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Passivity-based synchronous control of Markov jump systems with actuator saturation

Key words: Markov jump systems; Synchronous controller; Actuator saturation; Passivity; Linear matrix inequalities

Corresponding author: Liqing WANG
E-mail: liqingwang@zju.edu.cn
ORCID: https://orcid.org/0000-0002-0224-3909

Motivation

1. Markov jump systems (MJSs), capable of simultaneously characterizing both external environmental variations and internal parameter transitions, have been widely applied in networked control systems.

2. Actuator saturation, as a common nonlinear constraint in practical systems, may lead to performance degradation or even instability.

3. While most existing studies focus on H_∞ control or asynchronous control, there has been relatively little research integrating saturation constraints with passivity theory, particularly under a synchronous control framework. This paper aims to develop a passivity-based synchronous control approach to ensure mean-square stability and stochastic passivity of MJSs subject to actuator saturation.

Main idea

1. A mode-dependent (MD) synchronization controller is proposed whose modes are fully synchronized with the system modes.

2. By introducing a saturation-dependent Lyapunov function and linear matrix inequality (LMI) techniques, sufficient conditions are derived, under which the system is mean-square stable and stochastically passive within the domain of attraction.

3. The domain of attraction of the system is estimated via a convex optimization problem.

4. The MD controller is degenerated into a mode-independent (MID) controller, which is analyzed as a special case for comparison.

Method

1. By introducing a Lyapunov function associated with the system modes and saturation function and combining LMI techniques, sufficient conditions are derived, ensuring that the system is mean-square stable and stochastically passive within the domain of attraction.

The stochastic Lyapunov function that depends on both saturation and system mode is

$$\mathcal{V}(k) = \boldsymbol{x}^{\mathrm{T}}(k)\left(\sum_{l=1}^{2^m}\eta_l(k)\boldsymbol{P}_{il}\right)\boldsymbol{x}(k)$$

Method (Cont'd)

2. An auxiliary matrix $\boldsymbol{G}_i$ is introduced to apply a congruence transformation, thereby decoupling the coupled variables and enhancing numerical tractability. In the original condition, the terms involving $\boldsymbol{P}_{jt}$ are multiplied by $\boldsymbol{K}_i$ and $\boldsymbol{H}_i$, separately. By defining $\boldsymbol{X}_i = \boldsymbol{K}_i\boldsymbol{G}_i$ and $\boldsymbol{Y}_i = \boldsymbol{H}_i\boldsymbol{G}_i$, these nonlinear terms are linearized. This reformulation yields a set of LMIs (22)−(24) that can be solved simultaneously in a single step.

Method (Cont'd)

Theorem 1 If there exist $P_{il} > 0$ and scalar $\gamma > 0$ satisfying expressions (10)–(12):

$$\tilde{A}_{il}^{\mathrm{T}} - \tilde{B}_{il}^{\mathrm{T}} \tilde{F}_i^{-1} \tilde{B}_{il} - P_{il} < 0, \tag{10}$$

$$\tilde{F}_i \leq 0, \tag{11}$$

$$\Omega(P_{il}) \subset \mathcal{L}(H_i), \tag{12}$$

the MJSs in Eq. (1) are mean-square stable and stochastically passive. Additionally, the set $\bigcap_{i=1}^{N} \bigcap_{l=1}^{2^m} \Omega(P_{il})$ is referred to as the estimation of the DoA-MSS, where

$$\tilde{A}_{il} = A_{il}^{\mathrm{T}} \sum_{j=1}^{N} \pi_{ij} P_{jt} A_{il},$$
$$\tilde{B}_{il}^{\mathrm{T}} = A_{il}^{\mathrm{T}} \sum_{j=1}^{N} \pi_{ij} P_{jt} F_i - C_{il}^{\mathrm{T}},$$
$$\tilde{F}_i = F_i^{\mathrm{T}} \sum_{j=1}^{N} \pi_{ij} P_{jt} F_i - \gamma I,$$
$$A_{il} = A_i + B_i(M_l K_i + M_l^{-} H_i),$$
$$C_{il} = C_i + D_i(M_l K_i + M_l^{-} H_i).$$

