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Near-field joint estimation of multi-targets’ position and velocity in a terahertz MIMO-OFDM system based on tensor decomposition

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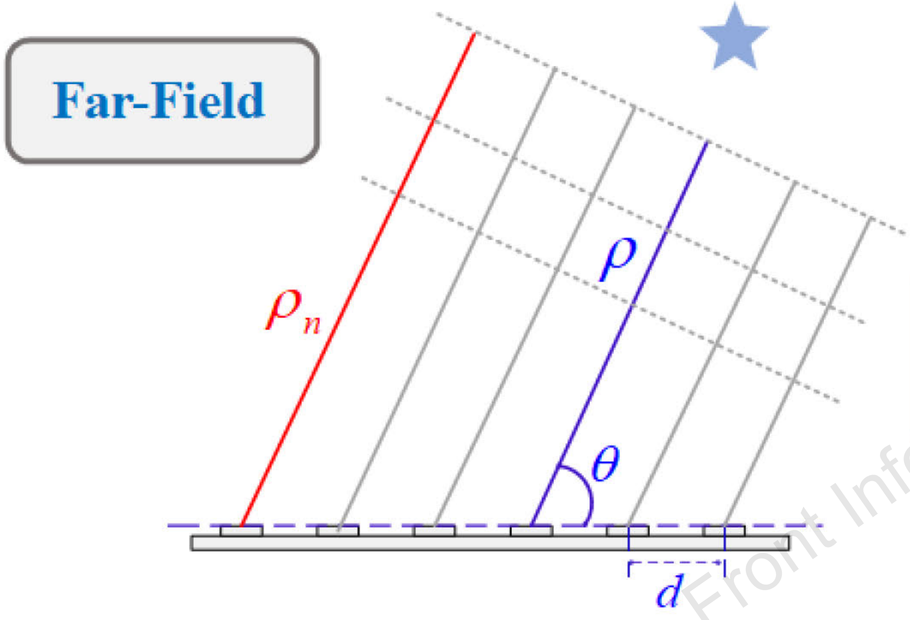
Outline

- ☐ Challenges and Opportunities
- ☐ Signal Model and Tensor Decomposition
- ☐ The Proposed CP-NFL Algorithm
- ☐ Simulation Results
- ☐ Conclusions

Challenges and Opportunities

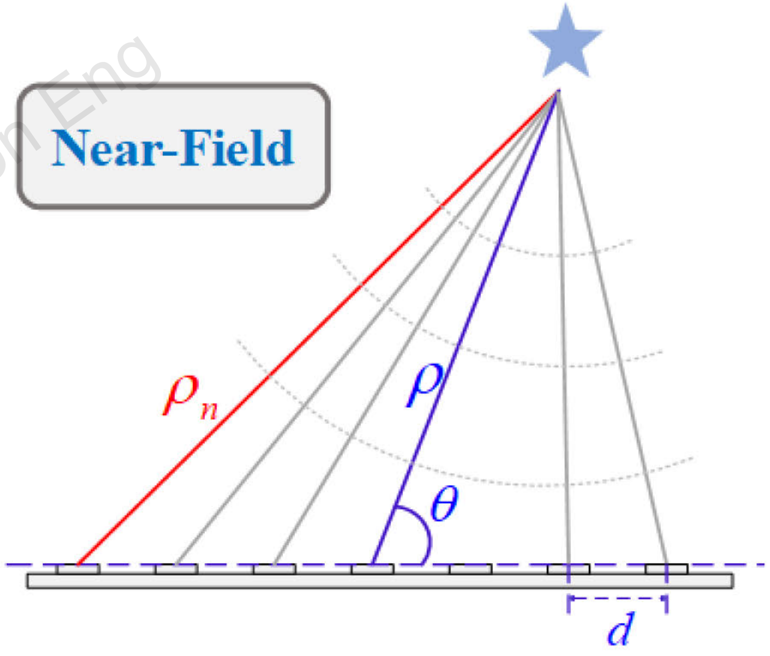
- Near-field and far-field propagation wave models



Distance: $\rho_n = \rho - nd\cos\theta$

Rayleigh distance:

$$\frac{2D^2}{\lambda} = \frac{2D^2 f}{c}$$

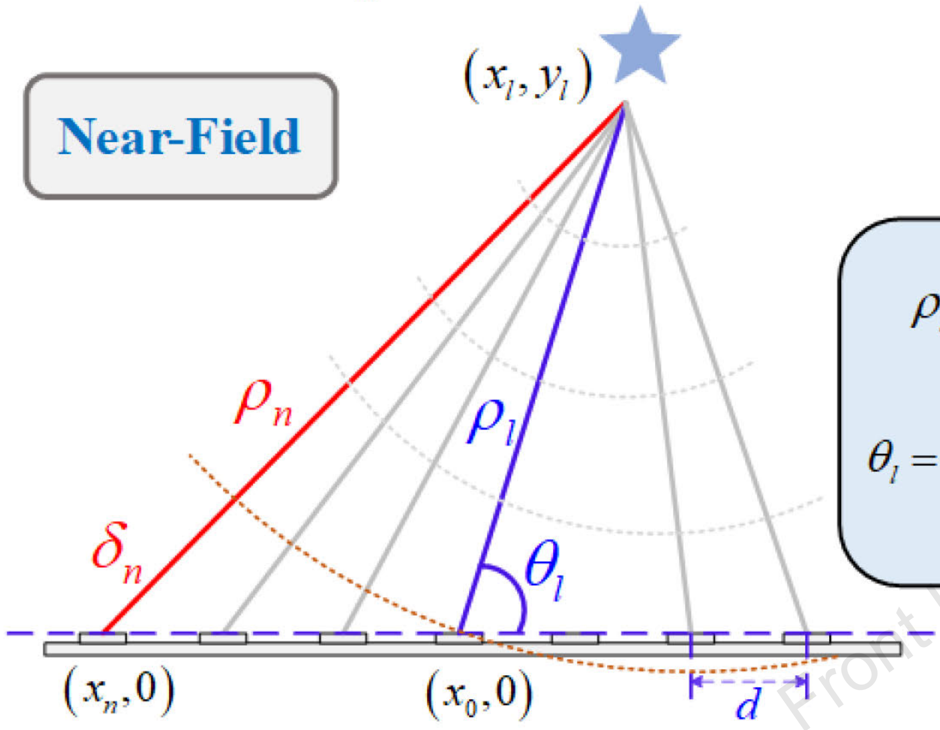


Distance: $\rho_n = \sqrt{\rho^2 + n^2 d^2 - 2\rho nd\cos\theta}$

Different wave models result in different calculation of *propagation distance* of each antenna elements.

