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Trajectory Poisson multi-Bernoulli filters with unknown detection probability

Key words: Trajectory Poisson multi-Bernoulli; Beta Gaussian; Detection probability; Alive trajectories; All trajectories

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Motivation

1. The current trajectory Poisson multi-Bernoulli (TPMB) filter provides the detection probability of the sensor as prior information and assumes that the detection probability of the sensor remains constant. However, in actual scenarios, the detection probability of the sensor may change with the increase in usage time.

2. For filters based on random finite sets with unknown detection probabilities, they cannot directly output the trajectory of the target but can only output the state of the tracked target at a certain moment.

3. For filters based on a random finite set of trajectories with unknown detection probability, only the alive target trajectories are considered, rather than all target trajectories.

Main idea

1. The detection probability is extended as a state variable to the trajectory state, and the alive trajectory state transition model considering the detection probability and the all trajectory state transition model are proposed.

2. The recursion of TPMB with unknown detection probability is derived.

3. The detection probability state is modeled as a Beta distribution, and the Beta-Gaussian implementation of TPMB for the alive trajectories and the Beta-Gaussian implementation of TPMB for all trajectories are given.

Method

Suppose that the parameters of the TPMB density for the augmented trajectory state are λ_{k-1} and $\left\{r_{k-1}^{i}, p_{k-1}^{i}\left(\hat{X}\right)\right\}_{i=1}^{n_{k-1}}$ at time step $k-1$. At time step k, the predicted density is a TPMB density with $n_{k|k-1}=n_{k-1}$ and

$$\lambda_{k|k-1}\left(\hat{X}\right)=\lambda_{k}^{\mathrm{born}}\left(\hat{X}\right)+\left\langle\lambda_{k-1}, g_{k}(\hat{X}|\cdot) p^{\mathrm{S}}(\cdot)\right\rangle, \tag{21}$$

$$r_{k|k-1}^{i}=r_{k-1}^{i}\left\langle p_{k-1}^{i}\left(\hat{X}\right), p^{\mathrm{S}}\right\rangle, \tag{22}$$

$$p_{k|k-1}^{i}\left(\hat{X}\right)=\frac{\left\langle p_{k-1}^{i}\left(\hat{X}\right), g_{k}\left(\hat{X}|\cdot\right) p^{\mathrm{S}}(\cdot)\right\rangle}{\left\langle p_{k-1}^{i}\left(\hat{X}\right), p^{\mathrm{S}}\right\rangle}, \tag{23}$$

where $\lambda_{k}^{\mathrm{born}}$, $g_{k}(\cdot|\cdot)$, and $p^{S}(\cdot)$ are chosen depending on the case in Section 3.1 for alive trajectories and all trajectories.

Method (Cont’d)

Suppose that the parameters of predicted TPMB density for the augmented trajectory state are $\lambda_{k|k-1}$ and $\left\{r^{i}_{k|k-1}, p^{i}_{k|k-1}\left(\hat{X}\right)\right\}_{i=1}^{n_{k|k-1}}$ at time step k and the measurement set $z_k = \left\{z^1_k, z^2_k, \cdots, z^m_k\right\}$. The parameters of TPMBM density are obtained as follows:

1. Missed detection for PPP:

$$\lambda_k\left(\hat{X}\right) = \left(1 - p_{\mathrm{D}k}\left(\hat{X}\right)\right)\lambda_{k|k-1}\left(\hat{X}\right). \qquad (24)$$

For each Bernoulli component $i, i \in \left(1, 2, \cdots, n_{k|k-1}\right)$, there are $h^i = m_k + 1$ local hypotheses.

2. Missed detection for the Bernoulli component:

$$w^{i,1}_k = 1 - r^{i}_{k|k-1}\left\langle p^{i}_{k|k-1}\left(\hat{X}\right), p_{\mathrm{D}k}\left(\hat{X}\right)\right\rangle, \qquad (25)$$

Method (Cont'd)

$$r_k^{i,1} = \frac{r_{k|k-1}^i \left\langle p_{k|k-1}^i\left(\hat{X}\right), 1 - p_{\mathrm{D}k}\left(\hat{X}\right)\right\rangle}{1 - r_{k|k-1}^i \left\langle p_{k|k-1}^i\left(\hat{X}\right), p_{\mathrm{D}k}\left(\hat{X}\right)\right\rangle}, \tag{26}$$

$$p_k^{i,1}\left(\hat{X}\right) = \frac{\left(1 - p_{\mathrm{D}k}\left(\hat{X}\right)\right) p_{k|k-1}^i\left(\hat{X}\right)}{\left\langle p_{k|k-1}^i\left(\hat{X}\right), 1 - p_{\mathrm{D}k}\left(\hat{X}\right)\right\rangle}, \tag{27}$$

