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Privacy-preserving bipartite consensus with cooperative–competitive interactions via a node decomposition strategy

Key words: Privacy-preserving; Bipartite consensus; Cooperative–competitive interactions; Multi-agent systems; Node decomposition

Corresponding author: Shuai LIU
E-mail: liushuai871030@163.com
ORCID: https://orcid.org/0000-0003-0523-022X

Motivation

- This paper describes our investigation of the privacy protection problem of multi-agent systems under cooperative–competitive networks. The purpose is to design a privacy-preserving consensus algorithm such that the privacy performance is guaranteed by using the node decomposition strategy, while the bipartite consensus is achieved for the cooperative–competitive multi-agent systems.

Main idea

- Node v_i is decomposed into n_i sub-nodes, termed as homologous sub-nodes, where n_i denotes the number of neighboring nodes. These homologous sub-nodes are sequentially connected in a chain-like structure. Sub-nodes derived from different nodes are referred to as non-homologous sub-nodes, each connecting to exactly one non-homologous sub-node under predefined rules.
- By designing inter-node weights, the initial value of each node is protected from honest-but-curious nodes and eavesdroppers without relying on external algorithms.

Method

Consider a cooperative–competitive multi-agent system with n discrete-time agents as follows:

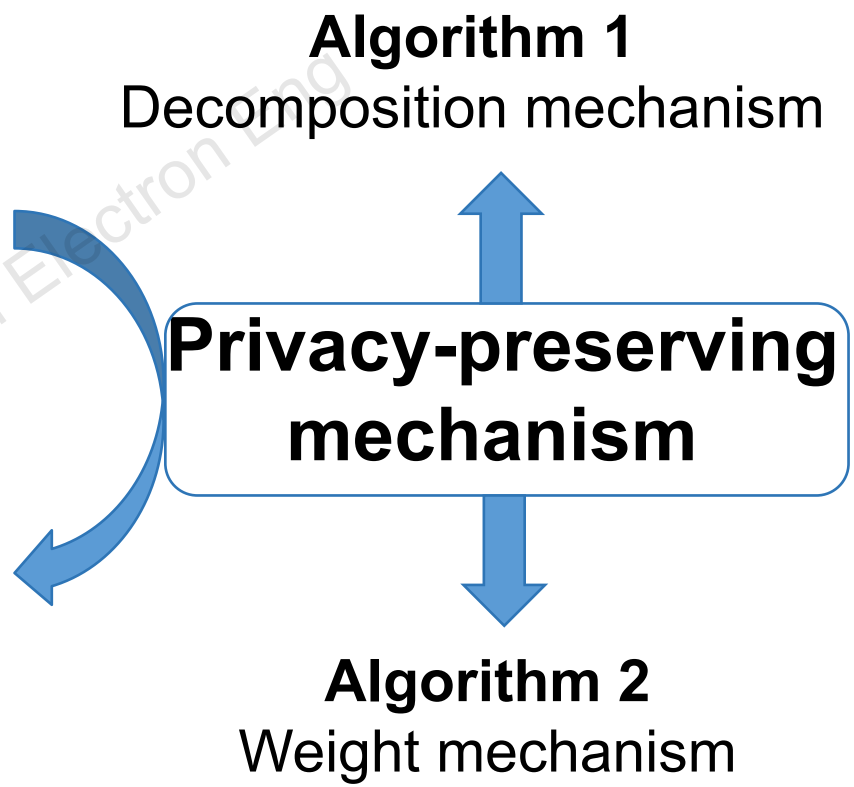
$$\boldsymbol{x}_i[k+1] = \boldsymbol{x}_i[k] + \varepsilon \boldsymbol{u}_i[k], \tag{1}$$

with the bipartite consensus protocol

$$\boldsymbol{u}_i[k] = -\sum_{v_j \in N_i} |a_{i,j}|(\boldsymbol{x}_i[k] - \operatorname{sgn}(a_{i,j})\boldsymbol{x}_j[k]), \tag{2}$$

Through this privacy-preserving mechanism, system (1) can be decomposed into the following form:

$$\begin{aligned} &x_{i,j}[k+1] \\ =&x_{i,j}[k] + \varepsilon \sum_{v_j \in N_i} |a_{i,j}|(\operatorname{sgn}(a_{i,j})x_{j,i}[k] - x_{i,j}[k]) \\ &+ \varepsilon \sum_{\substack{v_p \in N_i, \\ v_{i,p} \in N(v_{i,j})}} a_i^{p,j}(x_{i,p}[k] - x_{i,j}[k]). \end{aligned} \tag{4}$$



Method

Algorithm 1 Decomposition mechanism

Step 1: for any node v_i with $|N_i| = n_i$, the neighboring nodes of node v_i are sequentially represented as $v_1, v_2, \ldots, v_{n_i}$ and $1 < 2 < \ldots < n_i$. Now we decompose node v_i into n_i sub-nodes, represented as $v_{i,1}, v_{i,2}, \ldots, v_{i,n_i}$ and those n_i sub-nodes are called homologous nodes. Then, we can totally get $2N$ sub-nodes.

