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Waveform design based on mutual information upper bound for joint detection and estimation

Key words: Radar waveform design; Mutual information upper bound; Target detection; Parameter estimation; Constant modulus constraint

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Motivation

1. In practice, detection and estimation are interdependent, motivating joint optimization.
2. Multi-objective formulations provide partial solutions, while learning-based approaches suffer from complexity and generalization issues.
3. Information-theoretic designs have been explored, e.g., a two-stage scheme using mutual information (MI) and Kullback–Leibler (KL) divergence, yet a unified foundation remains lacking.

Main idea

1. Unified variational framework: Joint detection-estimation waveform design is formulated via mutual information upper bound (MIUB) maximization, unifying MI- and KL-based criteria.

2. Flexible statistical modeling: Gaussian mixture distributions (GMDs) are adopted for target and clutter, combining fidelity with analytical tractability.

3. Phase-coded dream optimization algorithm (PC-DOA): A constant modulus synthesis algorithm with hybrid initialization and adaptive dynamics is developed to enhance convergence and solution quality.

Method

1. The received signal y is modeled by the following binary hypothesis test.

$$\begin{cases} \mathcal{H}_0: & y = w, \\ \mathcal{H}_1: & y = Sx + w, \end{cases}$$

The corresponding likelihoods of y under two hypothesis:

$$p_0(y) \triangleq p(y \mid \mathcal{H}_0) = \sum_{k=1}^{K} \alpha_k \mathcal{CN}(y;\ 0,\ R_k),$$

$$p_1(y) \triangleq p(y \mid \mathcal{H}_1) = \sum_{\ell=1}^{L} \gamma_\ell \mathcal{CN}(y;\ 0,\ \Sigma_\ell),$$

The constant modulus constraint:

$$|s_n| = c = \sqrt{E_\text{s} / N}, \quad n = 1, 2, \ldots, N,$$

MIUB decomposition:

$$\text{MIUB}(s) = \underbrace{I(x; y)}_{\mathsf{E}(s)} + \underbrace{\mathcal{D}_{\text{KL}}\big(p_1(y) \| p_0(y)\big)}_{\mathsf{D}(s)},$$

Method (Cont’d)

2. The exact evaluation of MIUB under GMD likelihoods is generally intractable. Tractable approximation strategies: to circumvent this computational challenge, we adopt the following tractable approximation strategies.

- MI approximation (Lemma 2): the MI term can be approximated as $\overline{\mathsf{E}}(s) = \Xi\left(\{\alpha_k,\ R_k\}_{k=1}^{K}\right) - \Xi\left(\{\gamma_\ell,\ \Sigma_\ell\}_{\ell=1}^{L}\right)$,
- KL divergence approximation (Lemma 3): the KL divergence term admits the approximation $\overline{\mathsf{D}}(s) = \sum_{\ell=1}^{L} \gamma_\ell \left[\ln \frac{\gamma_\ell}{\alpha_{k^\star(\ell)}} + \mathcal{L}(\Sigma_\ell,\ R_{k^\star(\ell)})\right]$,

Final optimization problem (P1):

$$(\mathcal{P}_1): \begin{array}{ll} \max_s & F(s) \triangleq \overline{\mathsf{E}}(s) + \overline{\mathsf{D}}(s) \\ \text{s.t.} & |s_n| = c, \quad n = 1, 2, \ldots, N \end{array}$$

Method (Cont'd)

3. We propose the PC-DOA, a metaheuristic specifically designed for MIUB-based waveform design under constant modulus constraints.

Phase-coded representation: $s(\theta) = c \cdot [e^{j\theta_1}, e^{j\theta_2}, \ldots, e^{j\theta_N}]^T$.

Hybrid initialization strategy:

- LFM-inspired initialization: $[\theta^0_{\mathrm{LFM},i}]_n = \Psi\left(\beta_i \pi \frac{(n-1)^2}{N-1} + \Delta_{i,n}\right)$,
- Chaotic initialization: $\rho_n^k = 4\rho_n^{k-1}(1-\rho_n^{k-1})$, $[\theta^0_{\mathrm{chaos},i}]_n = \Psi\left(-\pi + 2\pi\rho_n^{K_{\mathrm{chaos}}}\right)$.
- Random initialization: $[\theta^0_{\mathrm{rand},i}]_n \sim \mathcal{U}(-\pi, \pi)$.

Exploration and exploitation:

- Exploration phase: $\theta_g^{\mathrm{best}} = \arg\max_{\theta \in \mathcal{G}_g} f(\theta)$.
- Exploitation phase: $[\theta_{\mathrm{new}}]_n = \Psi\left([\theta^{\mathrm{global}}]_n + \eta(t) \cdot \Delta_n\right), \quad n \in \mathcal{I}$.