$\mathcal{X}_i = K_i G_i$

$\mathcal{Y}_i = H_i G_i$

→ Congruence transformation

Theorem 2 If there exist $\bar{P}_{ij} > 0$, $\bar{R}_{ij} > 0$, $G_i > 0$, and scalar $\gamma > 0$ satisfying

$$\begin{bmatrix} \bar{R}_{il} - G_i^{\mathrm{T}} - G_i & \mathcal{C}_{il}^{\mathrm{T}} & \hat{\mathcal{A}}_{il}^{\mathrm{T}} \\ * & -\gamma I & \hat{\mathcal{F}}_i^{\mathrm{T}} \\ * & * & \bar{\mathcal{P}}_{it} \end{bmatrix} < 0, \tag{22}$$

$$\begin{bmatrix} -\bar{P}_{il} & \bar{P}_{il}^{\mathrm{T}} \\ * & -\bar{R}_{il} \end{bmatrix} < 0, \tag{23}$$

$$\begin{bmatrix} \bar{P}_{il} - G_i^{\mathrm{T}} - G_i & y_{iq}^{\mathrm{T}} \\ * & -1 \end{bmatrix} \leq 0, \tag{24}$$

the MJSs in Eq. (1) are mean-square stable and stochastically passive. Here, y_{iq} denotes the q^{th} row of $\mathcal{Y}_i$. The set $\bigcap_{i=1}^{N} \bigcap_{l=1}^{2^m} \Omega(P_{il})$ is referred to as the estimation of DoA-MSS, where

$$\hat{\mathcal{A}}_{il}^{\mathrm{T}} = \begin{bmatrix} \mathcal{A}_{il}^{\mathrm{T}} & \mathcal{A}_{il}^{\mathrm{T}} & \ldots & \mathcal{A}_{il}^{\mathrm{T}} \end{bmatrix},$$
$$\hat{\mathcal{F}}_i^{\mathrm{T}} = \begin{bmatrix} F_i^{\mathrm{T}} & F_i^{\mathrm{T}} & \ldots & F_i^{\mathrm{T}} \end{bmatrix},$$
$$\bar{\mathcal{P}}_{it} = -\mathrm{diag}\left\{ \pi_{i1} \bar{P}_{1t} \quad \pi_{i2} \bar{P}_{2t} \quad \ldots \quad \pi_{iN} \bar{P}_{Nt} \right\},$$
$$\bar{P}_{il} = P_{il}^{-1}, \ \bar{R}_{il} = R_{il}^{-1},$$
$$\mathcal{A}_{il}^{\mathrm{T}} = G_i^{\mathrm{T}} A_i^{\mathrm{T}} + \mathcal{X}_i^{\mathrm{T}} M_l^{\mathrm{T}} B_i^{\mathrm{T}} + \mathcal{Y}_i^{\mathrm{T}} (M_l^{-})^{\mathrm{T}} B_i^{\mathrm{T}},$$
$$\mathcal{C}_{il}^{\mathrm{T}} = -G_i^{\mathrm{T}} C_i^{\mathrm{T}} - \mathcal{X}_i^{\mathrm{T}} M_l^{\mathrm{T}} D_i^{\mathrm{T}} - \mathcal{Y}_i^{\mathrm{T}} (M_l^{-})^{\mathrm{T}} D_i^{\mathrm{T}},$$
$$\mathcal{X}_i = K_i G_i, \ \mathcal{Y}_i = H_i G_i.$$

Method (Cont'd)

3. To estimate the attraction domain of the system, the conditions are transformed into an optimization problem, where the estimate of the domain attraction is maximized by minimizing ρ.

Optimization problem 2:

$$\inf \rho$$

$$\bar{\boldsymbol{P}}_{il} > 0,\ \bar{\boldsymbol{R}}_{il} > 0,\ \boldsymbol{G}_i > 0,\ \boldsymbol{\mathcal{X}}_i > 0,\ \boldsymbol{\mathcal{Y}}_i > 0$$

$$\text{s.t. (a)} \begin{bmatrix} -\rho & \boldsymbol{x}^{\mathrm{T}}(0) \\ * & -\bar{\boldsymbol{P}}_{il} \end{bmatrix} \leq 0,$$

(b) LMIs (22)–(24).

