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Event-triggered finite-time guaranteed cost control of asynchronous switched systems under the round-robin protocol via an AED-ADT method

Key words: Switched systems; Event-triggered scheme; Round-robin protocol; Asynchronous switching; Admissible edge-dependent average dwell time (AED-ADT); Guaranteed cost control

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Motivation

- Event-triggered scheme (ETS) reduces unnecessary data transmission and controller updates by triggering control actions based on system state changes. Round-robin protocol (RRP) transmits data in a predetermined order. To address data redundancy and congestion in sensor nodes, it is necessary to study the integration of ETS and RRP.
- System mode may undergo asynchronous switching within event intervals, affecting stability and performance. Existing literature often assumes known maximum asynchronous switching delay, which can be conservative. Therefore, a more flexible approach is needed to handle such asynchronous switching.
- The admissible edge-dependent average dwell time (AED-ADT) method is an effective tool for designing switching signals to accommodate different needs of the system in terms of finite-time stability and finite-time instability.

Main idea

- A joint ETS and RRP design method is proposed, which ensures good data filtering while avoiding sensor node congestion.
- In switching signal design that takes into account the significance of finite-time control, the method with modal memory, namely AED-ADT, is applied. This method better reflects the switching characteristics of modal signals.
- By implementing slow AED-ADT switching and fast AED-ADT switching techniques, the challenges of finite-time instability in the controlled system during asynchronous switching intervals are addressed, thereby diminishing the impact of unstable conditions on the control system and ensuring finite-time stability.
- Based on RRP and ETS, sufficient criteria are obtained to ensure the finite-time boundedness of the switched system and the establishment of the guaranteed cost control index.

Framework

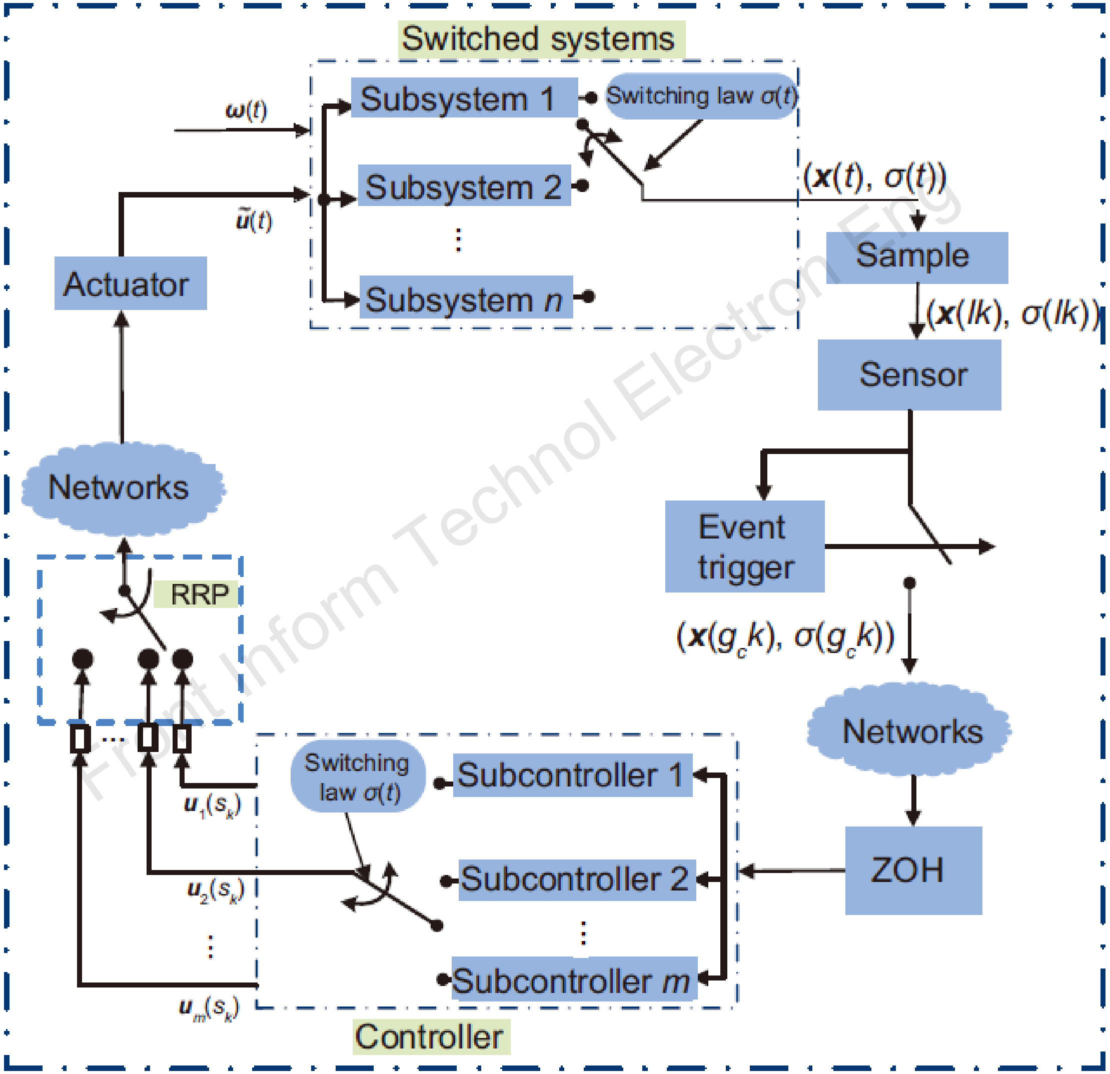
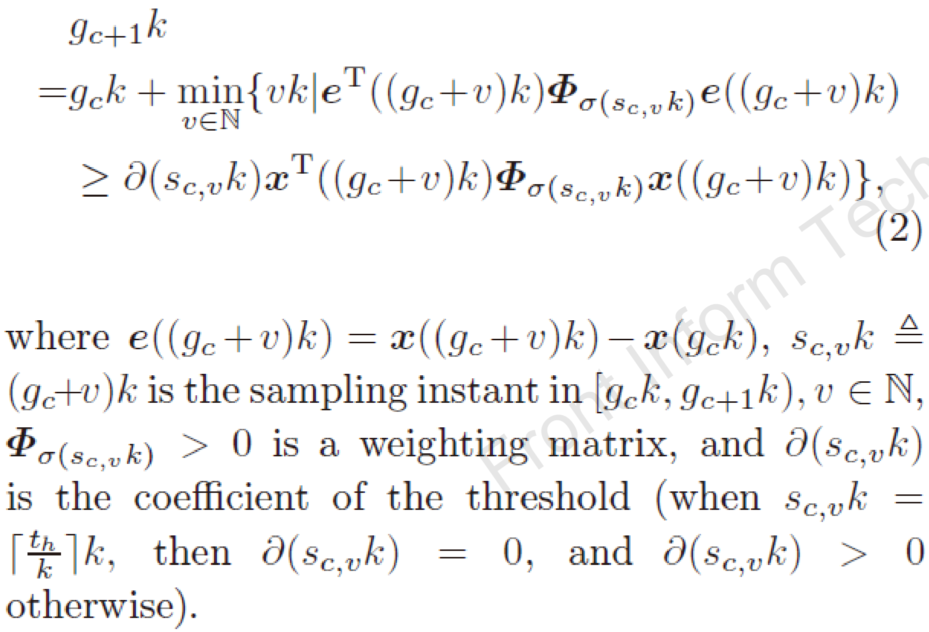