Challenges and Opportunities

☐ Accurate spherical model and the position estimation



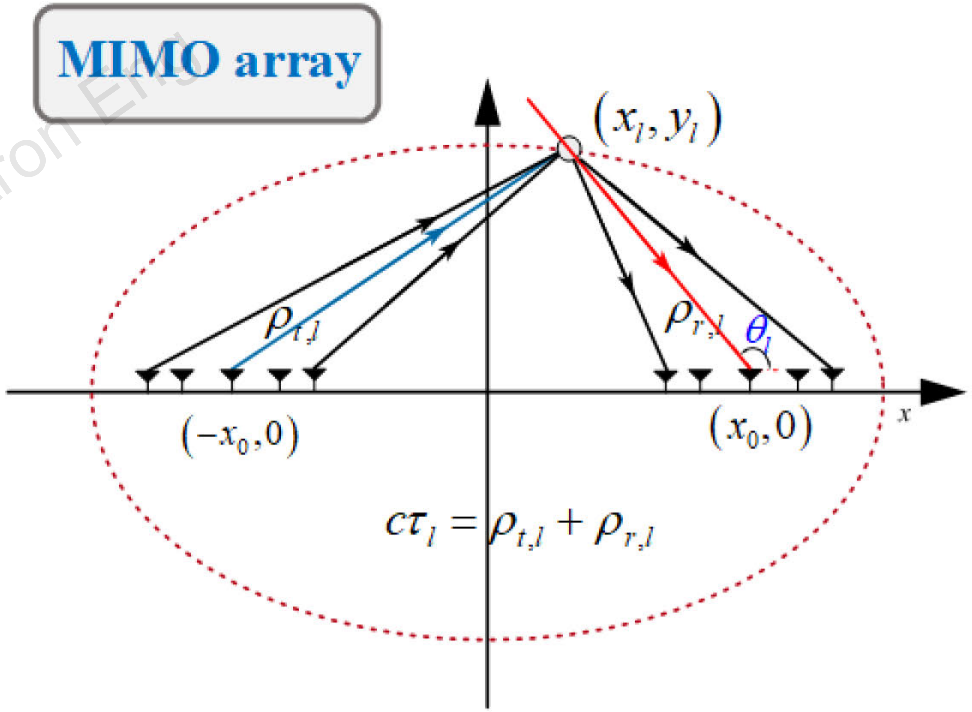
$$\delta_n = \sqrt{\rho_l^2 - 2nd\,\rho_l \cos\theta_l + n^2 d^2} - \rho_l$$

$$(\delta_n + \rho_l)^2 = \rho_l^2 - 2nd(x_l - x_0) + n^2 d^2$$

Coupled

$$\rho_l = \sqrt{(x_l - x_0)^2 + (y_l)^2}$$

$$\theta_l = \cos^{-1}\left(\frac{x_l - x_0}{\sqrt{(x_l - x_0)^2 + (y_l)^2}}\right)$$

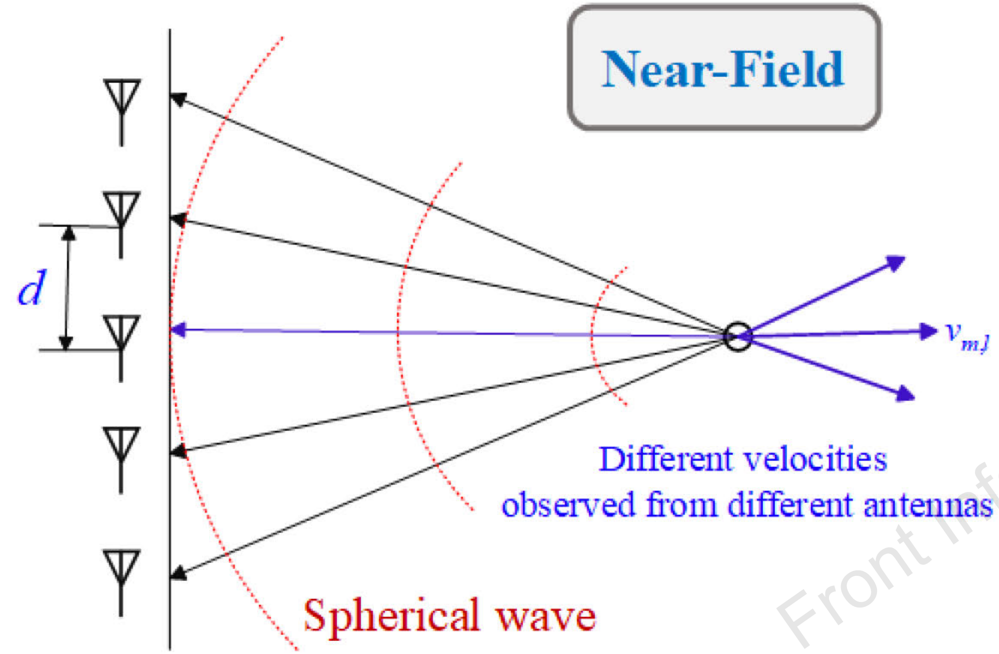


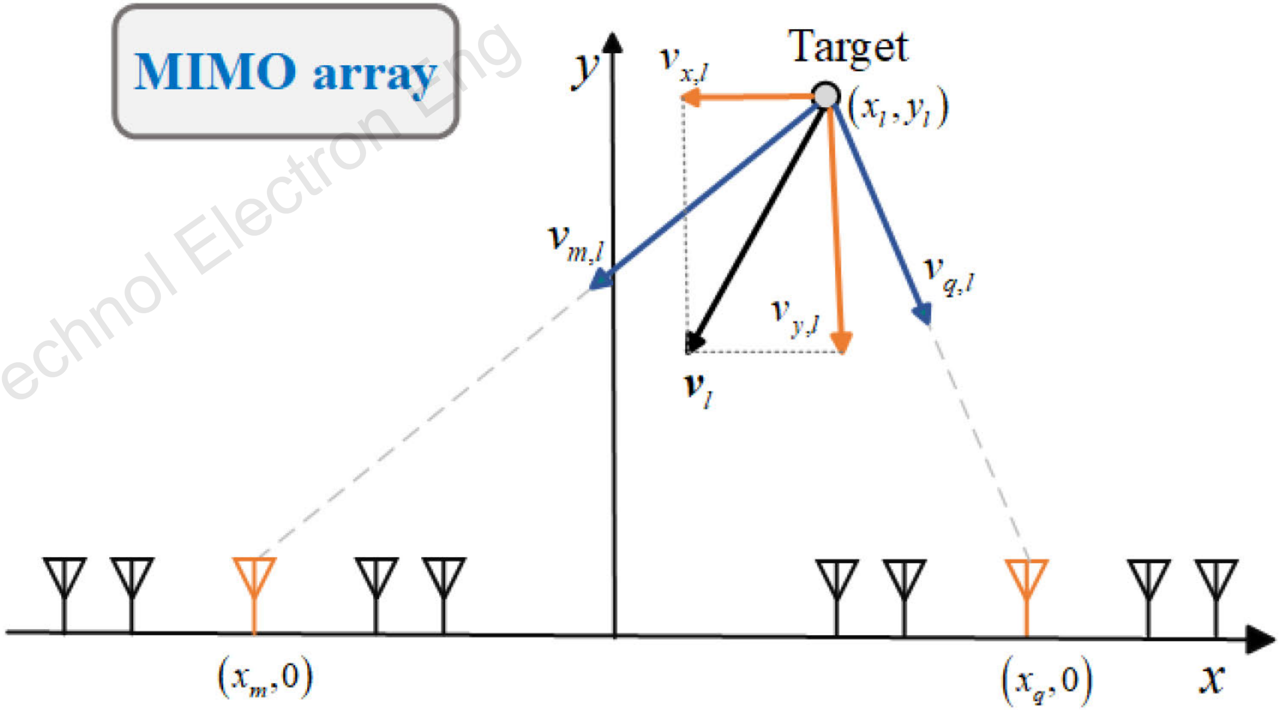
$$c\tau_l = \rho_{t,l} + \rho_{r,l}$$

The target locates at the intersection of the ellipse and the red line

Challenges and Opportunities

- ☐ Accurate spherical model and the velocity estimation





In the far-field model, the radial velocities observed from different antennas are the same; but in the near-field, the observed radial velocities are different.