Method (Cont'd)

4. The PC-DOA algorithm initializes a population using a hybrid strategy and finds the initial best solution θ^*. It then iterates T times, splitting the search into two phases: first, a broad exploration phase, followed by an exploitation phase that focuses the search around the current θ^* In each iteration, it evaluates the new candidates, selects the next generation, and updates θ^* if a better solution is found. Finally, it computes and returns the optimal waveform s^* based on the final θ^*

Algorithm 1 Phase-coded dream optimization algorithm

Input: Objective function $F : \mathcal{M} \to \mathbb{R}$, waveform parameters $\{N, c\}$, and algorithm parameters $\{N_\mathrm{p}, T, G, \eta, \alpha\}$
Output: Optimal phase vector $\boldsymbol{\theta}^*$ and waveform $\boldsymbol{s}^*$

Initialize Population $\mathcal{P}_0 = \{\boldsymbol{\theta}_1^0, \boldsymbol{\theta}_2^0, \ldots, \boldsymbol{\theta}_{N_\mathrm{p}}^0\}$ via hybrid strategy Eqs. (21)–(24), fitness $f(\boldsymbol{\theta}) \leftarrow F(\boldsymbol{s}(\boldsymbol{\theta}))$ for all $\boldsymbol{\theta} \in \mathcal{P}_0$, global best $\boldsymbol{\theta}^* \leftarrow \arg\max_{\boldsymbol{\theta}\in\mathcal{P}_0} f(\boldsymbol{\theta})$, and $f^* \leftarrow f(\boldsymbol{\theta}^*)$
for $t = 1$ to T **do**
  Initialize $\mathcal{P}_\mathrm{candidate} \leftarrow \varnothing$
  // $\mathcal{P}_\mathrm{candidate}$ denotes the candidate population
  if $t \le \alpha T$ **then**
    Partition $\mathcal{P}_{t-1}$ into G subgroups
    for each $\boldsymbol{\theta}_\mathrm{old} \in \mathcal{P}_{t-1}$ **do**
      Generate $\boldsymbol{\theta}_\mathrm{new}$ from $\boldsymbol{\theta}_\mathrm{old}$ via Eq. (28)
      $\mathcal{P}_\mathrm{candidate} \leftarrow \mathcal{P}_\mathrm{candidate} \cup \{\boldsymbol{\theta}_\mathrm{new}\}$
    end for
  else
    for each $\boldsymbol{\theta}_\mathrm{old} \in \mathcal{P}_{t-1}$ **do**
      Generate $\boldsymbol{\theta}_\mathrm{new}$ guided by $\boldsymbol{\theta}^*$ via Eq. (30)
      $\mathcal{P}_\mathrm{candidate} \leftarrow \mathcal{P}_\mathrm{candidate} \cup \{\boldsymbol{\theta}_\mathrm{new}\}$
    end for
  end if
  Evaluate fitness $f(\boldsymbol{\theta})$ for all $\boldsymbol{\theta} \in \mathcal{P}_\mathrm{candidate}$
  Select next-generation population $\mathcal{P}_t$ from $\mathcal{P}_{t-1} \cup \mathcal{P}_\mathrm{candidate}$
  if $\max_{\boldsymbol{\theta}\in\mathcal{P}_t} f(\boldsymbol{\theta}) > f^*$ **then**
    $\boldsymbol{\theta}^* \leftarrow \arg\max_{\boldsymbol{\theta}\in\mathcal{P}_t} f(\boldsymbol{\theta})$
    $f^* \leftarrow f(\boldsymbol{\theta}^*)$
  end if
end for
Compute optimal waveform $\boldsymbol{s}^* \leftarrow c \cdot \exp(\mathrm{j}\boldsymbol{\theta}^*)$
Return $\boldsymbol{\theta}^*$ and $\boldsymbol{s}^*$

