Yu GUAN, Xiaoyu JIANG, Yanpeng ZHENG, Zhaolin JIANG, 2025. An optimized formula for the two-point resistance of a cobweb resistance network and its potential application. *Frontiers of Information Technology & Electronic Engineering*, 26(6):946-958. https://doi.org/10.1631/FITEE.2400613

An optimized formula for the two-point resistance of a cobweb resistance network and its potential application

Key words: Resistance network; Equivalent resistance; Potential function; Chebyshev polynomials; Path planning

Xiaoyu JIANG
E-mail: jxy19890422@sina.com
ORCID: https://orcid.org/0000-0001-5857-9148
Yanpeng ZHENG
E-mail: zhengyanpeng0702@sina.com
ORCID: https://orcid.org/0000-0002-6874-9187

Motivation

- **Challenge**: The existing equivalent resistance formula for a cobweb resistance network (e.g., Tan and Fang, 2015) is computationally inefficient for large-scale networks due to scalability and performance limitations.
- **Need**: An optimized equivalent resistance formula is urgently needed to enhance computational efficiency under complex boundary conditions in a cobweb resistance network.
- **Application**: Harnessing the unique potential function of a cobweb resistance network enables efficient robot path planning in complex cobweb environments with obstacles.
- **Goal**: Utilize mathematical tools, such as Chebyshev polynomials, to simplify formula derivation and significantly boost computational performance.

Main idea

- **Optimized Formula**: Re-express the equivalent resistance formula for an $m \times n$ cobweb resistance network using Chebyshev polynomials of the second kind combined with hyperbolic functions.
- **Key Innovation**: Leverage the approximation properties of Chebyshev polynomials to simplify derivation and enhance computational efficiency for large-scale networks.
- **Practical Application**: Develop a heuristic path planning algorithm for robots in a cobweb resistance network, utilizing its potential function for efficient navigation in environments with obstacles.
- **Core Outcome**: Achieve over three-fold improvement in computational efficiency and realize the application of cobweb resistance network potential function in path planning.

Method – Formula Optimization

1. Starting Point: Original equivalent resistance formula for an *m*×*n* cobweb network derived via the Recursive Transformation (RT) method (Eq. (1)).

2. Optimization Approach: Reformulate the expression using Chebyshev polynomials of the second kind and hyperbolic functions for computational efficiency.

- New formula (Eq. (7)):

$$R_{m\times n}\big((0,y_1),(x,y_2)\big)=\frac{2r}{m}\sum_{j=1}^{m}\frac{U_{n-1}^{(j)}(S_{1,j}^2+S_{2,j}^2)-2(U_{n-1}^{(j)}+U_{n-x-1}^{(j)})S_{1,j}S_{2,j}}{U_n^{(j)}-U_{n-2}^{(j)}-2}$$

where

$$U_k^{(j)}=\frac{\sinh\big((k+1)\psi_j\big)}{\sinh\psi_j},\quad \cosh\psi_j=\frac{\kappa_j}{2},\quad \kappa_j=2+2\frac{r}{r_0}\left(1-\cos\frac{(2j-1)\pi}{2m}\right)$$

3. Benefit: Simplify computations and enhances scalability for large-scale networks.

Method – Derivation Process

1. Horadam Sequence: Represent recursive relations in the formula using the Horadam sequence, defined as

$$W_k = dW_{k-1} - qW_{k-2},$$

- Expressed via Chebyshev polynomials of the second kind:

$$W_k = (\sqrt{q})^k \left(\frac{B}{\sqrt{q}} U_{k-1}\left(\frac{d}{2\sqrt{q}}\right) - A\, U_{k-2}\left(\frac{d}{2\sqrt{q}}\right)\right).$$

2. Key Derivations:

- Link to Chebyshev polynomials: $F_k^{(j)} = \frac{\lambda_j^k - \bar{\lambda}_j^k}{\lambda_j - \bar{\lambda}_j} = U_{k-1}^{(j)}\left(\frac{\kappa_j}{2}\right),$
- Simplify denominator: $B_n^{(j)} = \lambda_j^n + \bar{\lambda}_j^n = U_n\left(\frac{\kappa_j}{2}\right) - U_{n-2}\left(\frac{\kappa_j}{2}\right).$

