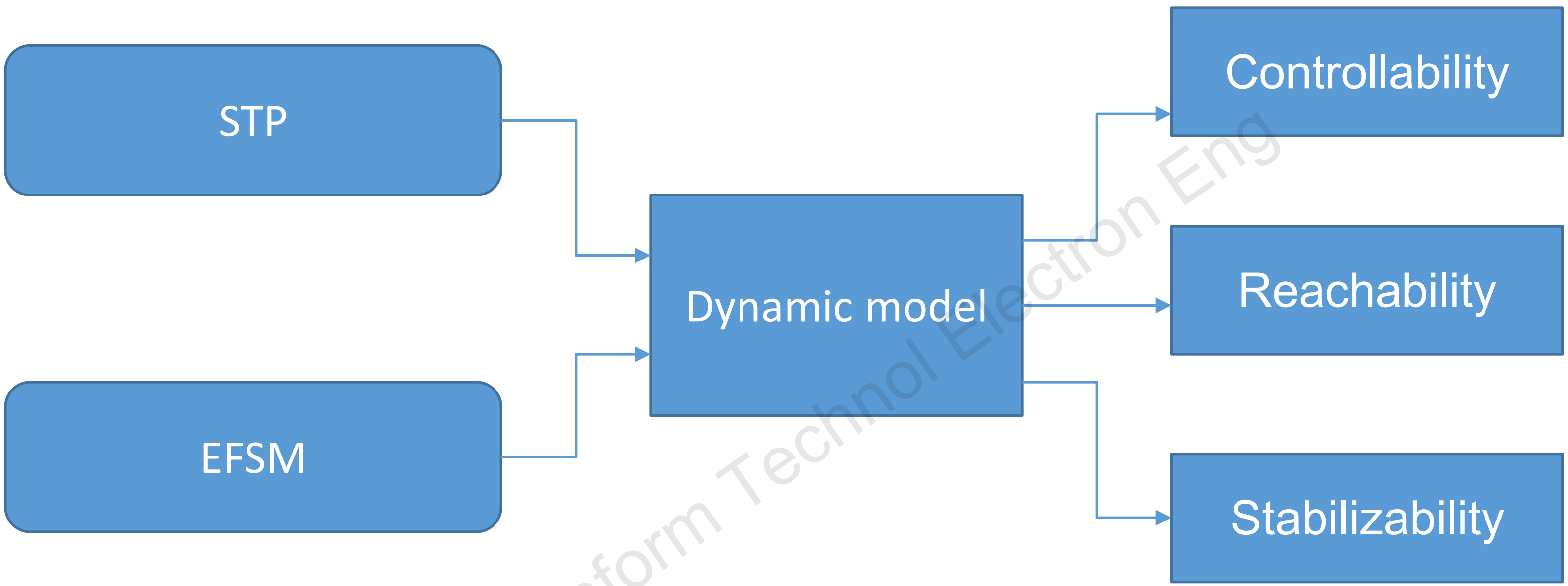
Motivation

- Controllability, reachability, and stabilizability are essential characteristics in finite state machines (FSMs) that enable optimal control strategies and ensure system stability.
- Most existing studies on FSMs focus on these properties using traditional formal language methods, which are effective for conventional FSMs but not suitable for extended finite state machines (EFSMs).
- To address this limitation, this study introduces the semi-tensor product (STP) as a novel mathematical tool for analyzing the controllability, reachability, and stabilizability of EFSMs using an algebraic approach.

Main idea

- This work constructs a bilinear dynamic system model of the EFSM, allowing analysis similar to that of control theory systems.
- Using STP, the study generalizes matrix multiplication for matrices of any dimension, facilitating efficient handling of EFSMs' multi-dimensional data and enabling an algebraic framework for controllability, reachability, and stabilizability.
- An algorithm based on STP is designed to determine these properties for the EFSM model, avoiding complex symbolic operations and simplifying the analysis through straightforward algebraic calculations.