Major results

Comparison of optimization objectives

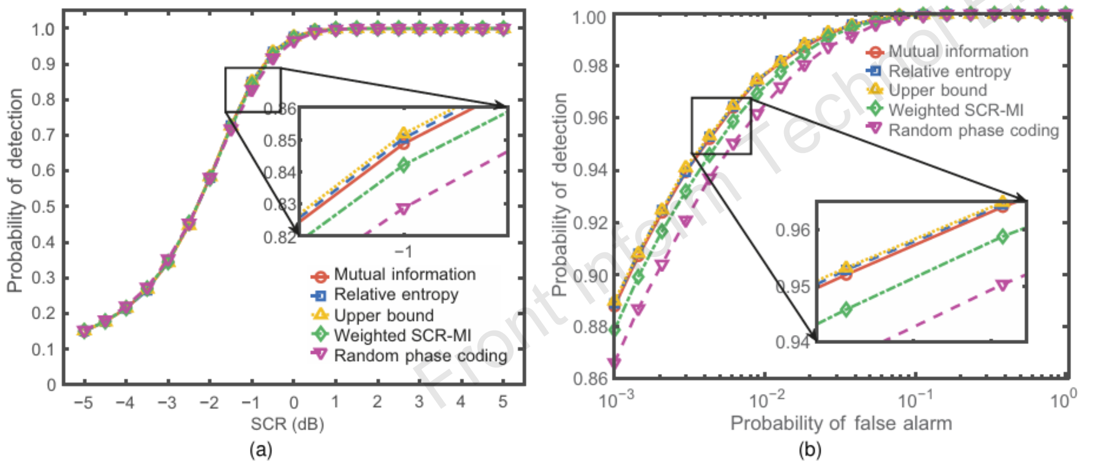

Fig. 2 Detection performance of waveforms using different criteria: (a) detection probability P_{D} vs. SCR ($P_{\mathrm{FA}} = 10^{-2}$); (b) ROC curves ($P_{\mathrm{D}}$ vs. P_{FA}) at SCR = 0 dB

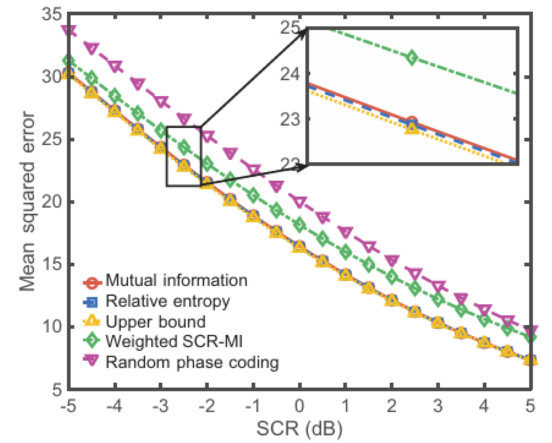

Fig. 3 MSE of TIR estimation vs. SCR for waveforms using different criteria

Major results (Cont'd)