It means that $\boldsymbol{x}(0)$ is in the domain of attraction of the mean-square scene (DoA-MSS), if the optimal value $\rho_{\min} < 1$. Moreover, the smaller the value of $\rho_{\min}$, the closer the DoA-MSS to the real one.

Major results

Open-loop

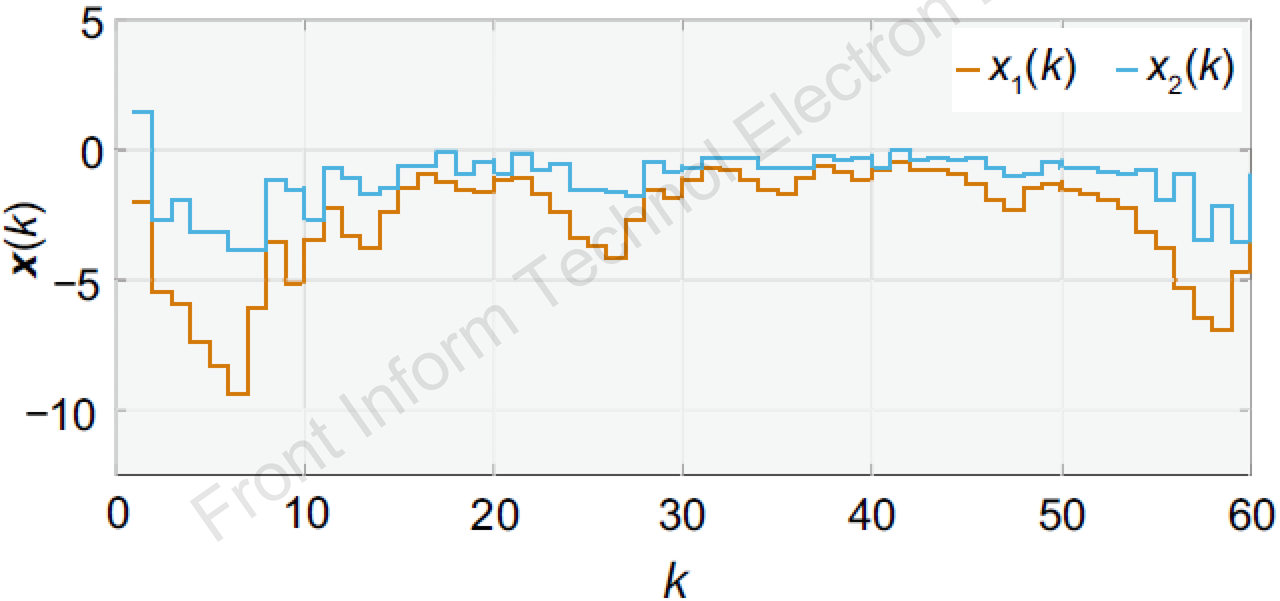

Fig. 1 System state without controller

Major results (Cont'd)