3. Update for the detection Bernoulli component:

$$w_k^{i,1+j} = r_{k|k-1}^i \left\langle p_{k|k-1}^i\left(\hat{X}\right), p_{\mathrm{D}k}\left(\hat{X}\right)\phi\left(z_k^j|\cdot\right)\right\rangle, \tag{28}$$

$$r_k^{i,1+j} = 1, \tag{29}$$

$$p_k^{i,1+j}\left(\hat{X}\right) = \frac{p_{k|k-1}^i\left(\hat{X}\right) p_{\mathrm{D}k}\left(\hat{X}\right)\phi\left(z_k^j|\cdot\right)}{\left\langle p_{k|k-1}^i\left(\hat{X}\right), p_{\mathrm{D}k}(\hat{X})\phi\left(z_k^j|\cdot\right)\right\rangle}, \tag{30}$$

Method (Cont'd)

4. New Bernoulli component for the first time

Each measurement generates a new Bernoulli component. Let new Bernoulli component be $i = n_{k|k-1} + j$, $j \in \left(1, 2, \cdots, n_{k|k-1}\right)$, which is caused by measurement z_k^j.

$$w_k^{i,1} = 1,\ r_k^{i,1} = 0,$$

$$w_k^{i,2} = \lambda^{\mathrm{C}}\left(z_k^j\right) + \left\langle \lambda_{k|k-1}, p_{\mathrm{D}k}\left(\hat{X}\right)\phi\left(z_k^j|\cdot\right)\right\rangle, \tag{31}$$

$$r_k^{i,2} = \frac{\left\langle \lambda_{k|k-1}, p_{\mathrm{D}k}\left(\hat{X}\right)\phi\left(z_k^j|\cdot\right)\right\rangle}{\lambda^{\mathrm{C}}\left(z_k^j\right) + \left\langle \lambda_{k|k-1}, p_{\mathrm{D}k}\left(\hat{X}\right)\phi\left(z_k^j|\cdot\right)\right\rangle}, \tag{32}$$

$$p_k^{i,2}\left(\hat{X}\right) = \frac{\lambda_{k|k-1} p_{\mathrm{D}k}\left(\hat{X}\right)\phi\left(z_k^j|\cdot\right)}{\left\langle \lambda_{k|k-1}, p_{\mathrm{D}k}\left(\hat{X}\right)\phi\left(z_k^j|\cdot\right)\right\rangle}, \tag{33}$$

Major results

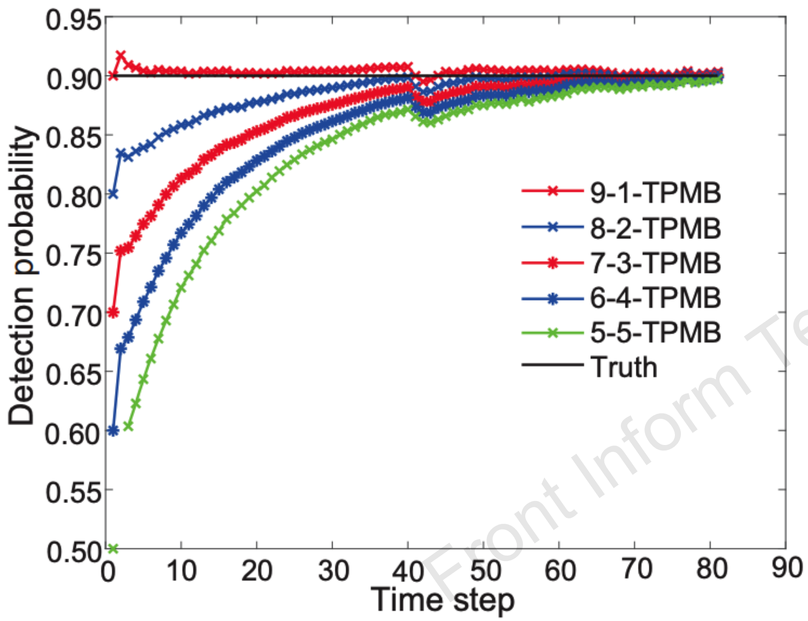

Fig. 2 Estimation of detection probability for all trajectories under different beta parameters