Step 2: according to the subscripts of the decomposed sub-nodes $1, 2, \ldots, n_i$, connect those sub-nodes in ascending order to obtain a chain graph, where those n_i sub-nodes are sequentially connected.

Step 3: perform the above two steps for each node in sequence.

Step 4: connect $v_{i,j}$ and $v_{j,i}$ where node $v_{i,j}$ is decomposed by node v_i and node $v_{j,i}$ is decomposed by node v_j. $v_{i,j}$ and $v_{j,i}$ are non-homologous sub-nodes. So, we can connect all the nodes with their neighbor nodes.

Step 5: define the sum of homologous child nodes that form one node as $\frac{2N}{n}$ of the original node, that is, $\sum\limits_{q \in N_i} x_{i,q}[0] = \frac{2N}{n} x_i[0]$.

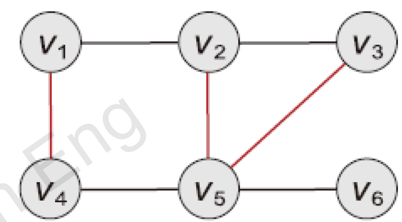


Fig. 1 Before node decomposition. References to color refer to the online version of this figure

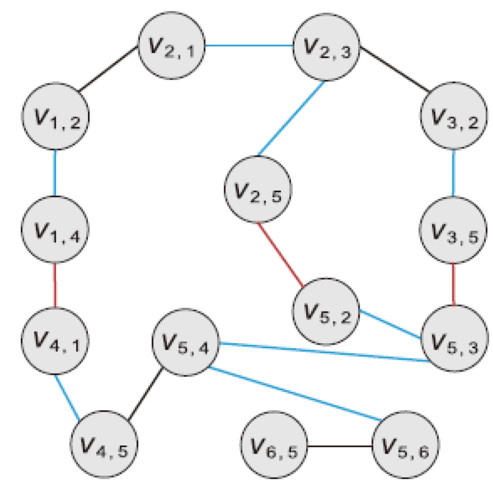


Fig. 2 After node decomposition. References to color refer to the online version of this figure

Method

Algorithm 2 Weight mechanism

Step 1: after node decomposition, the weight between non-homologous child nodes $v_{i,j}$ remains consistent with the weight $a_{i,j}$ between nodes v_i and v_j before node decomposition, represented by $a_{i,j}$. Because the graph studied here is undirected, $a_{i,j} = a_{j,i} \in (-\frac{1}{\sqrt{3\varepsilon}}, \frac{1}{\sqrt{3\varepsilon}})$.

Step 2: the weight between the homologous subnodes $v_{i,s}$ and $v_{i,r}$ is represented as $a_i^{r,s}$. The specific value of $a_i^{r,s}$ is designed in Section 4. In particular, the graph $\mathcal{G}$ is undirected, $a_i^{s,r} = a_i^{r,s} \in (0, \frac{1}{3\varepsilon})$.

Method

$$
\begin{cases}
\bar{a}_i^{m,j}[0] = \dfrac{a_i^{m,j}[0](x_{i,m}[0] - x_{i,j}[0])}{\frac{2N}{n}\bar{x}_i[0] - 2x_{i,j}[0]}, \\[2ex]
\bar{a}_m^{i,j}[0] = \dfrac{a_m^{i,j}[0](x_{m,i}[0] - x_{m,j}[0])}{\frac{2N}{n}\bar{x}_m[0] - 2x_{m,j}[0]}, \\[2ex]
\bar{a}_{i,m}[0] = \dfrac{\varepsilon a_{i,m}[0](x_{m,i}[0] - x_{i,m}[0])}{\varepsilon(\frac{2N}{n}\vec{x}[0] - x_{m,j}[0] + x_{i,j}[0])} \\[2ex]
\qquad\qquad + \dfrac{x_{i,m}[0] - \bar{x}_i[0] + x_{i,j}[0]}{\varepsilon(\frac{2N}{n}\vec{x}[0] - x_{m,j}[0] + x_{i,j}[0])}, \\[2ex]
\bar{a}_i^{p,q} = a_i^{p,q},\ p,q \neq m,j, \\
\bar{a}_m^{p,q} = a_m^{p,q},\ p,q \neq i,j, \\
\bar{a}_j^{p,q} = a_j^{p,q},\ v_p, v_q \in N_j, \\
\bar{a}_{p,q} = a_{p,q},\ p,q \neq i,m,
\end{cases}
$$