3. Outcome: Streamlined derivation reduces computational complexity.

Method – Path Planning Algorithm

1. Potential Function: Model the potential distribution in a cobweb resistance network (based on Zhao et al. (2023)):

$$\frac{\mathbf{U}_{m\times n}(x,y)}{J}=\frac{4r}{2m+1}\sum_{\iota=1}^{m}\frac{\mu_{x_1,x}^{(\iota)}S_{y_1,\iota}-\mu_{x_2,x}^{(\iota)}S_{y_2,\iota}}{U_n^{(\iota)}-U_{n-2}^{(\iota)}-2}S_{y,\iota},$$

2. Algorithm Steps:

① Map the cobweb environment as a grid, identifying start, target, and obstacle nodes.

② Compute node potentials using the potential function.

③ Assign higher potential weights to obstacle nodes (e.g., add 0.3 $\frac{U(x,y)}{J}$).

④ Move the robot to the neighbor node with the minimum potential.

⑤ Repeat until the target is reached.

3. Validation: Simulate in a 10×16 cobweb network with obstacles to test obstacle avoidance.

Major Results – Computational Efficiency

◇ **Performance Comparison**: Compare the original (Eq. (1)) and optimized (Eq. (7)) equivalent resistance formulas across various network scales (m×n) with h=r/r_0=1, 10, 0.1.

Table 1 Comparison of calculation efficiency for equivalent resistance formulas (1) and (7) at $h = 1^*$

$m \times n$	r	r_0	t_1 (s)	t_2 (s)
100 × 100	1	1	0.7799	0.2353
500 × 500	1	1	92.0133	24.0147
1000 × 500	1	1	380.6860	98.6743
800 × 800	1	1	357.9222	91.4097
1000 × 1000	1	1	652.8197	180.2885
1500 × 1000	1	1	–	404.4679

Table 2 Comparison of calculation efficiency for equivalent resistance formulas (1) and (7) at $h = 10^*$

$m \times n$	r	r_0	t_1 (s)	t_2 (s)
100 × 100	1	0.1	0.7827	0.2290
500 × 500	1	0.1	78.4835	21.8411
1000 × 500	1	0.1	316.9167	88.1708
800 × 800	1	0.1	300.1308	84.5293
1000 × 1000	1	0.1	577.8685	187.3093
1500 × 1000	1	0.1	–	369.5674

Table 3 Comparison of calculation efficiency for equivalent resistance formulas (1) and (7) at $h = 0.1^*$

$m \times n$	r	r_0	t_1 (s)	t_2 (s)
100 × 100	0.1	1	0.7750	0.3874
500 × 500	0.1	1	113.9321	25.6996
1000 × 500	0.1	1	385.4453	101.4138
800 × 800	0.1	1	400.6745	103.6280
1000 × 1000	0.1	1	786.0421	206.0119
1500 × 1000	0.1	1	–	458.0859

Major Results – Path Planning

- **Path Planning Simulation**: Perform the test in a 10×16 cobweb network with obstacles. Robot successfully navigated from start to target, avoiding obstacles using potential-weighted nodes.