$$v_{m,l} = \frac{v_{x,l}(x_m - x_l) - v_{y,l} y_l}{\sqrt{(x_m - x_l)^2 + y_l^2}} \qquad v_{q,l} = \frac{v_{x,l}(x_q - x_l) - v_{y,l} y_l}{\sqrt{(x_q - x_l)^2 + y_l^2}}$$

Signal Model

☐ Near-field MIMO model

- Path difference

$$\delta^{T}_{m_{u_t},l} = \sqrt{\left(x_l - x_{T,m_{u_t}}\right)^2 + \left(y_l\right)^2} - \rho^{T}_{M_T/2,l}$$

$$\delta^{R}_{q_{u_r},l} = \sqrt{\left(x_l - x_{R,q_{u_r}}\right)^2 + \left(y_l\right)^2} - \rho^{R}_{Q_R/2,l}$$

- Time delay

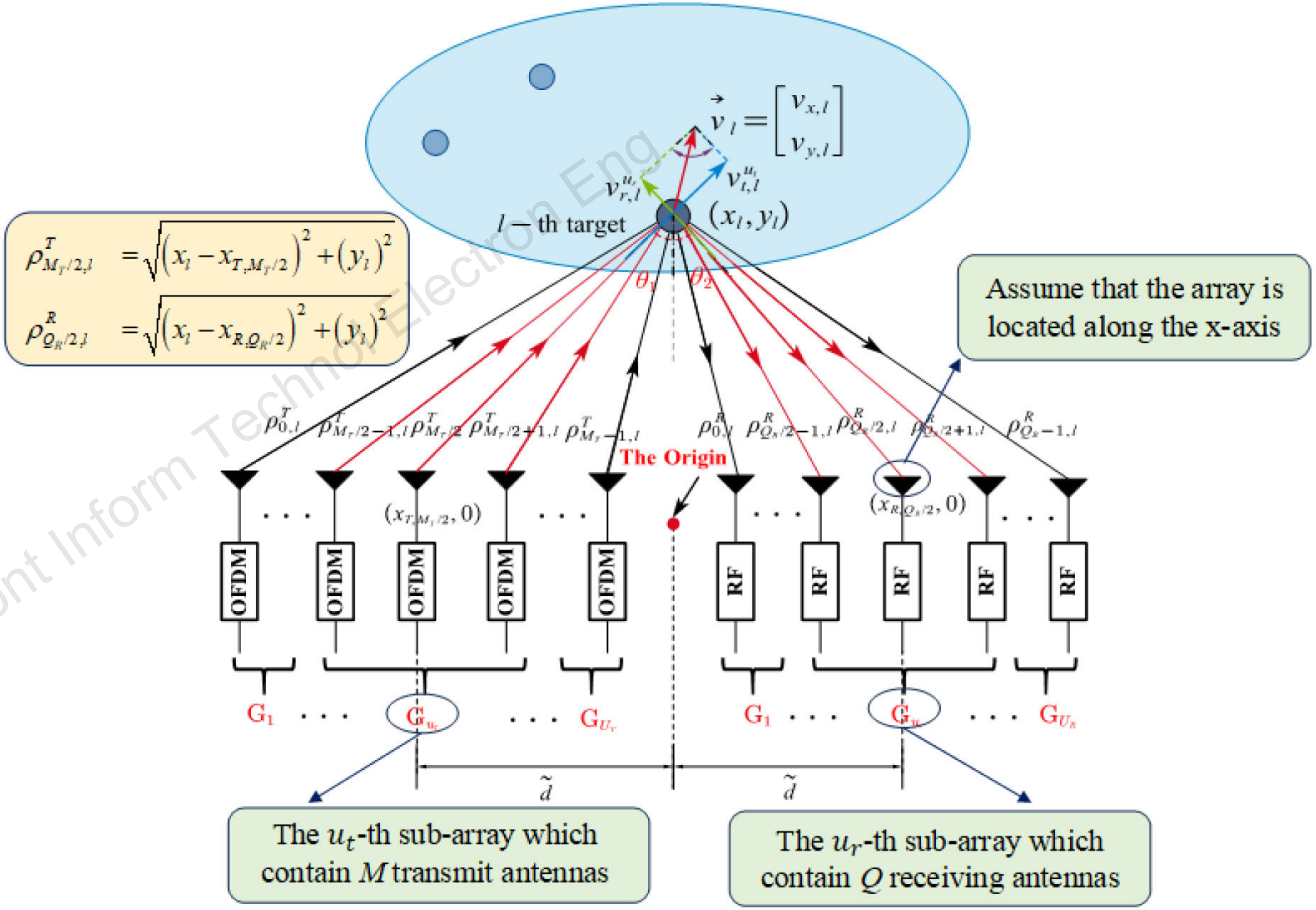
$$\tau_{m_{u_t},q_{u_r},l} = \tau_{M_T/2,Q_R/2,l} + (\delta^{T}_{m_{u_t},l} + \delta^{R}_{q_{u_r},l}) / c$$

$$\tau_{M_T/2,Q_R/2,l} = (\rho^{T}_{M_T/2,l} + \rho^{R}_{Q_R/2,l}) / c$$

- Velocity

$$v_l^{u_t,u_r} = v_l^{u_t} + v_l^{u_r}$$

$$\begin{cases} v_l^{u_t} = \dfrac{v_{x,l}(x_{T,(u_t-\frac{1}{2})M} - x_l) - v_{y,l} y_l}{\rho^{T}_{(u_t-\frac{1}{2})M,l}} \\ v_l^{u_r} = \dfrac{v_{x,l}(x_{R,(u_r-\frac{1}{2})Q} - x_l) - v_{y,l} y_l}{\rho^{R}_{(u_r-\frac{1}{2})Q,l}} \end{cases}$$



$$\rho^{T}_{M_T/2,l} = \sqrt{\left(x_l - x_{T,M_T/2}\right)^2 + \left(y_l\right)^2}$$

$$\rho^{R}_{Q_R/2,l} = \sqrt{\left(x_l - x_{R,Q_R/2}\right)^2 + \left(y_l\right)^2}$$

Assume that the array is located along the x-axis

The u_t-th sub-array which contain M transmit antennas

The u_r-th sub-array which contain Q receiving antennas

Signal Model

- The m_{u_t}-th element transmitted signal (FO-OFDM)

$K > N$ such that the waveform from each antenna do not intersect in the frequency domain

$$s_{m_{u_t}}(t) = e^{j2\pi f_c t} \sum_{\boxed{p}=0}^{P-1} \sum_{\boxed{n}=0}^{N-1} \tilde{s}_{m_{u_t},n,p} e^{j2\pi\left(n+m_{u_t}\boxed{K}\right)\Delta f\left(t-pT_o-T_{cp}\right)} \text{rect}\left[\frac{t-pT_o}{T_o}\right]$$

Symbol index

Sub-carrier index

Modulated symbol

- Received signal at the q_{u_r}-th element in time domain

$$y_{m_{u_t},q_{u_r}}(t) = \sum_{l=1}^{L} \alpha_l e^{j2\pi(f_c+\boxed{f_l^D})\left(t-\tau_{m_{u_t},q_{u_r},l}\right)} s_{m_{u_t}}\left(t-\tau_{m_{u_t},q_{u_r},l}\right) + z_{m_{u_t},q_{u_r}}(t)$$