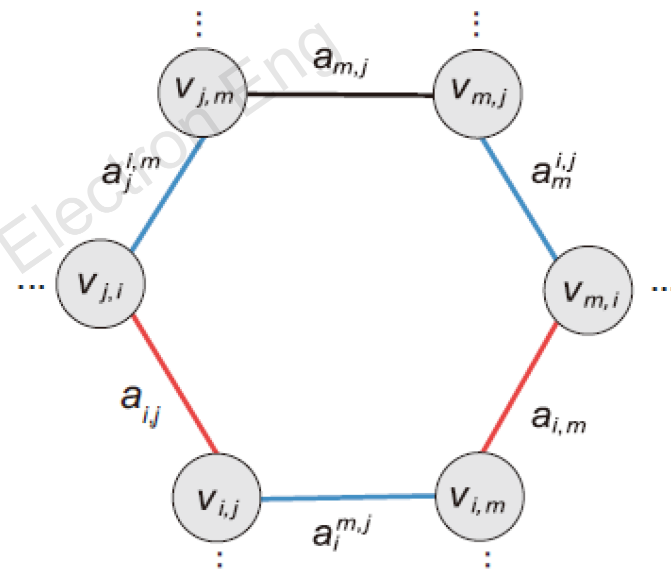


Fig. 8 Topological graph of the original structure decomposed by Algorithm 1

Results

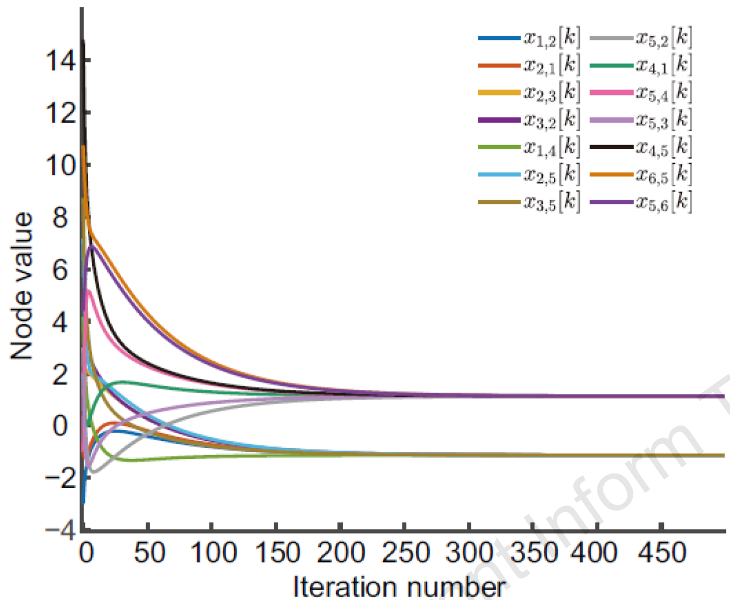


Fig. 9 The bipartite consensus after the node decomposing strategy. The 14 nodes obtained from node decomposition can still achieve bipartite consensus, which is the same as the bipartite consensus before decomposition

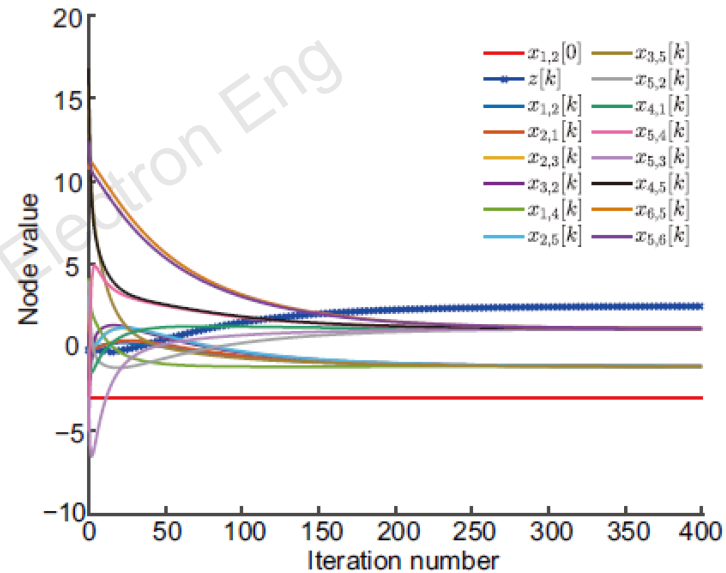


Fig. 11 The bipartite consensus after the privacy-preserving mechanism with the honest-but-curious node. The red horizontal solid line represents the initial value of $v_{1,2}$. Honest-but-curious node z cannot infer the initial value $x_{1,2}[0]$ under Algorithms 1 and 2. References to color refer to the online version of this figure

