Virtual internal thermal work evaluation in the multifield variational statements for the analysis of multilayered structures

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Main goal of this paper

The virtual internal thermal work in the principle of virtual displacements (PVD) extended to elasto-thermo-electric problems changes its form depending on the analysis performed and the load applied.

Results about multilayered plates and shells suggest the appropriate extension of the variational statement for each analysis, and they give an exhaustive explanation for several forms of the PVD proposed.

Thermal effects are evaluated in the cases of free vibration and static analysis (applied mechanical load or imposed electric potential) of multilayered structures.

Computational model and results

General form of PVD for the thermo-electro-mechanical case

$$\int_V (\delta \boldsymbol{\varepsilon}^{\mathrm{T}} \boldsymbol{\sigma} - \delta \boldsymbol{E}^{\mathrm{T}} \boldsymbol{D} - \delta\theta\eta - \delta \boldsymbol{\Theta}^{\mathrm{T}} \boldsymbol{h})\mathrm{d}V = \delta L_{\mathrm{e}} - \delta L_{\mathrm{in}},$$

Governing equation for the thermo-electro-mechanical case

$$\begin{cases} \delta \boldsymbol{u} : \boldsymbol{K}_{uu} \boldsymbol{u} + \boldsymbol{K}_{u\phi} \phi + \boldsymbol{K}_{u\theta} \theta = \boldsymbol{p}_u - \boldsymbol{M}_{uu} \ddot{\boldsymbol{u}}, \\ \delta\phi : \boldsymbol{K}_{\phi u} \boldsymbol{u} + \boldsymbol{K}_{\phi\phi} \phi + \boldsymbol{K}_{\phi\theta} \theta = \boldsymbol{p}_\phi, \\ \delta\theta : \boldsymbol{K}_{\theta u} \boldsymbol{u} + \boldsymbol{K}_{\theta\phi} \phi + \boldsymbol{K}_{\theta\theta} \theta = \boldsymbol{p}_\theta, \end{cases}$$

Constitutive equations for the thermo-electro-mechanical case

$$\begin{aligned} \boldsymbol{\sigma}^k &= \boldsymbol{Q}^k \boldsymbol{\varepsilon}^k - \boldsymbol{e}^{k\mathrm{T}} \boldsymbol{E}^k - \boldsymbol{\lambda}^k \theta^k, \\ \boldsymbol{D}^k &= \boldsymbol{e}^k \boldsymbol{\varepsilon}^k + \boldsymbol{\mu}^k \boldsymbol{E}^k + \boldsymbol{p}^k \theta^k, \\ \eta^k &= \boldsymbol{\lambda}^{k\mathrm{T}} \boldsymbol{\varepsilon}^k + \boldsymbol{p}^{k\mathrm{T}} \boldsymbol{E}^k + \chi^k \theta^k, \\ \boldsymbol{h}^k &= \boldsymbol{\kappa}^k \boldsymbol{\Theta}^k, \end{aligned}$$

Table 5 Problem 5. Transverse displacement w at z=0 for transverse mechanical pressure applied in the static analysis of the two-layered piezoelectric shell (unit: 10^{-9} m)

PVD-u	PVD-$u\phi$	PVD-$u\theta$-1	PVD-$u\theta$-2
3.6580	3.5001	3.6571	3.6580
PVD-$u\theta$-3	**PVD-$u\phi\theta$-1**	**PVD-$u\phi\theta$-2**	**PVD-$u\phi\theta$-3**
3.6573	3.4993	3.5001	3.4994

PVD-u: pure mechanical case
PVD-uϕ: electro-mechanical case
PVD-uθ-1: first version of thermo-mechanical case
PVD-uθ-2: second version of thermo-mechanical case
PVD-uθ-3: third version of thermo-mechanical case
PVD-uϕθ-1: first version of thermo-electro-mechanical case
PVD-uϕθ-2: second version of thermo-electro-mechanical case
PVD-uϕθ-3: third version of thermo-electro-mechanical case

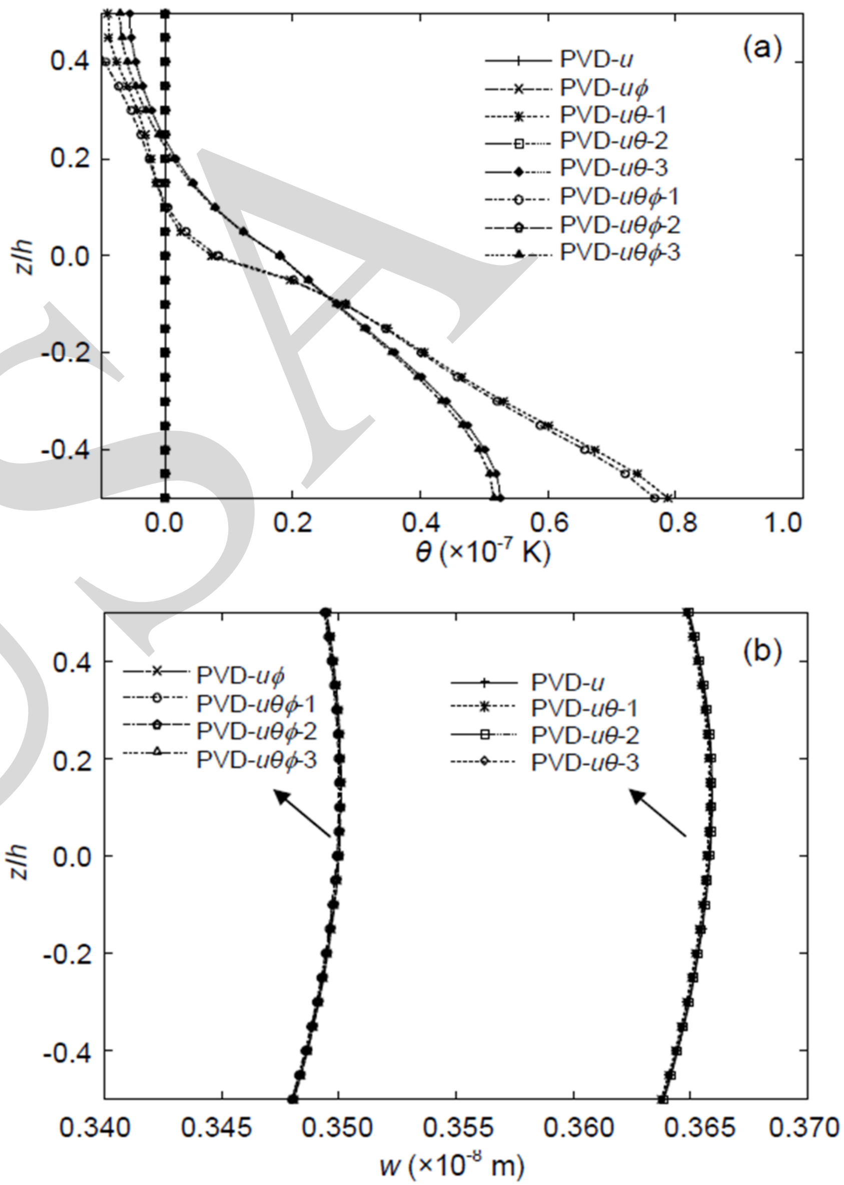

Fig. 5 Problem 5. Over-temperature (a) and transverse displacement (b) through the thickness