Doppler shift: $f_l^D = \frac{2v_{m_{u_t},q_{u_r},l} f_c}{c}$

Additive Gaussian White Noise

- Removing guard interval and sampling the received signal at time of the p-th FO-OFDM symbol

$$y_{m_{u_t},q_{u_r},p}(i) = y_{m_{u_t},q_{u_r}}(t)\big|_{t=pT_0+T_{cp}+iT/N}$$

Signal Model

☐ Assumptions

- **A1** The antenna aperture $D_T + D_R$ is small enough as compared with $\frac{c}{N\Delta f}$, On the other hand, since $\delta^R_{q_{u_r},l} + \delta^T_{m_{u_t},l} \le D_T + D_R$, it holds true that

$$\boxed{N\Delta f \frac{\delta^R_{q_{u_r},l} + \delta^T_{m_{u_t},l}}{c} \ll 1} \Rightarrow \boxed{e^{j2\pi n\Delta f \frac{\delta^R_{q_{u_r},l}+\delta^T_{m_{u_t},l}}{c}} \text{ can be ignored!}}$$

- **A2** The propagation time delay is small enough as compared with the symbol duration T_0

$$\boxed{\tau_{m_{u_t},q_{u_r},l} \ll T_0} \Rightarrow \boxed{e^{-j2\pi \frac{2v_{m_{u_t},q_{u_r},l} f_c \tau_{m_{u_t},q_{u_r},l}}{c}} \text{ can be ignored!}}$$

- **A3** The speed of the targets relative to the same subarray antenna is roughly equal

$$\boxed{v_{m_{u_t},q_{u_r},l} \approx v_l^{u_t,u_r} = v_l^{u_t} + v_l^{u_r}}$$

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[1] D. Cohen, D. Cohen, Y. C. Eldar and A. M. Haimovich, "SUMMeR: Sub-Nyquist MIMO Radar," in *IEEE Transactions on Signal Processing*, vol. 66, no. 16, pp. 4315-4330, 15 Aug.15, 2018.

Tensor Form of the Signal Model

- ☐ With the assumptions and performing Discrete Fourier transform(DFT), we get

$$y_{m_{u_t},q_{u_r},p,n}=\frac{1}{N}\sum_{i=0}^{N-1}y_{m_{u_t},q_{u_r},p}(i)e^{-j2\pi\frac{ni}{N}}$$

$$\approx\sum_{l=1}^{L}\beta_l e^{-j2\pi n\Delta f\tau_{M_T/2,Q_R/2,l}}e^{j2\pi f_c(\frac{2v_l^{u_t,u_r}pT_0}{c})}\gamma_{m_{u_t},q_{u_r},l}+z_{m_{u_t},q_{u_r},p,n}$$

$$\boldsymbol{b}_{u_t,u_r,l}(p)=e^{j2\pi f_c\left(\frac{2v_l^{u_t,u_r}pT_0}{c}\right)},p=0,1,\cdots,P-1.$$

$$\mathrm{X}_{u_t,u_r}=\sum_{l=1}^{L}\tilde{\beta}_l\boldsymbol{a}_{u_t,u_r,l}\circ\boldsymbol{b}_{u_t,u_r,l}\circ\boldsymbol{c}_{u_t,u_r,l}$$

in which

$$\boldsymbol{a}_{u_t,u_r,l}(n)=e^{-j2\pi n\Delta f\tau_{M_T/2,Q_R/2,l}},n=0,1,\cdots,N-1.$$

$$\beta_l=\alpha_l e^{-j2\pi f_c\tau_{M_T/2,Q_R/2,l}}\tilde{s}_{m_{u_t},n,p}$$

A scalar independent of the estimated parameter

$$\gamma_{m_{u_t},q_{u_r},l}=e^{-j2\pi Km_{u_t}\Delta f\left(\tau_{M_T/2,Q_R/2,l}+\frac{\delta^T_{m_{u_t},l}}{c}+\frac{\delta^R_{q_{u_r},l}}{c}\right)}e^{-j2\pi f_c\left(\frac{\delta^T_{m_{u_t},l}}{c}+\frac{\delta^R_{q_{u_r},l}}{c}\right)}$$

$$c_{u_t,u_r,l}(\tilde{m}_{u_t,u_r})=\gamma_{\tilde{m}_{u_t,u_r},l},\tilde{m}_{u_t,u_r}=0,1,\cdots,MQ.$$

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[1] L. Gaudio, M. Kobayashi, B. Bissinger and G. Caire, "Performance Analysis of Joint Radar and Communication using OFDM and OTFS," *2019 IEEE International Conference on Communications Workshops (ICC Workshops)*, Shanghai, China, 2019.

Tensor Decomposition

☐ CP-decomposition

➢ *Lemma* of CP-decomposition

If $\mathcal{A} \in \mathbb{C}^{I_1 \times I_2 \times \dots \times I_M}$ denote an M-th order tensor with its $(i_1, i_2, \cdots, i_M)$-th entry denoted by $\mathcal{A}_{i_1,\cdots,i_M}$., the order M of a tensor is the number of dimensions. The CP decomposition decomposes a tensor into a sum of rank-one component tensors

$$\mathcal{A} = \sum_{l=1}^{L_0} x_l^1 \circ x_l^2 \circ \cdots \circ x_l^M$$

where $x_l^m \in \mathbb{C}^{I_m}$. The minimum achievable L_0 is referred to as the rank of the tensor, and $X^m = [x_1^m, \cdots, x_{L_0}^m] \in \mathbb{C}^{I_m \times L_0}$ denotes the factor along the m-th mode.

➢ *Kruskal Condition*

When the *Kruskal Condition* holds true, the CP decomposition is unique, namely satisfied

$$\min(I_1, L_0) + \min(I_2, L_0) + \cdots + \min(I_M, L_0) \geq 2L_0 + 2$$

According to the *Kruskal Condition* and the work of (De Lathauwer, 2006), The CP-decomposition of receiving tensor data $\mathcal{Y}_{u_t, u_r}$ is unique!

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[1] De Lathauwer L. A link between the canonical decomposition in multilinear algebra and simultaneous matrix diagonalization[J]. SIAM journal on Matrix Analysis and Applications, 2006, 28(3): 642-666.

Tensor Decomposition

- Estimate the time delay $\tau_{M_T/2,Q_R/2,l}$, path difference $\delta^{R}_{q_{u_r},l}$, velocity $v_l^{u_t,u_r}$

After CP-decomposition ,the estimated factor matrices composed of $\hat{\boldsymbol{a}}_{u_t,u_r,l},\hat{\boldsymbol{b}}_{u_t,u_r,l},\hat{\boldsymbol{c}}_{u_t,u_r,l}$ can be obtained, and the time delay $\tau_{M_T/2,Q_R/2,l}$, path difference $\delta^{R}_{q_{u_r},l}$, velocity $v_l^{u_t,u_r}$ can be estimated by $\hat{\boldsymbol{a}}_{u_t,u_r,l},\hat{\boldsymbol{b}}_{u_t,u_r,l},\hat{\boldsymbol{c}}_{u_t,u_r,l}$.