Results

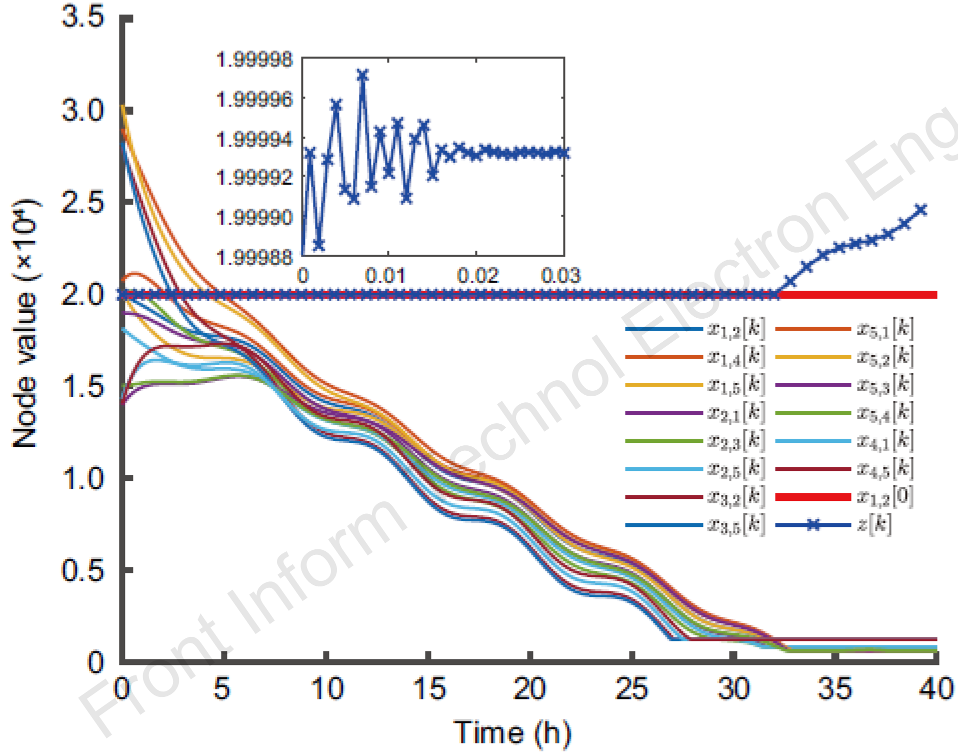


Fig. 13 The bipartite consensus after the privacy-preserving mechanism for BESSs with the eavesdropper. The red horizontal solid line represents the initial value of $\mathrm{BESS}_{1,2}$. Eavesdropper z can infer the initial state $x_{1,2}[0]$. References to color refer to the online version of this figure

Conclusions

This study addresses the privacy-protection-based consensus problem for cooperative–competitive multi-agent systems. In undirected signed graphs, a privacy-preserving mechanism has been devised to safeguard node privacy while ensuring bipartite consensus, thus thwarting honest-but-curious nodes and eavesdroppers from accessing the nodes' initial values. The proposed privacy-preserving mechanism encompasses the decomposition mechanism and the weight mechanism.

Licheng WANG received his PhD degree in control science and control engineering from the University of Shanghai for Science and Technology, Shanghai, China. From November 2016 to November 2018, he was a visiting PhD student with the Department of Electronic and Computer Engineering, Brunel University London, Uxbridge, UK. He is currently with the College of Automation Engineering, Shanghai University of Electric Power. His research interests include nonlinear stochastic control and filtering, distributed optimization theory, and their applications in power systems.

Yongling CHEN received her BS degree from Xichang University, Sichuang, China, in 2021. She is currently pursuing her MS degree in operations research and cybernetics with the University of Shanghai for Science and Technology. Her research interests include privacy-preserving problems in multi-agent systems.

Shuai LIU received her PhD degree in system analysis and integration from the University of Shanghai for Science and Technology in 2018. From 2017 to 2018, she was a visiting PhD student at the Department of Computer Science, Brunel University London, Uxbridge, UK. She is currently an associate research professor with the College of Science, University of Shanghai for Science and Technology. She was a recipient of the National Postdoctoral Innovative Talent Scholars of China. She has authored or co-authored more than 30 articles in refereed international journals. Her current research interests include sensor networks, complex networks, information fusion, and Kalman filtering.