Framework



The framework illustrates an approach for analyzing the controllability, reachability, and stabilizability of dynamic systems by combining the STP method with EFSMs. The STP method provides algebraic tools to support EFSM analysis, enabling the construction of a bilinear dynamic system model. Through this model, it is possible to systematically study the properties of controllability, reachability, and stabilizability, thereby establishing a mathematical foundation for examining EFSMs within the framework of control theory.

Method

Algorithm 1 Finding controllable, returnable, and stabilizable states

Input: STSM $\boldsymbol{F}$ of EFSM A

Output: The controllable state set S_{C}, the one-step returnable state set S_{O}, and the stabilizable state set S

1. Let i=1. For l_i, if there exists a finite input sequence $e_j \sim \delta_m^j \in \Sigma$ such that $\boldsymbol{F} \ltimes \delta_m^j \ltimes \delta_n^j \ltimes \delta_2^1 = \delta_n^i$, then l_i is controllable; otherwise l_i is not controllable.
2. Let $i=i+1$, and repeat step 1 until $i=n$. Check whether l_i is controllable. If so, put it into the controllable state set S_{C}.
3. Let $i=1$, and consider l_i. If there exists a set of states such that $\boldsymbol{F} \ltimes \delta_m^j \ltimes \delta_n^i \ltimes \delta_2^1 = \delta_n^{i+1}$, $\boldsymbol{F} \ltimes \delta_m^j \ltimes \delta_n^{i+1} \ltimes \delta_2^1 = \delta_n^{i+2}$, $\cdots$, $\boldsymbol{F} \ltimes \delta_m^j \ltimes \delta_n^{i+t} \ltimes \delta_2^1 = \delta_n^i$, then these states are mutually reachable.
4. Let $i=i+1$, and repeat step 3 until $i=n$. If all $l_\alpha \sim \delta_n^\alpha$ $(1 \le \alpha \le n)$ in L_1 are mutually reachable, then put them into the one-step returnable state set S_{O}.
5. The stabilizable state set of EFSM is $S = S_{\mathrm{C}} \cap S_{\mathrm{O}}$.

To determine the properties of the EFSM, we design Algorithm 1. This algorithm takes the state transition structure matrix (STSM) of the EFSM as input and sequentially evaluates each state to determine its controllability, one-step returnability, and stabilizability. First, the algorithm examines the controllability of each state by testing a series of input sequences, adding those that meet the criteria to the controllable state set. Next, it establishes the one-step returnable state set by verifying mutual reachability among states. Finally, the stabilizable state set of the EFSM is defined as the intersection of the controllable state set and the one-step returnable state set, thereby providing a comprehensive assessment of the system properties of the EFSM.

Results

Step 1: based on Theorem 3, we can determine whether the state of the EFSM A_3 is controllable: $\boldsymbol{F} \ltimes \boldsymbol{\delta}_2^1 \ltimes \boldsymbol{\delta}_6^1 \ltimes \boldsymbol{\delta}_2^1 = \boldsymbol{\delta}_6^2$, $\boldsymbol{F} \ltimes \boldsymbol{\delta}_2^2 \ltimes \boldsymbol{\delta}_6^1 \ltimes \boldsymbol{\delta}_2^1 = \boldsymbol{\delta}_6^4$, $\boldsymbol{F} \ltimes \boldsymbol{\delta}_2^2 \ltimes \boldsymbol{\delta}_6^2 \ltimes \boldsymbol{\delta}_2^1 = \boldsymbol{\delta}_6^3$, $\boldsymbol{F} \ltimes \boldsymbol{\delta}_2^1 \ltimes \boldsymbol{\delta}_6^2 \ltimes \boldsymbol{\delta}_2^1 = \boldsymbol{\delta}_6^5$, $\boldsymbol{F} \ltimes \boldsymbol{\delta}_2^1 \ltimes \boldsymbol{\delta}_6^3 \ltimes \boldsymbol{\delta}_2^1 = \boldsymbol{\delta}_6^1$, $\boldsymbol{F} \ltimes \boldsymbol{\delta}_2^2 \ltimes \boldsymbol{\delta}_6^4 \ltimes \boldsymbol{\delta}_2^1 = \boldsymbol{\delta}_6^5$, $\boldsymbol{F} \ltimes \boldsymbol{\delta}_2^1 \ltimes \boldsymbol{\delta}_6^4 \ltimes \boldsymbol{\delta}_2^1 = \boldsymbol{\delta}_6^6$, $\boldsymbol{F} \ltimes \boldsymbol{\delta}_2^2 \ltimes \boldsymbol{\delta}_6^5 \ltimes \boldsymbol{\delta}_2^1 = \boldsymbol{\delta}_6^3$, $\boldsymbol{F} \ltimes \boldsymbol{\delta}_2^1 \ltimes \boldsymbol{\delta}_6^6 \ltimes \boldsymbol{\delta}_2^1 = \boldsymbol{\delta}_6^5$.