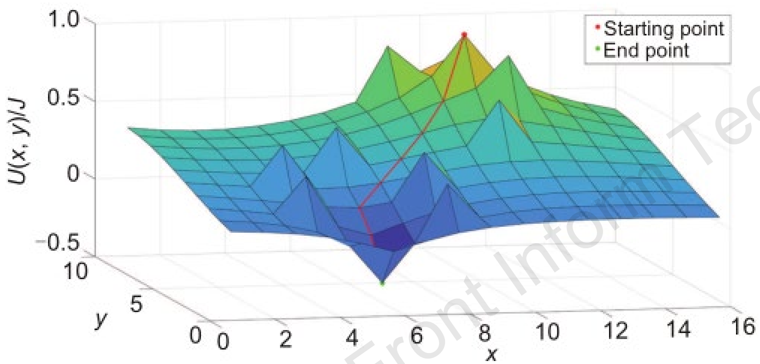


Fig. 13 Path planning in a node-weighted potential distribution map

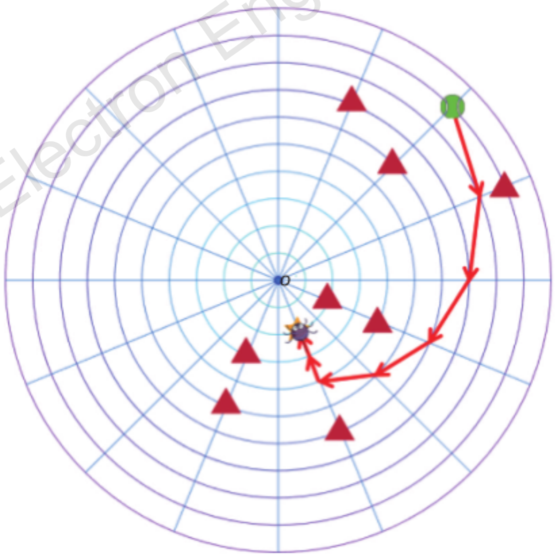


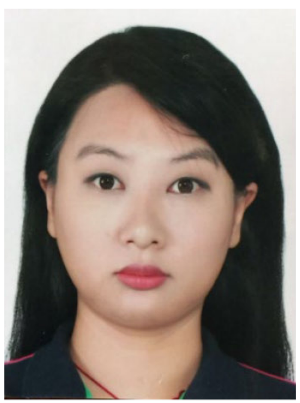
Fig. 14 Robot path planning in a physical cobweb environment. The green ball represents the starting point, the yellow five-pointed star represents the end point, and the red triangles represent the obstacles. The purple spider represents the robot, and the solid lines with red arrows represent the robot's path.

Conclusions

- **Formula Optimization**: Develop an efficient equivalent resistance formula for an $m \times n$ cobweb resistance network using Chebyshev polynomials, achieving over three-fold computational efficiency improvement.
- **Practical Impact**: Propose a heuristic path planning algorithm, leveraging the potential function of a cobweb resistance network, enabling effective robot navigation in environments with obstacles.
- **Validation**: Demonstrate scalability through large-scale network computations and successful path planning in a 10×16 cobweb simulation.
- **Future Directions**: Explore gradient descent or subgoal strategies for enhanced path planning and extend the application to other complex network topologies.

Author Bio

Yu GUAN received the B.S. degree in computer application engineering, from Shandong Vocational University of Foreign Affairs, China, in 2023. She is pursuing the M.S. degree in Department of Computer Science, Linyi University, China. Her research focuses on resistor networks, large-scale scientific engineering calculation, and machine learning.



Xiaoyu JIANG received the B.S. degree in information communication and the Ph.D. degree in computer science from the University of Suwon, Hwaseong, South Korea, in 2013 and 2018, respectively. She is currently an associate professor in School of Information Science and Engineering, Linyi University, China. Her research interests focus mainly on the intersection of computer and mathematics, such as resistor networks, large-scale scientific engineering calculation, and image processing.

Author Bio

Yanpeng ZHENG received his B.S. degree in information communication and M.S. and Ph.D. degrees in computer science, all from the University of Suwon, in 2013 to 2018. He is now an associate professor in School of Automation and Electrical Engineering of Linyi University, China. His research interests focus mainly on the intersection of computer and mathematics, such as large-scale scientific engineering calculation, big data analysis and processing, image processing, and resistor network.

Zhaolin JIANG received his B.S. and M.S. degrees from Qufu Normal University, China in 1986 and 1993, respectively. He received the Ph.D. degree from Xidian University, China in 2003. From 2004 to 2006, he was a postdoctoral researcher in Xian Jiaotong University, China. He has been a professor since 2001. He has been working at Linyi University, China from 1986 to present. He focuses on the theory, algorithm and application about special matrix, numerical linear algebra, resistor network, image and signal processing, and so on.