An iterative tolerance analysis procedure to deal with linearized behavior models

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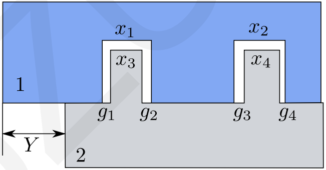
Outline

1. Introduction : tolerance analysis problem formulation
2. Linearization procedure
3. Solution methods
    - Monte Carlo simulation
    - Iterative algorithm
4. Industrial application

Formalization

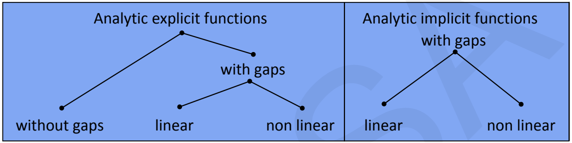
Geometrical model

- Geometrical deviations $\mathbf{x}$: Random variables whose distributions are known.
- Gaps $\mathbf{g}$: Free variables depending on the configuration of the mechanism.



Formalization

Behavior model



Analytic explicit functions

with gaps

without gaps

linear

non linear

Analytic implicit functions with gaps

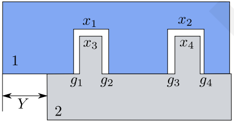
linear

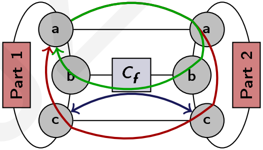
non linear

$$Y = f(\mathbf{x})$$
$$C_i(Y) \leq 0$$
$$C_f(Y) \leq 0$$

$$Y = f(\mathbf{x}, \mathbf{g})$$
$$C_i(Y) \leq 0$$
$$C_f(Y) \leq 0$$

$$C_c(\mathbf{x}, \mathbf{g}) = 0$$
$$C_i(\mathbf{x}, \mathbf{g}) \leq 0; \; C_{i*}(\mathbf{x}, \mathbf{g}) = 0$$
$$C_f(\mathbf{x}, \mathbf{g}) \leq 0$$





Compatibility equations

$$C_c(\mathbf{x}, \mathbf{g}) = 0$$

Interface constraints

$$C_i(\mathbf{x}, \mathbf{g}) \leq 0 \text{ and } C_{i*}(\mathbf{x}, \mathbf{g}) = 0$$

Functional condition

$$C_f(\mathbf{x}, \mathbf{g}) \leq 0$$

Formalization

Quantifiers based conditions

Assembly condition

There exists an admissible gap configuration of the mechanism such that the assembly requirement (interface constraints) and the compatibility equations are respected.

$$\exists \mathbf{g} \in \mathbb{R}^m \left\{ C_c\left(\mathbf{x}, \mathbf{g}\right) = 0 \cap C_i\left(\mathbf{x}, \mathbf{g}\right) \leq 0 \cap C_{i*}\left(\mathbf{x}, \mathbf{g}\right) = 0 \right\}$$

Use of an optimization algorithm to check the existence of an admissible configuration

$$\begin{aligned} R(\mathbf{x}) = \min_{\mathbf{g} \in \mathbb{R}^m} \quad & C_i^1(\mathbf{x}, \mathbf{g}) \\ \text{Subject to} \quad & C_c(\mathbf{x}, \mathbf{g}) = 0 \\ & C_i(\mathbf{x}, \mathbf{g}) \leq 0 \\ & C_{i*}(\mathbf{x}, \mathbf{g}) = 0 \end{aligned}$$

- $R(\mathbf{x})$ has a solution means that there exists a configuration of gaps satisfying all constraints.
- $R(\mathbf{x})$ has no solution means the mechanism cannot be assembled.

$$P_{fa} = \text{Prob}\left[\nexists R(\mathbf{X})\right]$$

Optimization algorithm

Use of an optimization algorithm to check the existence of an admissible configuration

$$\begin{aligned} R(\mathbf{x}) = \min_{\mathbf{g}\in\mathbb{R}^m} \quad & C_i^1(\mathbf{x},\mathbf{g}) \\ \text{Subject to} \quad & C_c(\mathbf{x},\mathbf{g}) = 0 \\ & C_i(\mathbf{x},\mathbf{g}) \leq 0 \\ & C_{i*}(\mathbf{x},\mathbf{g}) = 0 \end{aligned}$$