Comparison of optimization objectives

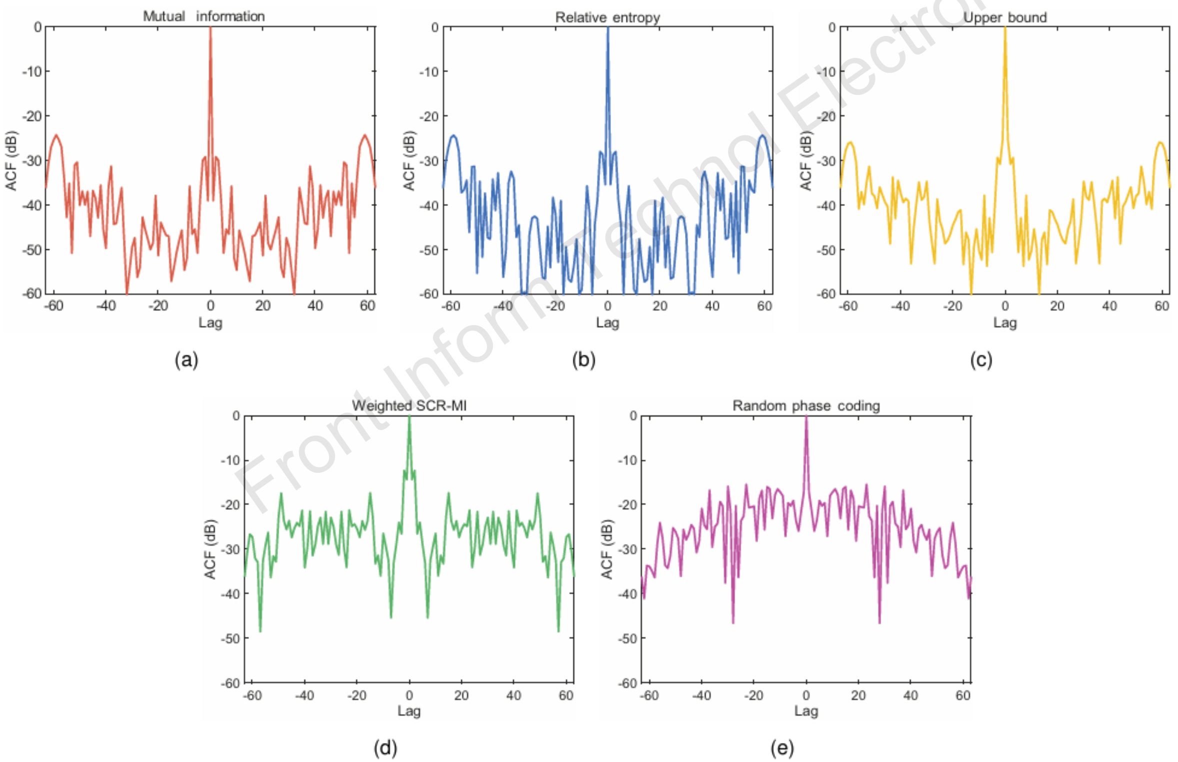

Fig. 4 Autocorrelation functions of waveforms using different criteria: (a) MI; (b) RE; (c) MIUB; (d) WSM; (e) RPC

Major results (Cont'd)

Comparison of optimization objectives

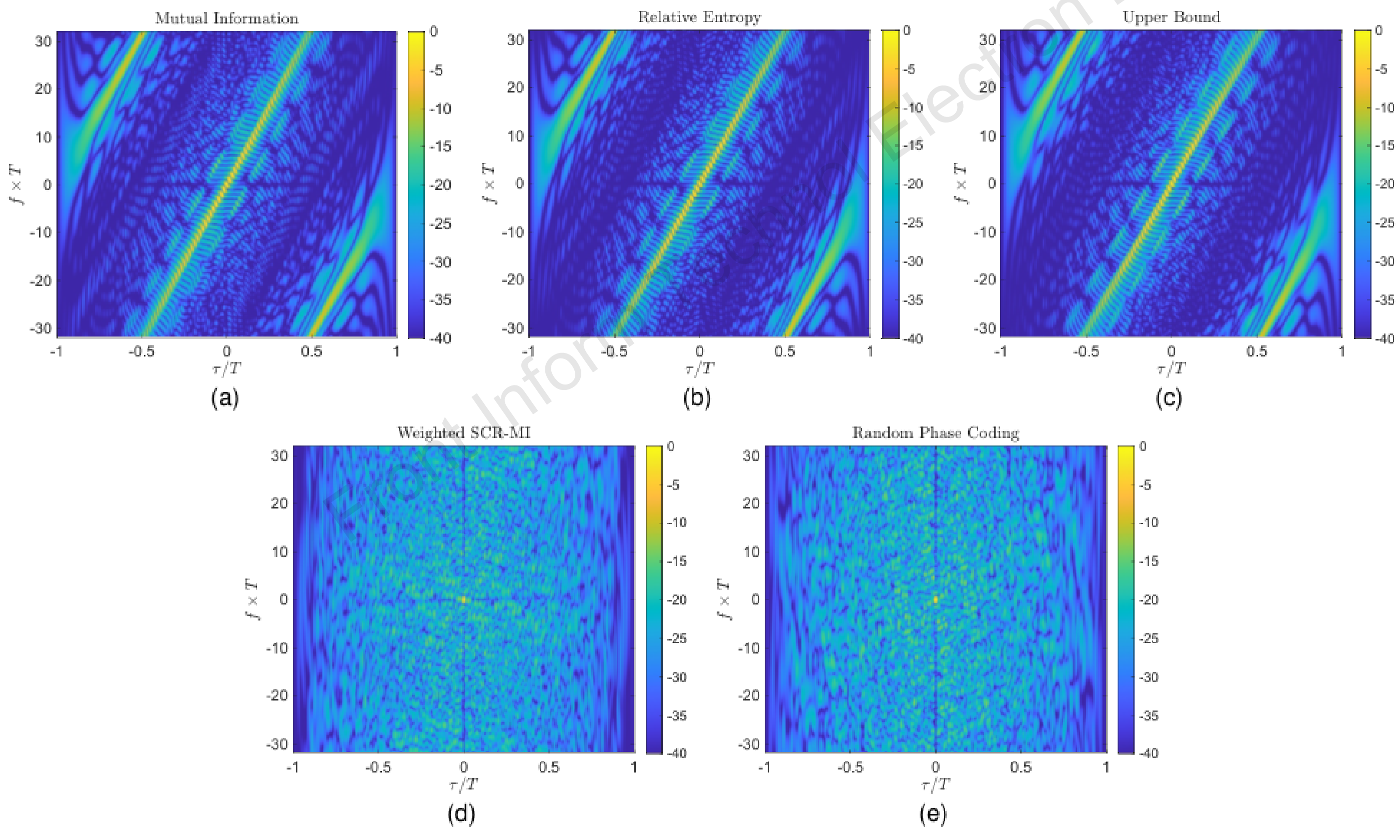

Fig. 5 Ambiguity functions of waveforms using different criteria: (a) MI; (b) RE; (c) MIUB; (d) WSM; (e) RPC