Fig. 1 Structure of a closed-loop switched system

Method

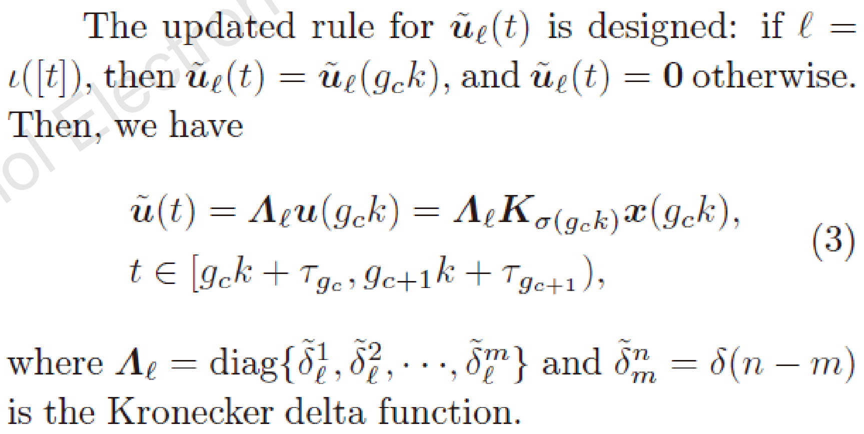
- To reduce the transmission rate of redundant information and avoid Zeno phenomenon, a hybrid ETS is designed, as follows:



$$\begin{aligned} &g_{c+1}k \\ =&g_ck+\min_{v\in\mathbb{N}}\{vk|\boldsymbol{e}^{\mathrm{T}}((g_c+v)k)\boldsymbol{\Phi}_{\sigma(s_{c,v}k)}\boldsymbol{e}((g_c+v)k) \\ &\geq \partial(s_{c,v}k)\boldsymbol{x}^{\mathrm{T}}((g_c+v)k)\boldsymbol{\Phi}_{\sigma(s_{c,v}k)}\boldsymbol{x}((g_c+v)k)\}, \end{aligned} \tag{2}$$