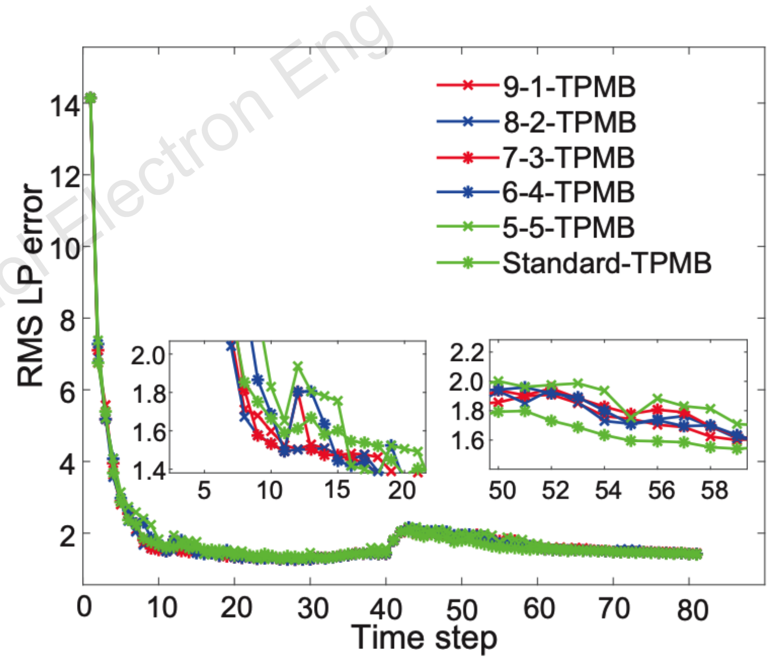

Fig. 3 Trajectory metric errors for all trajectories

Major results (Cont'd)

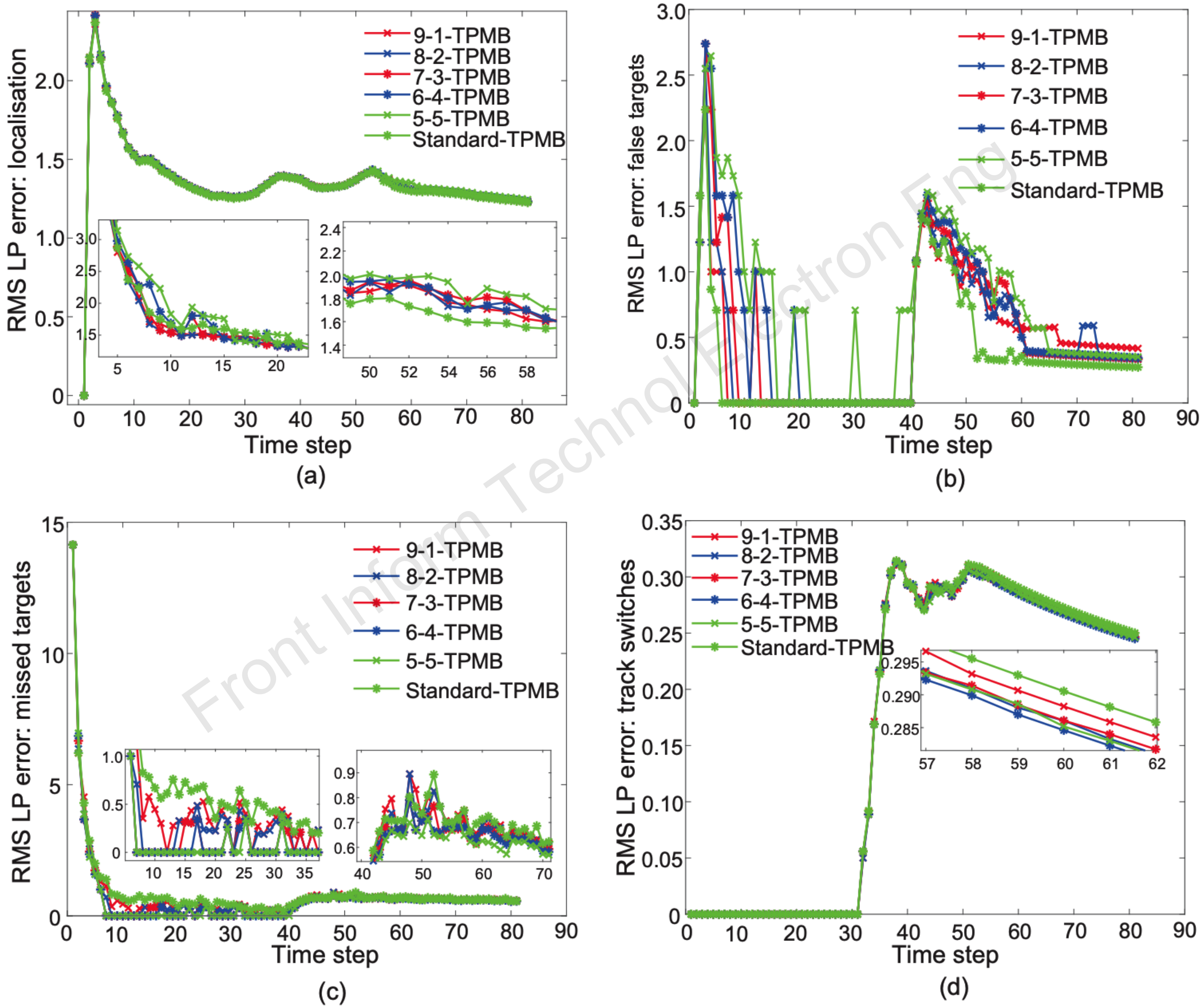

Fig. 4 Decomposition of trajectory metric errors for all trajectories: (a) local error; (b) false error; (c) miss error; (d) switch error