Step 2: based on Theorem 4, combined with step 1, we know that nonempty sets of states $S_{C1}=L_1$ and $S_{C2}=L_2$ are controllable.

Step 3: based on Corollary 1, we can determine whether the state of the EFSM A_3 is reachable: $\boldsymbol{F} \ltimes \boldsymbol{\delta}_2^1 \ltimes \boldsymbol{\delta}_6^1 \ltimes \boldsymbol{\delta}_2^1 = \boldsymbol{\delta}_6^2$, $\boldsymbol{F} \ltimes \boldsymbol{\delta}_2^2 \ltimes \boldsymbol{\delta}_6^1 \ltimes \boldsymbol{\delta}_2^1 = \boldsymbol{\delta}_6^4$, $\boldsymbol{F} \ltimes \boldsymbol{\delta}_2^2 \ltimes \boldsymbol{\delta}_6^2 \ltimes \boldsymbol{\delta}_2^1 = \boldsymbol{\delta}_6^3$, $\boldsymbol{F} \ltimes \boldsymbol{\delta}_2^1 \ltimes \boldsymbol{\delta}_6^2 \ltimes \boldsymbol{\delta}_2^1 = \boldsymbol{\delta}_6^5$, $\boldsymbol{F} \ltimes \boldsymbol{\delta}_2^1 \ltimes \boldsymbol{\delta}_6^3 \ltimes \boldsymbol{\delta}_2^1 = \boldsymbol{\delta}_6^1$, $\boldsymbol{F} \ltimes \boldsymbol{\delta}_2^2 \ltimes \boldsymbol{\delta}_6^4 \ltimes \boldsymbol{\delta}_2^1 = \boldsymbol{\delta}_6^5$, $\boldsymbol{F} \ltimes \boldsymbol{\delta}_2^1 \ltimes \boldsymbol{\delta}_6^4 \ltimes \boldsymbol{\delta}_2^1 = \boldsymbol{\delta}_6^6$, $\boldsymbol{F} \ltimes \boldsymbol{\delta}_2^2 \ltimes \boldsymbol{\delta}_6^5 \ltimes \boldsymbol{\delta}_2^1 = \boldsymbol{\delta}_6^3$, $\boldsymbol{F} \ltimes \boldsymbol{\delta}_2^1 \ltimes \boldsymbol{\delta}_6^6 \ltimes \boldsymbol{\delta}_2^1 = \boldsymbol{\delta}_6^5$.