where $\boldsymbol{e}((g_c+v)k)=\boldsymbol{x}((g_c+v)k)-\boldsymbol{x}(g_ck)$, $s_{c,v}k \triangleq (g_c+v)k$ is the sampling instant in $[g_ck, g_{c+1}k)$, $v\in\mathbb{N}$, $\boldsymbol{\Phi}_{\sigma(s_{c,v}k)}>0$ is a weighting matrix, and $\partial(s_{c,v}k)$ is the coefficient of the threshold (when $s_{c,v}k=\lceil\frac{t_h}{k}\rceil k$, then $\partial(s_{c,v}k)=0$, and $\partial(s_{c,v}k)>0$ otherwise).

- To prevent data collisions, an RRP is designed to regulate sensor communications as follows:



The updated rule for $\tilde{\boldsymbol{u}}_\ell(t)$ is designed: if $\ell=\iota([t])$, then $\tilde{\boldsymbol{u}}_\ell(t)=\tilde{\boldsymbol{u}}_\ell(g_ck)$, and $\tilde{\boldsymbol{u}}_\ell(t)=\boldsymbol{0}$ otherwise. Then, we have

$$\begin{aligned} &\tilde{\boldsymbol{u}}(t)=\boldsymbol{\Lambda}_\ell\boldsymbol{u}(g_ck)=\boldsymbol{\Lambda}_\ell\boldsymbol{K}_{\sigma(g_ck)}\boldsymbol{x}(g_ck), \\ &t\in[g_ck+\tau_{g_c}, g_{c+1}k+\tau_{g_{c+1}}), \end{aligned} \tag{3}$$

where $\boldsymbol{\Lambda}_\ell=\mathrm{diag}\{\tilde{\delta}_\ell^1,\tilde{\delta}_\ell^2,\cdots,\tilde{\delta}_\ell^m\}$ and $\tilde{\delta}_m^n=\delta(n-m)$ is the Kronecker delta function.

Results

Theorem 2 Given positive scalars c_1, c_2 $(c_2 \geq c_1)$, $k, \varrho_0, \varepsilon_{\mathrm{m}}, \varepsilon_{\mathrm{M}}, d, T_{\mathrm{f}}$, and matrix $\boldsymbol{R} > 0$, for values $\alpha_i > 0$ with $0 < \upsilon_{i,j} < 1, (i,j) \in \boldsymbol{\Theta}_{\mathrm{s}}, \gamma_{i,j} > 0$ with $0 < \mu_{i,j} < 1, (i,j) \in \boldsymbol{\Theta}_{\mathrm{u}\downarrow}$, and $\gamma_{i,j} < 0$ with $\mu_{i,j} > 1, (i,j) \in \boldsymbol{\Theta}_{\mathrm{u}\uparrow}$, if there exist matrices $\boldsymbol{S}_i > 0, \check{\boldsymbol{\Phi}}_i > 0, \check{\boldsymbol{P}}_i > 0, \check{\boldsymbol{H}}_i^{\imath} > 0, \check{\boldsymbol{M}}_i^{\jmath} > 0, (i,j) \in \boldsymbol{\Theta}_{\mathrm{s}}$, $\check{\boldsymbol{\Phi}}_j > 0, \check{\boldsymbol{P}}_{i,j} > 0, \check{\boldsymbol{H}}_{i,j}^{\imath} > 0, \check{\boldsymbol{M}}_{i,j}^{\jmath} > 0, (i,j) \in \boldsymbol{\Theta}_{\mathrm{u}}, \jmath, \imath \in \{1,2\}$, and real matrices $\boldsymbol{Y}_i, \check{\boldsymbol{Z}}_i, \check{\boldsymbol{Z}}_{i,j}, \forall i,j \in \mathcal{W}$ $(i \neq j)$, such that inequality (18) and the following inequalities hold:

$$\begin{bmatrix} \check{\mho}_{11} & \check{\mho}_{12} \\ * & \check{\mho}_{22} \end{bmatrix} < 0,$$

$$\begin{bmatrix} \check{\Xi}_0 & \check{\Xi}_1 \\ * & \check{\Xi}_2 \end{bmatrix} < 0,$$

$$\begin{bmatrix} \check{\boldsymbol{M}}_i^2 & \check{\boldsymbol{Z}}_i \\ \check{\boldsymbol{Z}}_i^{\mathrm{T}} & \check{\boldsymbol{M}}_i^2 \end{bmatrix} > 0,$$