Example 1

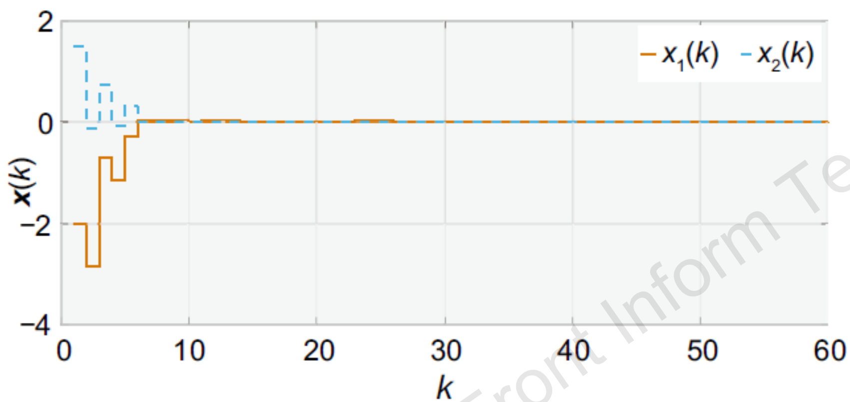

Fig. 2 System state under the MD controller in Example 1

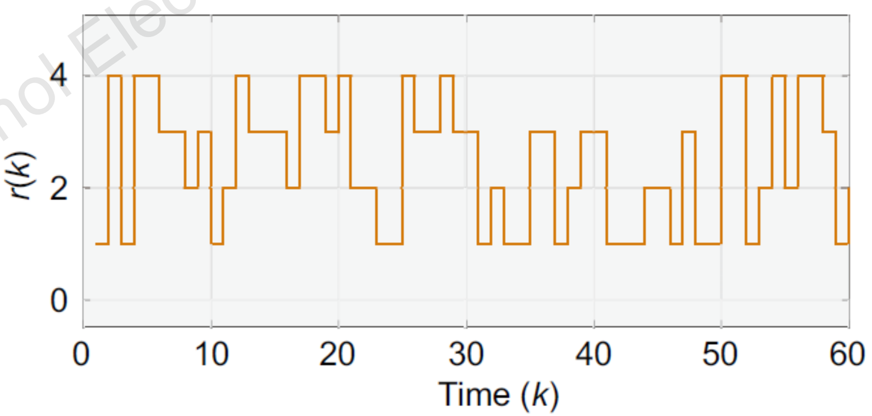

Fig. 3 System's mode evolution

Major results (Cont'd)

Example 2

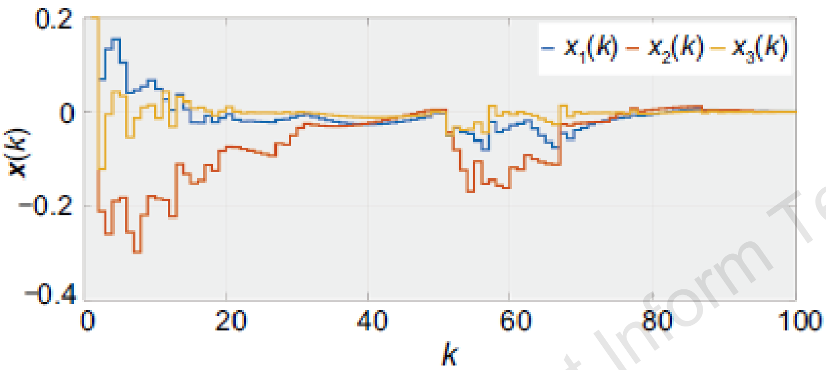

Fig. 4 System state under the MID controller in Example 2

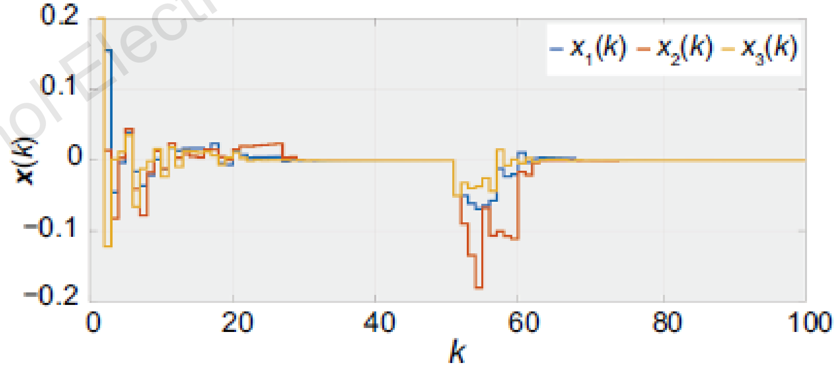

Fig. 5 System state under the MD controller in Example 2

Conclusions

1. A passivity-based synchronization control method is proposed, effectively addressing the robust control problem of MJSs with actuator saturation.

2. The domain of attraction is estimated through LMI and optimization techniques, which reduces the conservatism.

3. Simulation results demonstrate that the MD control outperforms the MID control, validating the effectiveness of the proposed theory.

Liqing WANG is an associate professor and master's supervisor at Zhejiang Sci-Tech University. She received her Ph.D. degree from the College of Control Science and Engineering, Zhejiang University in 2022. Her research direction is networked control and optimization. As the first author, she has published 12 SCI papers in IEEE Transactions, and has presided over one project of the National Natural Science Foundation of China (Class C) and one youth project of the Zhejiang Provincial Natural Science Foundation. She won the second prize for the doctoral dissertation in engineering of the 2024 Chinese Association of Automation Graduate Thesis.