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Analytical solutions of stresses and displacements for deep circular tunnels with liners in saturated ground

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Introduction and problem description

To investigate the displacement and stress distributions for deep circular tunnels with liners in saturated ground, an analytical model is proposed (Fig. 1). For a deep tunnel with drainage conditions, plane strain conditions at any cross-section of the tunnel and the elastic regime of the linear elasticity for the remaining liner are assumed, while the ground is assumed to be linearly elastic and perfectly plastic with a failure surface defined by the Mohr-Coulomb criterion. The post-yield behavior of the ground follows the non-associated flow rule defined by the dilation angle.

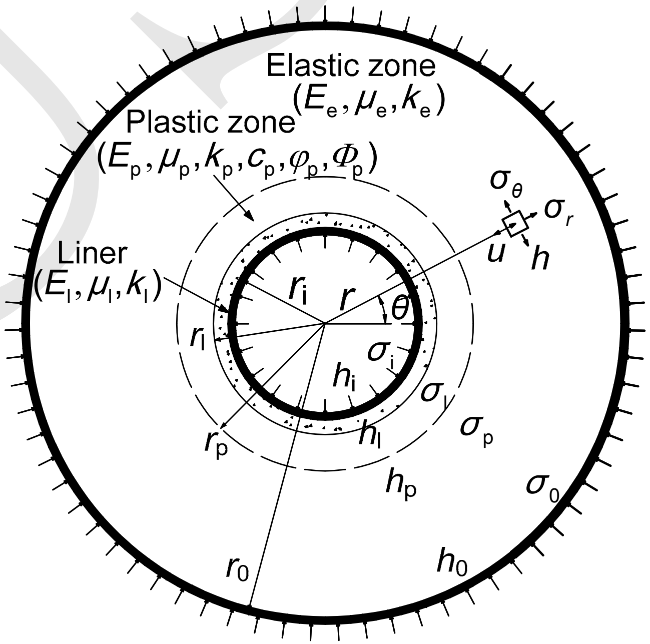


Fig. 1 Schematic illustration of the analytical model

Proposed analytical model

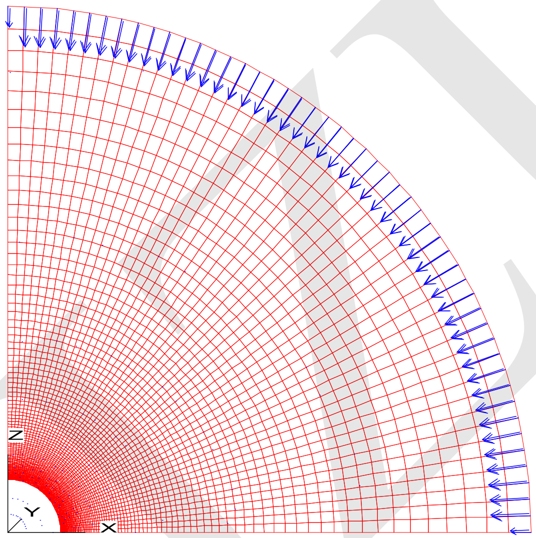
In order to solve the proposed problem, two procedures are presented. An axisymmetric model for a deep circular tunnel with a steady-state seepage condition is considered, and then a simple closed-form analytical solution is obtained with a common theoretical framework for the boundary conditions of a constant total head along the tunnel circumference (Eq.(1)). Assuming that certain ground displacements along the tunnel circumference have occurred before the installation of the liner, analytical solutions of stresses and displacements are derived with particular emphasis on the seepage and the stress release effect induced by tunnelling (Eq.(2)).

$$h=\begin{cases}\dfrac{h_{\mathrm{i}}\ln(r_1/r)+h_1\ln(r/r_{\mathrm{i}})}{\ln(r_1/r_{\mathrm{i}})}, & r_{\mathrm{i}}\le r\le r_1,\\[2ex] \dfrac{h_1\ln(r_{\mathrm{p}}/r)+h_{\mathrm{p}}\ln(r/r_1)}{\ln(r_{\mathrm{p}}/r_1)}, & r_1<r\le r_{\mathrm{p}},\\[2ex] \dfrac{h_{\mathrm{p}}\ln(r_0/r)+h_0\ln(r/r_{\mathrm{p}})}{\ln(r_0/r_1)}, & r_{\mathrm{p}}<r\le r_0.\end{cases} \quad (1)$$