- Estimate time delay

$$\hat{\tau}_{M_T/2,Q_R/2,l}=\frac{1}{2\pi\Delta f(N-1)U_TU_R}\sum_{u_t=1}^{U_T}\sum_{u_r=1}^{U_R}\sum_{n=0}^{N-2}\angle\left(\frac{\hat{\boldsymbol{a}}_{u_t,u_r,l}(n)}{\hat{\boldsymbol{a}}_{u_t,u_r,l}(n+1)}\right)$$

- Estimate the velocity

$$\hat{v}_l^{u_t,u_r}=\frac{c}{4\pi f_cT_0(P-1)}\sum_{p=0}^{P-2}\angle\left(\frac{\hat{\boldsymbol{b}}_{u_t,u_r,l}(p)}{\hat{\boldsymbol{b}}_{u_t,u_r,l}(p+1)}\right)$$

- Estimate the path difference

$$\hat{\delta}^{R}_{q_{u_r},l}=\frac{c}{2\pi K\Delta fM_T}\sum_{u_t=1}^{U_T}\sum_{m_{u_t}=1}^{M}\angle\left(\frac{\hat{\gamma}_{m_{u_t},q_{u_r},l}}{\hat{\gamma}_{m_{u_t},Q_R/2,l}}\bigg/\frac{\hat{\gamma}_{m_{u_t}-1,q_{u_r},l}}{\hat{\gamma}_{m_{u_t}-1,Q_R/2,l}}\right)$$

$$\hat{\gamma}_{m_{u_t},q_{u_r},l}=\hat{\boldsymbol{c}}_{u_t,u_r,l}(\tilde{m}_{u_t,u_r})$$

Parameter Estimation

Angle and position estimation

Re-arranging the formulas of the path difference $\delta^R_{q_{u_r},l}$

$$2(x_{R,q_{u_r}} - x_{R,Q_R/2})x_l + 2\delta^R_{q_{u_r},l}\rho^R_{Q_R/2,l} = (x_{R,q_{u_r}})^2 - (x_{R,Q_R/2})^2 - (\delta^R_{q_{u_r},l})^2$$

$$\mathbf{P}\boldsymbol{w}_l = \boldsymbol{h}_l$$

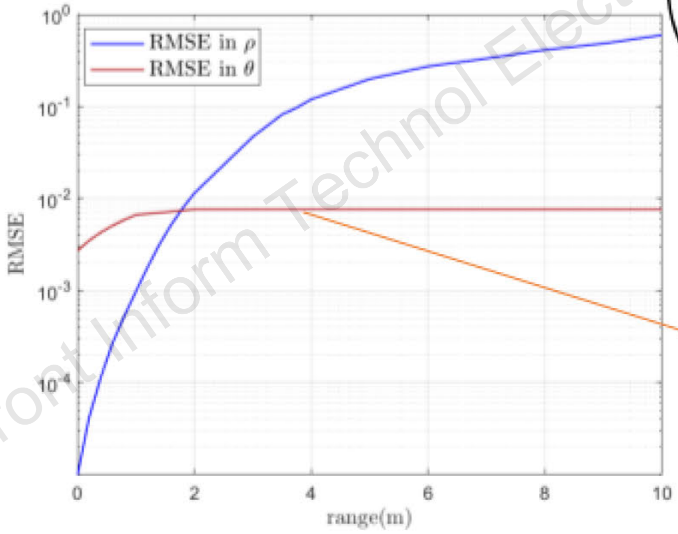
$$\boldsymbol{w}_l \triangleq \begin{bmatrix} x_l \\ \rho^R_{Q_R/2,l} \end{bmatrix}$$

Estimating $\boldsymbol{w}_l$ by LS

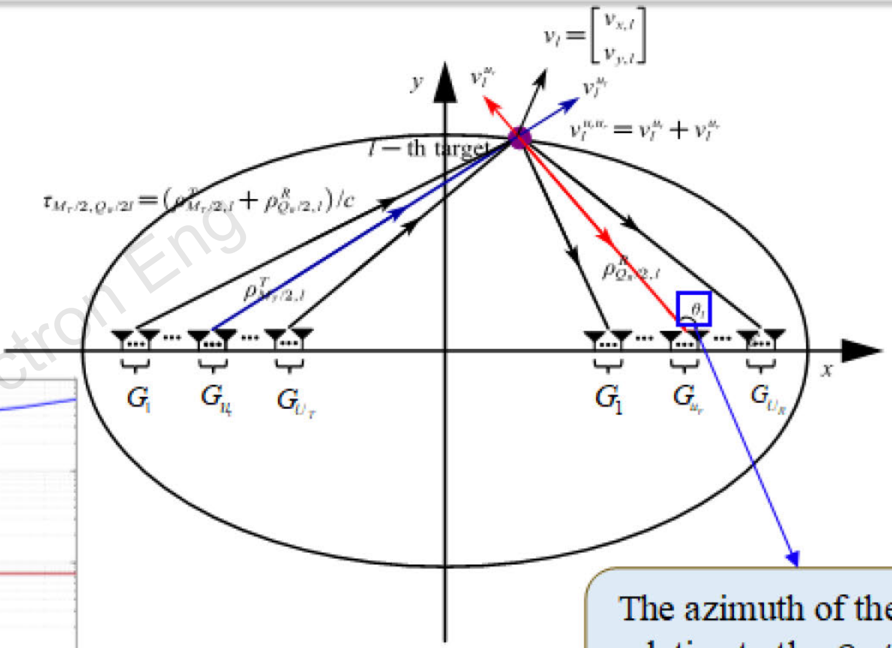
$$\begin{bmatrix} \bar{x}_l \\ \bar{\rho}^R_{M_R/2,l} \end{bmatrix} = \left(\mathbf{P}^T\mathbf{P}\right)^{-1}\mathbf{P}^T\boldsymbol{h}_l$$

Estimating of θ_l

$$\hat{\theta}_l = \cos^{-1}\left(\frac{\bar{x}_l - x_{R,Q_R/2}}{\bar{\rho}^R_{Q_R/2,l}}\right)$$



$\hat{\theta}_l$ has high accuracy



The azimuth of the target relative to the $Q_R/2$-th receiving antenna

$$\rho^R_{Q_R/2,l} + \rho^T_{M_T/2,l} = c\tau_{M_T/2,Q_R/2,l}$$

$$\begin{cases} \breve{\rho}^R_{Q_R/2,l} & = \dfrac{\left(c\hat{\tau}_{M_T/2,Q_R/2,l}\right)^2 + 8x^2_{R,Q_R/2}}{2c\hat{\tau}_{M_T/2,Q_R/2,l} - 4\cos\widehat{\theta}_l x_{R,Q_R/2}} \\ \breve{x}_l & = \breve{\rho}^R_{Q_R/2,l}\cos\left(\hat{\theta}_l\right) + x_{R,Q_R/2} \end{cases}$$

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[1] Podkurkov I, Hamidullina L, Traikov E, et al. Tensor-based near-field localization in bistatic MIMO radar systems[C]//WSA 2018; 22nd International ITG Workshop on Smart Antennas. VDE, 2018: 1-8.