Step 4: in an EFSM A_3, only $\boldsymbol{\delta}_6^1 \to \boldsymbol{\delta}_6^2 \to \boldsymbol{\delta}_6^3 \to \boldsymbol{\delta}_6^1$ ($\boldsymbol{F} \ltimes \boldsymbol{\delta}_2^1 \ltimes \boldsymbol{\delta}_6^1 \ltimes \boldsymbol{\delta}_2^1 = \boldsymbol{\delta}_6^2$, $\boldsymbol{F} \ltimes \boldsymbol{\delta}_2^2 \ltimes \boldsymbol{\delta}_6^2 \ltimes \boldsymbol{\delta}_2^1 = \boldsymbol{\delta}_6^3$, $\boldsymbol{F} \ltimes \boldsymbol{\delta}_2^1 \ltimes \boldsymbol{\delta}_6^3 \ltimes \boldsymbol{\delta}_2^1 = \boldsymbol{\delta}_6^1$), $\boldsymbol{\delta}_6^2 \to \boldsymbol{\delta}_6^3 \to \boldsymbol{\delta}_6^1 \to \boldsymbol{\delta}_6^2$ ($\boldsymbol{F} \ltimes \boldsymbol{\delta}_2^2 \ltimes \boldsymbol{\delta}_6^2 \ltimes \boldsymbol{\delta}_2^1 = \boldsymbol{\delta}_6^3$, $\boldsymbol{F} \ltimes \boldsymbol{\delta}_2^1 \ltimes \boldsymbol{\delta}_6^3 \ltimes \boldsymbol{\delta}_2^1 = \boldsymbol{\delta}_6^1$, $\boldsymbol{F} \ltimes \boldsymbol{\delta}_2^1 \ltimes \boldsymbol{\delta}_6^1 \ltimes \boldsymbol{\delta}_2^1 = \boldsymbol{\delta}_6^2$), $\boldsymbol{\delta}_6^3 \to \boldsymbol{\delta}_6^1 \to \boldsymbol{\delta}_6^2 \to \boldsymbol{\delta}_6^3$ ($\boldsymbol{F} \ltimes \boldsymbol{\delta}_2^1 \ltimes \boldsymbol{\delta}_6^3 \ltimes \boldsymbol{\delta}_2^1 = \boldsymbol{\delta}_6^1$, $\boldsymbol{F} \ltimes \boldsymbol{\delta}_2^1 \ltimes \boldsymbol{\delta}_6^1 \ltimes \boldsymbol{\delta}_2^1 = \boldsymbol{\delta}_6^2$, $\boldsymbol{F} \ltimes \boldsymbol{\delta}_2^2 \ltimes \boldsymbol{\delta}_6^2 \ltimes \boldsymbol{\delta}_2^1 = \boldsymbol{\delta}_6^3$) in L_1 are one-step returnable. That is, $S_O=L_1$ is one-step returnable.

Step 5: in this case, we can find that the stabilizable state set of the EFSM A_3 is $S=S_C \cap S_O=L_1$.

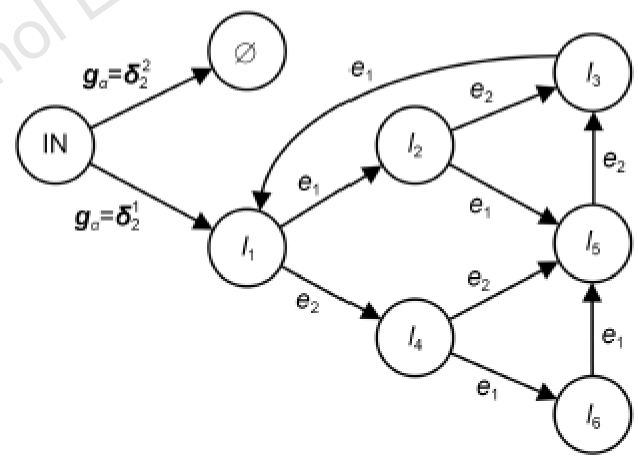


Fig. 3 State transition diagram of A_3

Conclusions

- An algebraic method was introduced to analyze the controllability, reachability, and stabilizability of EFSMs. A bilinear dynamic system model for EFSMs was constructed, providing a foundation for the analysis of these critical system properties.
- Key theorems were developed to establish clear criteria for assessing the controllability, reachability, and stabilizability of EFSMs, which are vital for the design and analysis of complex systems in control theory.
- An algorithm was also designed to determine the controllability and stabilizability of EFSMs, offering a practical tool for researchers and engineers in the field.

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