$$\begin{bmatrix} \check{\boldsymbol{M}}_{i,j}^2 & \check{\boldsymbol{Z}}_{ij} \\ \check{\boldsymbol{Z}}_{ij}^{\mathrm{T}} & \check{\boldsymbol{M}}_{ij}^2 \end{bmatrix} > 0,$$

$$\check{\boldsymbol{P}}_i \leq \upsilon_{i,j} \check{\boldsymbol{P}}_{i,j}, \check{\boldsymbol{H}}_i^{\imath} \leq \upsilon_{i,j} \check{\boldsymbol{H}}_{i,j}^{\imath}, \check{\boldsymbol{M}}_i^{\jmath} \leq \upsilon_{i,j} \check{\boldsymbol{M}}_{i,j}^{\jmath},$$

$$\check{\boldsymbol{P}}_{i,j} \leq \mu_{i,j} \check{\boldsymbol{P}}_j, \check{\boldsymbol{H}}_{i,j}^{\imath} \leq \mu_{i,j} \check{\boldsymbol{H}}_j^{\imath}, \check{\boldsymbol{M}}_{i,j}^{\jmath} \leq \mu_j \check{\boldsymbol{M}}_{i,j}^{\jmath},$$

then the augmented system (5) is FTB with respect to $(\boldsymbol{R}, T_{\mathrm{f}}, c_1, c_2)$ and the performance index J^* can be obtained. Moreover, the admissible controller gain is given by $\boldsymbol{K}_i = \boldsymbol{Y}_i \boldsymbol{S}_i^{-1}$.

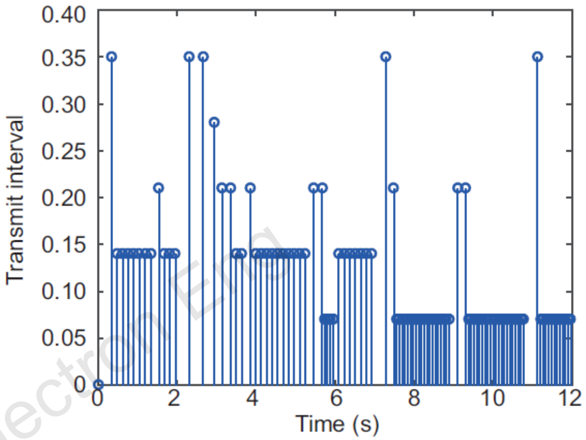


Fig. 4 Intervals of release for the hybrid event-triggered scheme (HETS)

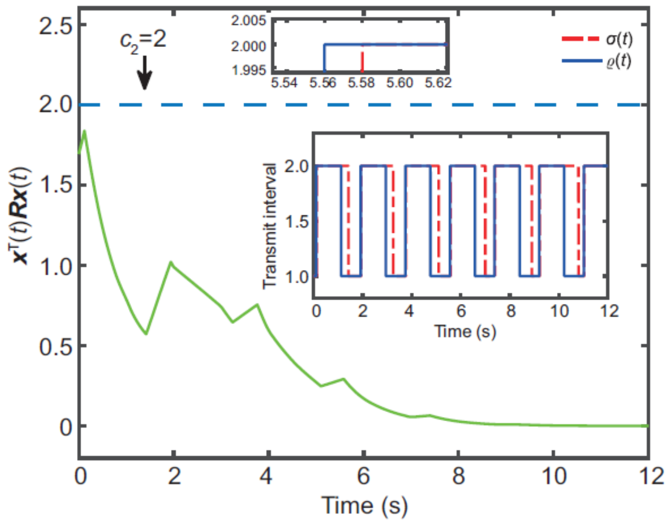


Fig. 5 Response of $x^{\mathrm{T}}(t)Rx(t)$ with inequalities (18) and (20)

Conclusions

This paper has studied the finite-time boundedness and guaranteed cost control of switched linear systems with asynchronous switching. By using ETS and RRP, the transmission of redundant information is reduced and the sensor's permission to receive data is more practical, thus easing the bandwidth pressure. The design of the switching signal adopts the AED-ADT method, including slow and fast AED-ADT. In this switching signal, the maximum delay limit for asynchronous switching is eliminated. By constructing the Lyapunov function related to the system mode and the controller mode, the controller gain and finite-time bounded criteria of the controlled switched system are obtained. Finally, the validity of the obtained results is verified by an example.

Hangli REN received her MS degree in operations research and cybernetics and her PhD degree in mathematics from Qufu Normal University in 2016 and 2019, respectively. She currently holds a position as an instructor at Zhengzhou University of Light Industry. Her research interests include stability analysis and control of complex switched systems, event-triggered control, and finite-time control.

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