Parameter Estimation

☐ Velocity estimation

- The observed velocity is related not only to the target's velocity $(v_{x,l}, v_{y,l})$ but also related to the target's position $(x_l\ , y_l)$. Let $\hat{\boldsymbol{v}}_l \triangleq \left[\hat{v}_l^{1,1}, \cdots \hat{v}_l^{U_T,U_R}\right]^T$, Then

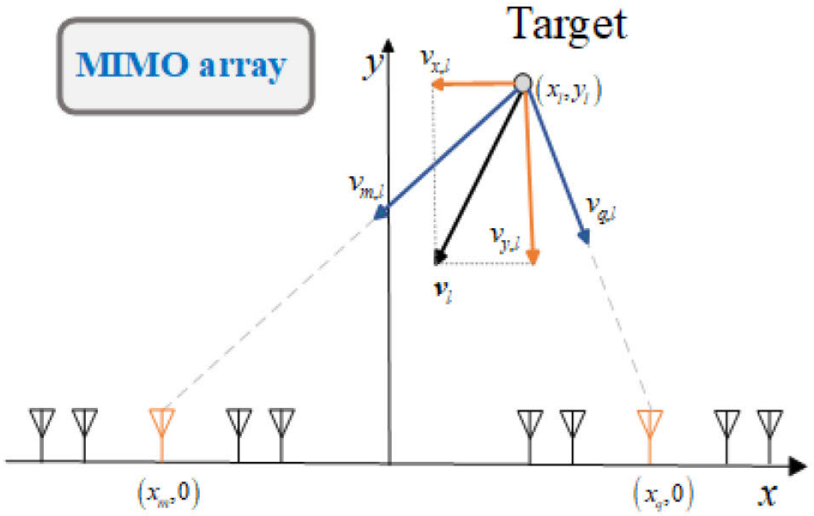
$$\boxed{\hat{\boldsymbol{v}}_l = \mathbf{G}\begin{bmatrix}\hat{v}_{x,l}\\ \hat{v}_{y,l}\end{bmatrix}}$$

where

$$\mathbf{G} \triangleq \begin{bmatrix} \dfrac{x_{T,\frac{M}{2}} - \hat{x}_l}{\rho^T_{\frac{M}{2},L}} + \dfrac{x_{R,\frac{Q}{2}} - \hat{x}_l}{\rho^R_{\frac{Q}{2},l}} & \dfrac{-\hat{y}_l}{\rho^T_{\frac{M}{2},L}} + \dfrac{-\hat{y}_l}{\rho^R_{\frac{Q}{2},l}} \\ \vdots & \vdots \\ \dfrac{x_{T,(U_T-\frac{1}{2})M} - \hat{x}_l}{\rho^T_{(U_T-\frac{1}{2})M,l}} + \dfrac{x_{R,(U_T-\frac{1}{2})Q} - \hat{x}_l}{\rho^R_{(U_R-\frac{1}{2})Q,l}} & \dfrac{-\hat{y}_l}{\rho^T_{(U_T-\frac{1}{2})M,l}} + \dfrac{-\hat{y}_l}{\rho^R_{(U_R-\frac{1}{2})Q,l}} \end{bmatrix}$$

- Least square algorithm estimate $(v_{x,l}, v_{y,l})$

$$\boxed{\begin{bmatrix}\hat{v}_{x,l}\\ \hat{v}_{y,l}\end{bmatrix} = \left(\mathbf{G}^T\mathbf{G}\right)^{-1}\mathbf{G}^T\hat{\boldsymbol{v}}_l}$$



$$v_{m,l} = \frac{v_{x,l}(x_m - x_l) - v_{y,l}y_l}{\sqrt{(x_m - x_l)^2 + y_l^2}} \qquad v_{q,l} = \frac{v_{x,l}(x_q - x_l) - v_{y,l}y_l}{\sqrt{(x_q - x_l)^2 + y_l^2}}$$

The Proposed CP-NFL Algorithm

Algorithm: CP-NFL Algorithm

Inputs: array groupings U_T and U_R, tensors of receiving data $\boldsymbol{y}_1$, $\boldsymbol{y}_2 \ldots \boldsymbol{y}_{U_T U_R}$, target number L;

Output: position $(\hat{x}_l, \hat{y}_l)$, velocity $\left(\hat{v}_{x,l}, \hat{v}_{y,l}\right), \forall l \in (1, \ldots, L)$;

```
for $u_t = 1 : U_T$ do
    for $u_r = 1 : U_R$ do
        Tensor decomposition of $\boldsymbol{y}_{u_t u_r}$;
        Yielding factor matrices composed of $\widehat{\boldsymbol{a}}_{u_t,u_r,l}$, $\widehat{\boldsymbol{b}}_{u_t,u_r,l}$, $\widehat{\boldsymbol{c}}_{u_t,u_r,l}$ ;
    end for
end for
for $l = 1 : L$ do
    Compute $\hat{\tau}_{M_T/2,Q_R/2,l}$, $\hat{v}_l^{u_t,u_r}$ and $\hat{\delta}^R_{u_r,l}$ by $\widehat{\boldsymbol{a}}_{u_t,u_r,l}$, $\widehat{\boldsymbol{b}}_{u_t,u_r,l}$, $\widehat{\boldsymbol{c}}_{u_t,u_r,l}$;
    Compute $\breve{x}_l$ and $\breve{\rho}^R_{Q_R/2,l}$ by step of coarse estimation;
    Estimate $\left(\hat{x}_l, \hat{\rho}^R_{Q_R/2,l}\right)$ by solving convex optimization problem with CVX tool;
    Compute the position parameter $\hat{y}_l$ by parameters $\hat{x}_l, \hat{\rho}^R_{Q_R/2,l}$;
    Compute the velocity $\left(\hat{v}_{x,l}, \hat{v}_{y,l}\right)$ by parameters $\hat{x}_l$, $\hat{y}_l$ with LS algorithm;
end for
```

Complexity Analysis

Operation	Computational complexities
CP decomposition	$O((P^3+N^3+(MQ)^3+L^3+LMQN^2+L(MQ)^2N+L^2PNMQ)U_TU_R)$
Estimation of $\hat{\tau}_{M_T/2,Q_R/2,l},\hat{v}_l^{u_t,u_r},\hat{\delta}_{q_{u_r},l}^R$	$O(LMQU_TU_R)$
Solving optimization problem	$O(T)$
Total	$O\{U_TU_R(P^3+N^3+L^3)+\frac{(M_TQ_R)^3}{(U_TU_R)^2}+L\frac{(M_TQ_R)^2}{U_TU_R}N\}$ $+O(LM_TQ_RN^2+L^2PNM_TQ_R)+O(T)$

The computational complexity varies with the number of groups. When M_TQ_R is greater than P, N and L, increasing the number of groups can reduce the complexity caused by the number of antennas, and thus reducing the overall complexity.

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[1] da Rosa Zanatta M, de Mendonça F L L, Antreich F, et al. Tensor-based time-delay estimation for second and third generation global positioning system[J]. Digital Signal Processing, 2019, 92: 1-19.