Major results (Cont'd)

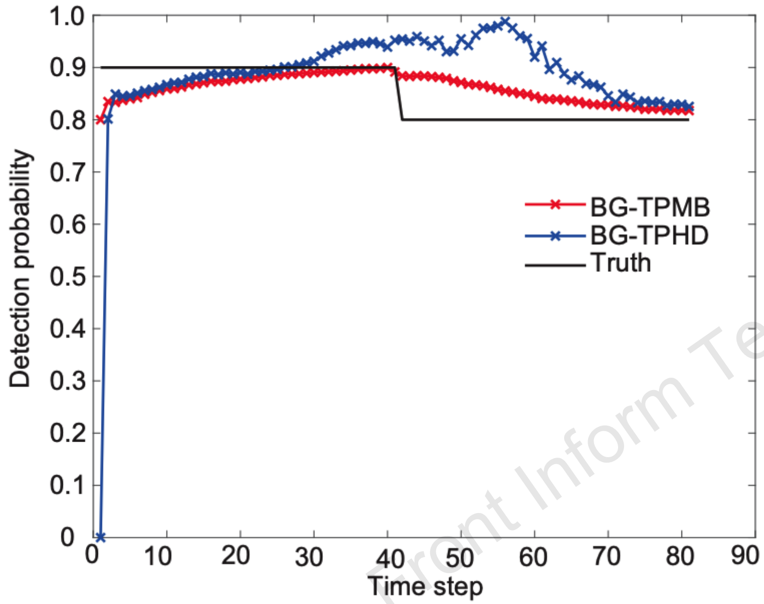

Fig. 5 Estimation of detection probability for alive trajectories

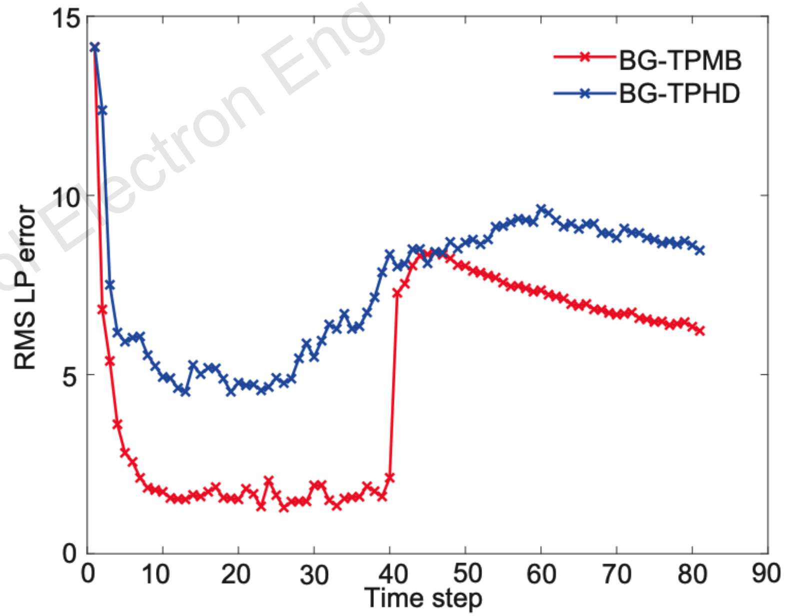

Fig. 6 Trajectory metric error for alive trajectories

Conclusions

1. This paper proposes two trajectory TPMB filters that perform robustly when the detection probability is unknown. These filters are implemented using the Beta-Gaussian (GB) mixture method, known as BG-TPMB filters.

2. The Poisson intensity is approximated as a BG mixture form, and the spatial probability density of each Bernoulli component is approximated as a single BG form.

3. The detailed recursive solutions are derived for TPMB filters with unknown detection probability, and closed forms for the BG implementation of alive and all trajectories are obtained.

Xiangfei ZHENG has been working toward the Ph.D. degree in the School of Mathematics and Physics at the China University of Geosciences (Wuhan) since 2023. His research interests include multi-target tracking & detection and multi-sensor information fusion.

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