Major results (Cont'd)

Comparison of optimization algorithms

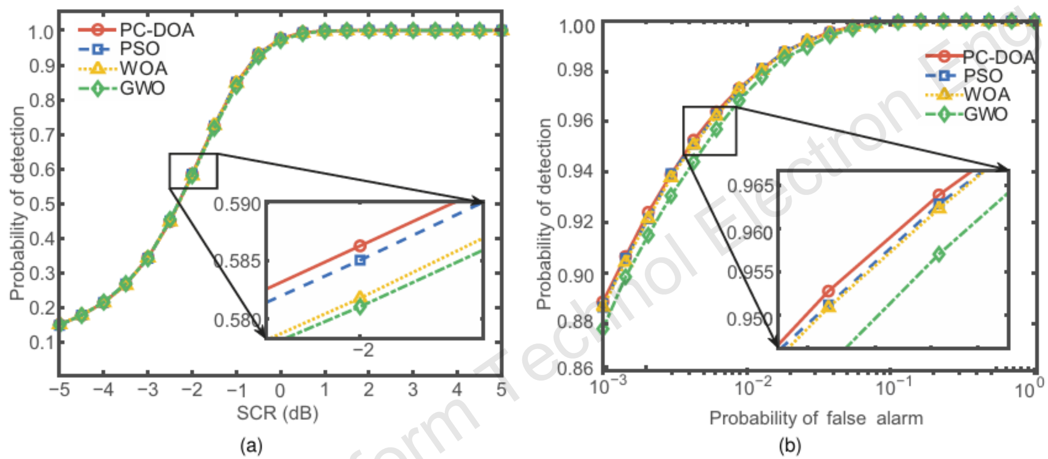

Fig. 6 Detection performance of waveforms using different optimization algorithms: (a) detection probability vs. SCR ($P_{FA} = 10^{-2}$); (b) ROC curves (P_D vs. P_{FA}) at SCR = 0 dB

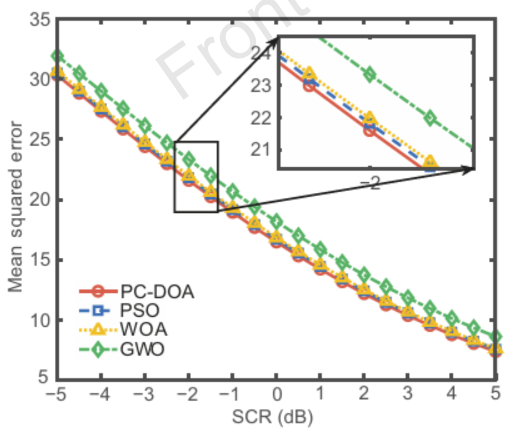

Fig. 7 MSE of TIR estimation vs. SCR of wavefor using different optimization algorithms

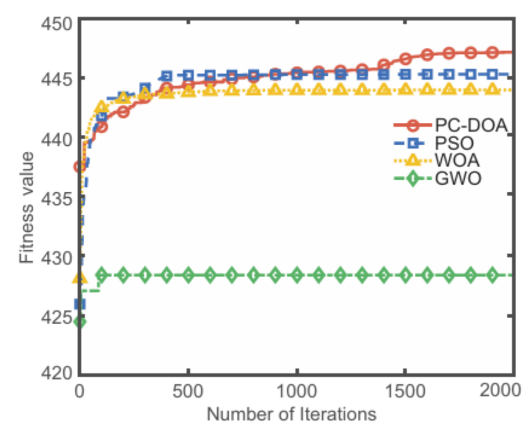

Fig. 8 Convergence of best fitness $F(s)$ optimized using different optimization algorithms

Conclusions

1. This work presents a unified information-theoretic framework for constant modulus radar waveform design, jointly optimizing detection and estimation via an MIUB under Gaussian mixture models.

2. To address the ensuing nonconvex optimization, we propose PC-DOA, which leverages hybrid initialization and adaptive bi-phase search on the complex circle manifold.

3. Numerical evaluations confirm that MIUB-based designs outperform conventional baselines, offering superior detection-estimation trade-offs and desirable ambiguity function properties, thereby establishing a principled foundation for future extensions.

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