Simulation Results

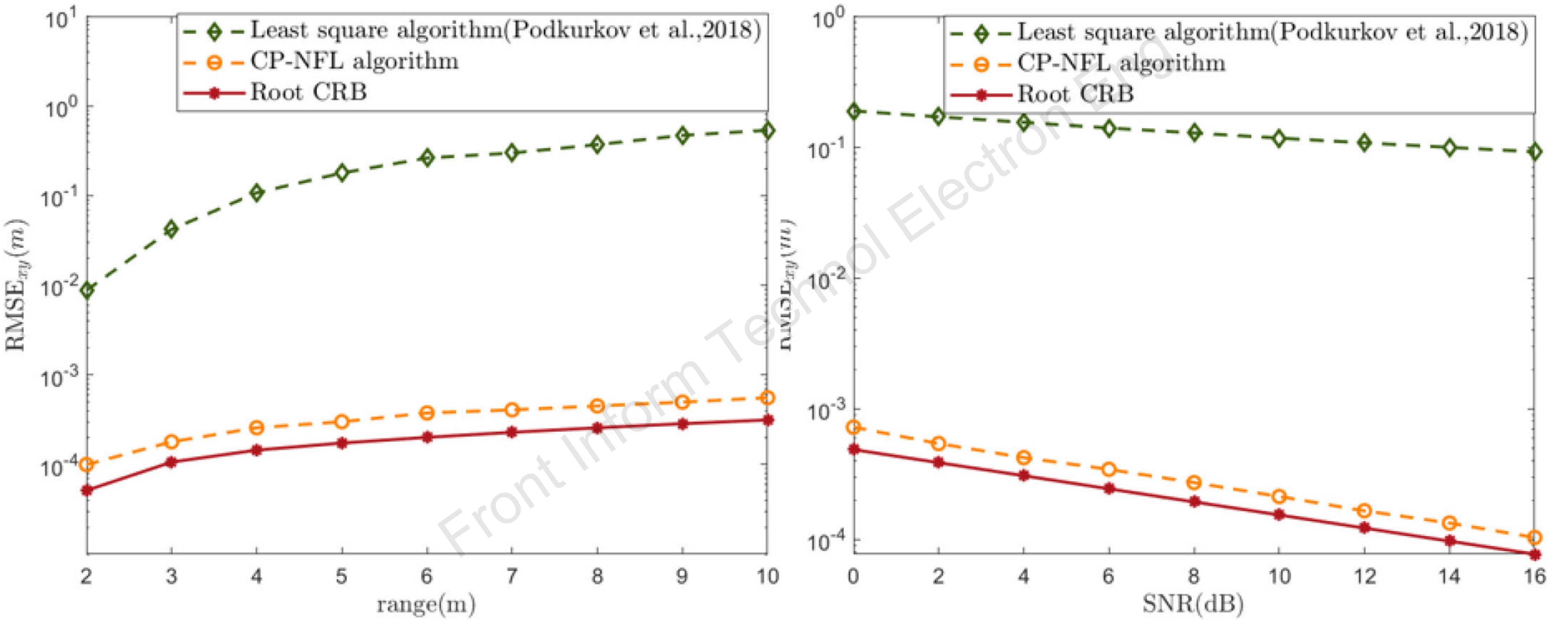
☐ Simulation settings

- In the following simulations, all simulations are carried out with $K_T = 1000$ trials and the other parameters are shown below.

Parameters	**Symbols**	**Value**
Subcarrier spacing	Δf	300kHz
Carrier frequency	f_c	300GHz
Spacing of transmitting/receiving array	d_t / d_r	0.5/5mm
Number of transmitting/receiving antennas	M_T / Q_R	31/30
A frame of OFDM symbols	P	60
Groups of transmitting/receiving array	U_T / U_R	1/6
Integer	K	401
Velocity of one/two target	v_1 / v_2	0.5/0.7m/s
Angle	θ_l	45°

Simulation Results

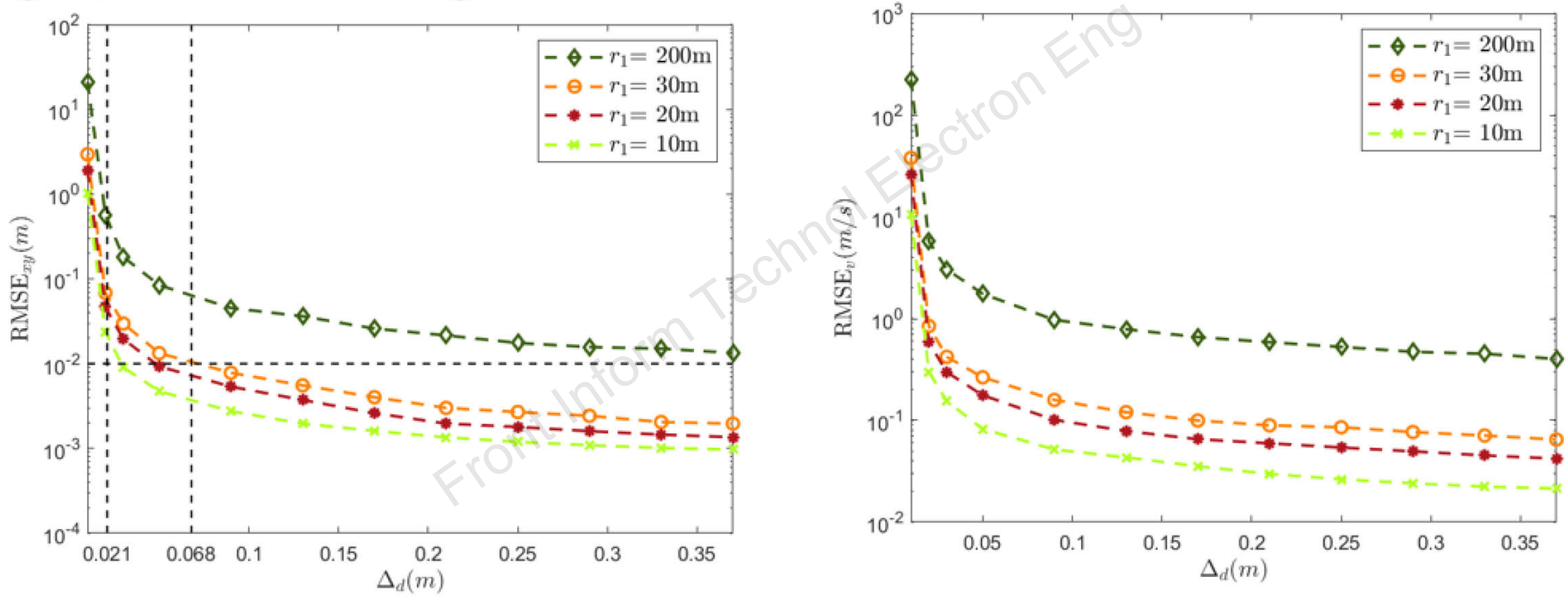
- ☐ The RMSE of the estimated location/velocity w.r.t. Range/SNR.



- ➢ From the left figure, one can see that the estimation performance of the proposed CP-NFL algorithm is much better than that of the LS algorithm.
- ➢ For the right figure, the target is set at 5m away from the array. one can see that the estimation performance achieved by the proposed CP-NFL algorithm is much better and it improves faster with the increase of SNR.