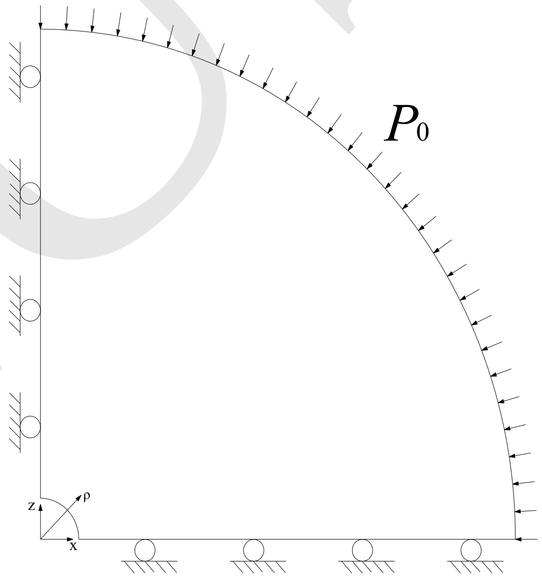
$$\begin{aligned}u^{\mathrm{p}}=\frac{(1-\mu_{\mathrm{p}}^2)r}{E_{\mathrm{p}}}\Bigg[\Bigg[&B-(\sigma_{r_{\mathrm{i}}}+B)\left(\frac{r}{r_1}\right)^{N_\varphi-1}\Bigg]\left(N_\varphi-\frac{\mu_{\mathrm{p}}}{1-\mu_{\mathrm{p}}}\right)\\ &+\frac{4\sin\varphi_{\mathrm{p}}(\sigma_{r_{\mathrm{i}}}+B)}{(1-\sin\varphi_{\mathrm{p}})^2(N_\varphi+N_\psi)}\left[\left(\frac{r}{r_1}\right)^{N_\varphi-1}-\left(\frac{r_p}{r_1}\right)^{N_\varphi-1}\left(\frac{r_p}{r}\right)^{1+N_\psi}\right]\\ &+\frac{2(c_{\mathrm{p}}\cos\varphi_{\mathrm{p}}-B\sin\varphi_{\mathrm{p}})}{(1-\mu_{\mathrm{p}})(1-\sin\varphi_{\mathrm{p}})(1+N_\psi)}\left[1-\left(\frac{r_{\mathrm{p}}}{r}\right)^{1+N_\psi}\right]\\ &-\frac{(1-2\mu_{\mathrm{p}})\sigma_0}{(1-\mu_{\mathrm{p}})}-\frac{2c_{\mathrm{p}}\cos\varphi_{\mathrm{p}}}{1-\sin\varphi_{\mathrm{p}}}\Bigg].\end{aligned} \quad (2)$$

Validation of the analytical solutions

To validate the analytical model, the results of the analytical model are compared with those obtained from numerical computations. The numerical computations are carried out by using a 3D finite difference code FLAC3D (Fig. 2) which can address 3D continuum problems with coupled groundwater flow.



(a)



(b)

Fig. 2 Calculation model and boundary conditions

Results of the calculation

Stresses and displacement distributions using the proposed analytical solutions are compared to the numerical solution of FLAC3D, which appears to match well.

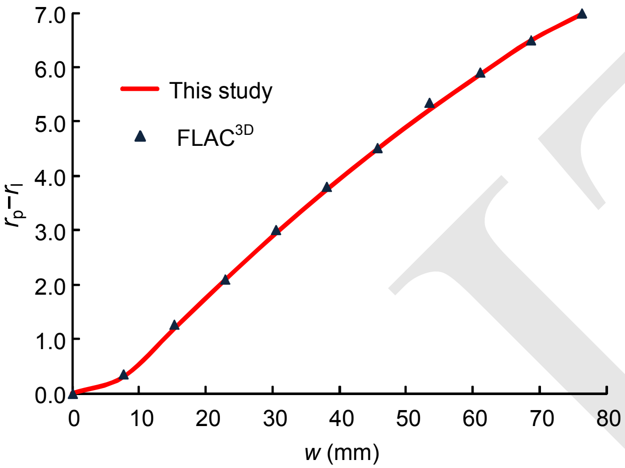


Fig. 3 Relation curves between r_p-r_l and w

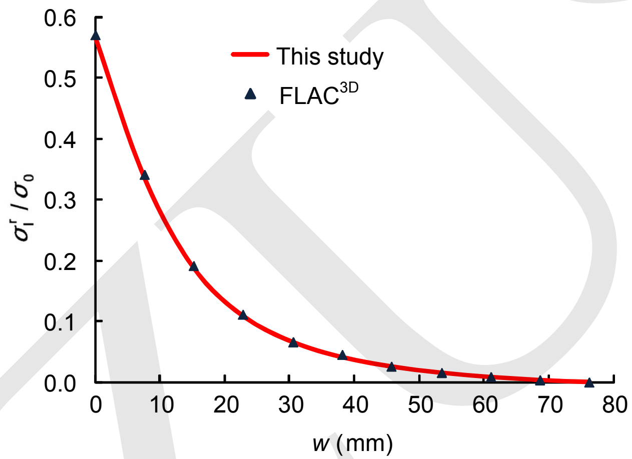


Fig. 4 Relation curves between σ_{rl}/σ_0 and w

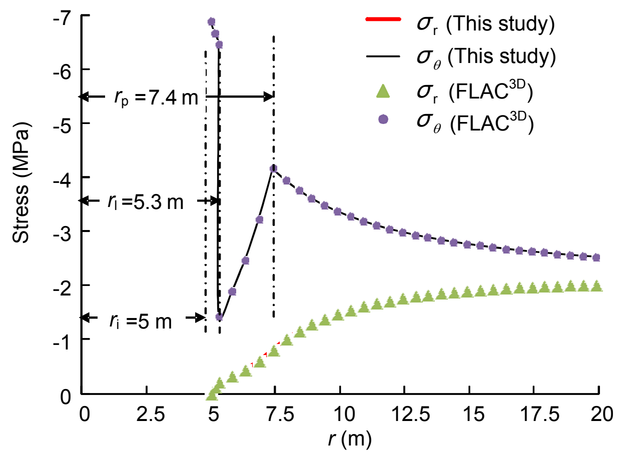


Fig. 5 Stress distributions around the tunnel when w=

Conclusions

- 1．Elasto-plastic plane strain solutions of the stress and displacement distributions around deep circular lined tunnels in isotropic saturated ground due to uniform ground loads and seepage forces are presented.
- 2．The analytical solutions are useful for a preliminary design of deep tunnels under high water level to predict the water pressure, stresses and displacements distribution around the tunnel.
- 3． Stresses and displacements distributions using the proposed analytical solutions are compared to the numerical solution of FLAC3D, and which appears to match well.
- 4．The theoretical expressions derived in this paper may be used in validating the numerical models.
- 5．The derivation process of the expression for displacements in the plastic zone is a highlight in this paper.