Simulation Results

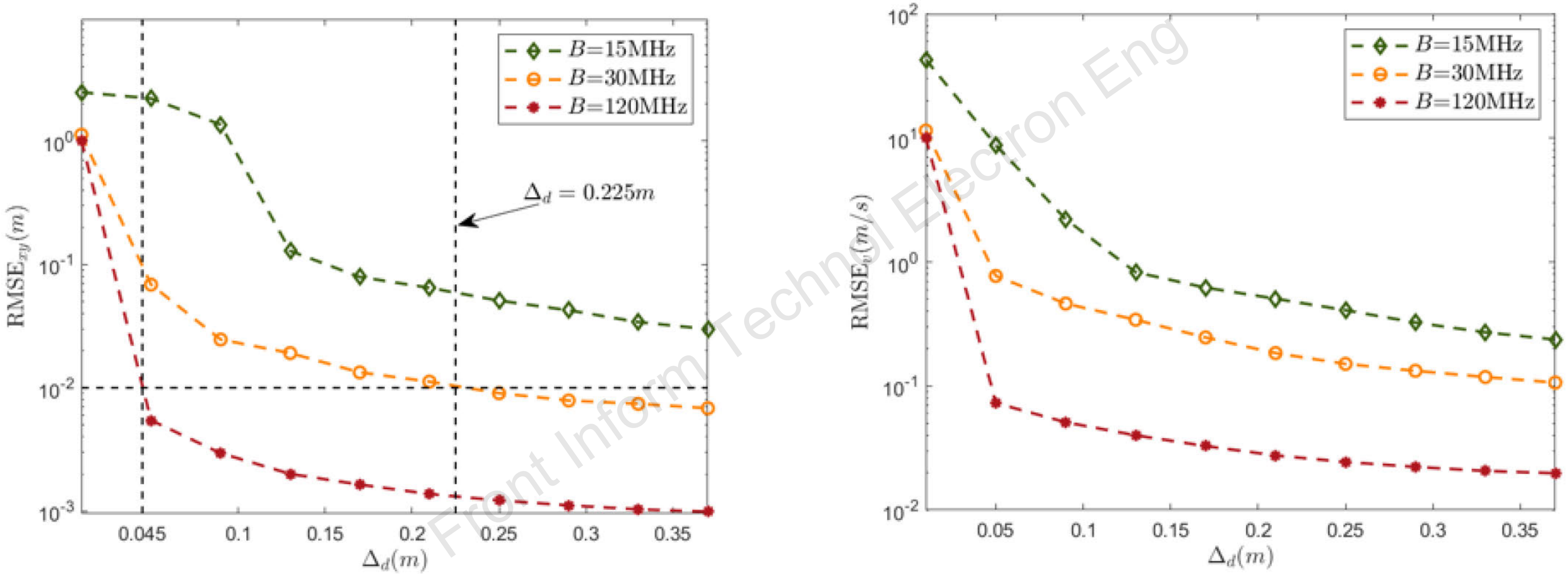
- ☐ The RMSE of the estimated location/velocity w.r.t. the range difference between two targets, for different ranges.



- ➢ The estimation accuracy increases as the distance of the first target r_1 decreases. The reason is that the near-field effect is more obvious when r_1 decreases.
- ➢ Since the estimation accuracy of the velocity is affected by that of the position, it can be seen that the variation trend of its estimation accuracy is the same as that in the left figure.

Simulation Results

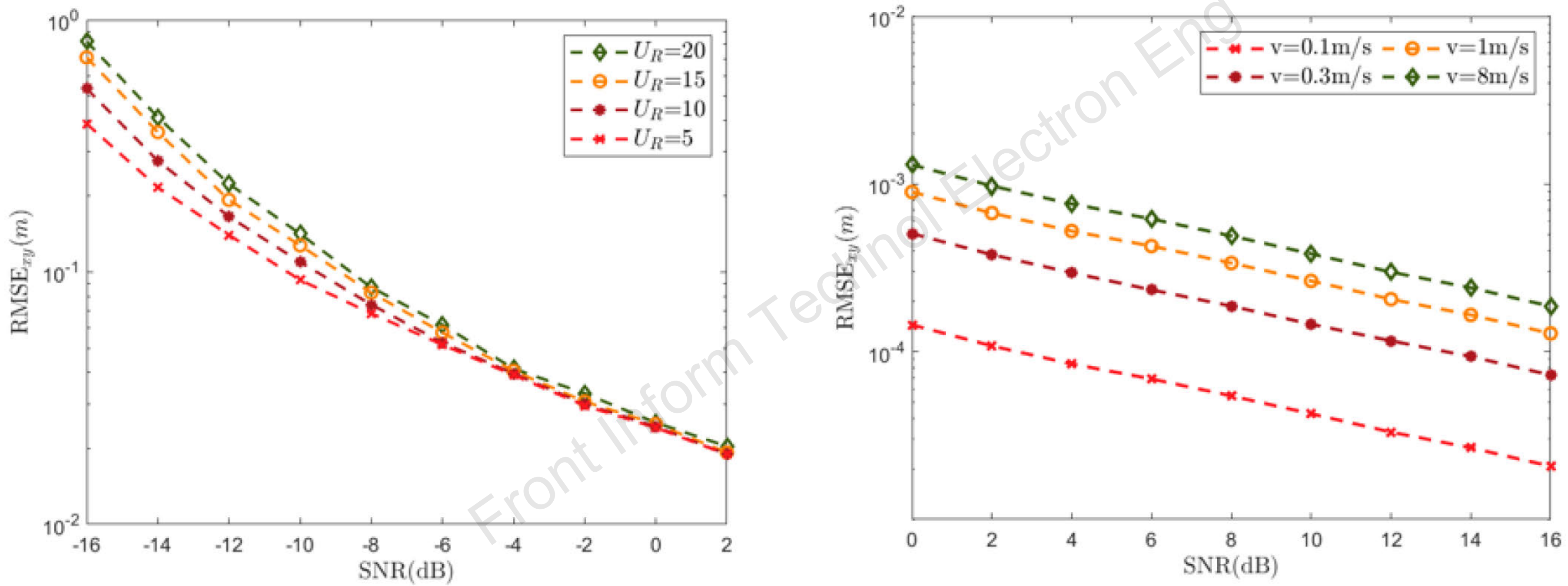
- The RMSE of the estimated location/velocity w.r.t. the range difference between two targets, for different available bandwith.



- With the increase of the available bandwidth, the estimation accuracy gradually improves. For example, for the cases where the alocated bandwith are 120MHz and 30MHz respetively, the estimation accuracy of the proposed CP-NFL algorithm reaches millimeter levels (we can differetiate two targets with the distance greater than 0.045m and 0.225m, respectively).
- The velocity estimation is independent of the estimated location information, so the accuracy of the estimated velocity estimation also increases.

Simulation Results

- ☐ The RMSE of the estimated location w.r.t. SNR, for different number of antenna groups or velocity.



- ➢ As the number of sub-array increases, the estimation accuracy deteriorates, but this deterioration disappears as the SNR is greater than -4dB.
- ➢ With the increase of the velocity, the localization accuracy gradually deteriorates. This is because as the velocity increases, Assumption **A3** gradually loses validity, which leads to the deterioration of localization performance.

Conclusions

- We proposed a CP decomposition-based method for the joint estimation of multi-targets' position and velocity via a THz MIMO-OFDM system operating in the near-field region, where the waveforms transmitted from each antenna carry communication messages and are orthogonal with each in the frequency domain.
- To further reduce the computational complexity of the proposed algorithm, we propose to divide the antennas into several sub-arrays. Our analysis showed that the computational complexity of the proposed method is linear to the sum of the third power of the number of sub-carriers, OFDM symbols, antennas and targets.
- We compared our proposed method with the existing LS-based parameter estimation method. Through simulations, the impact of systems parameters on the estimation accuracy was demonstrated, highlighting the advantage of the proposed CP-NFL method.

Thanks for your attention!