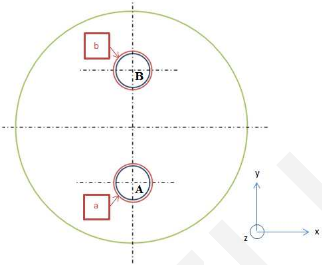
Difficulty :

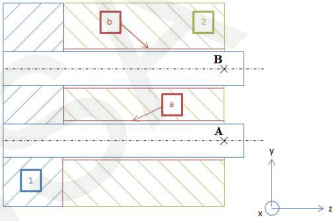
- Optimization step may be complex when taking into account non linear constraints.

Considered solution :

- Linearization of non linear equations.

Example of mechanism





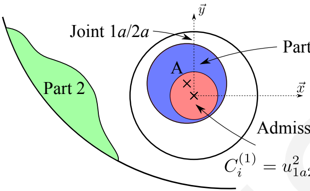
Interface constraints to be linearized

Both cylindrical joints on surfaces *a* and *b* : gap on each side must not exceed the radius difference ⇒ quadratic inequations :

$$u_{1a2a}^2 + v_{1a2a}^2 - \left(\frac{d_{1a} - d_{2a}}{2}\right)^2 \leq 0$$

$$u_{1b2b}^2 + v_{1b2b}^2 - \left(\frac{d_{1b} - d_{2b}}{2}\right)^2 \leq 0$$

Linearization procedure



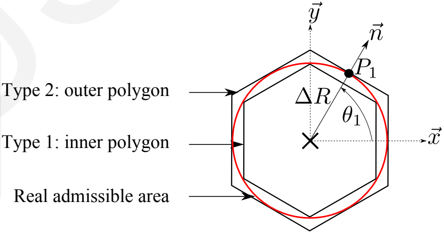
Admissible area satisfying the constraint

$$C_i^{(1)} = u_{1a2a}^2 + v_{1a2a}^2 - \left(\frac{d_{2a} - d_{1a}}{2}\right)^2 \leq 0$$

First order linearization to point $P_k(\Delta R \cos\theta_k, \Delta R \sin\theta_k)$

Two strategies :

- Inner polygon
- Outer polygon


Type 2: outer polygon

Type 1: inner polygon

Real admissible area

$$u_{1a2a} \cos\theta_k + v_{1a2a} \sin\theta_k - \left(\frac{d_{1a} - d_{2a}}{2}\right) - r_s \leq 0$$

with $\theta_k = \frac{2k\pi}{N_d}$ and $k = 1, \ldots, N_d$

Solution method based on Monte Carlo simulation

1. Random sampling of N geometrical deviations $\mathbf{X} = \{X_1(\omega), \ldots, X_n(\omega)\}$,
2. Finding an admissible configuration of gaps satisfying the behavior model using the optimization algorithm,
3. Estimating the probability of failure : $\tilde{P}_f = \dfrac{1}{N}\displaystyle\sum_{i=1}^{N} I_D(\mathbf{x}^{(i)})$, where $I_D(\mathbf{X})$ is the indicator function : $I_{D_{fa}}(\mathbf{X}) = \begin{cases} 1 & \text{if no solution can be provided} \\ 0 & \text{if a solution is found} \end{cases}$

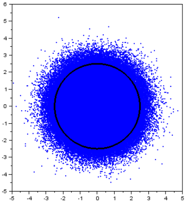
Difficulty :

- No information about the required number of linearizations N_d to get accurate enough result.
- Performing several Monte Carlo simulations is not conceivable due to prohibitive computing time.

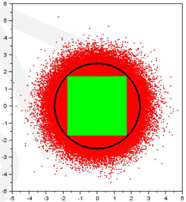
Considered solution :

- Use of an iterative algorithm.

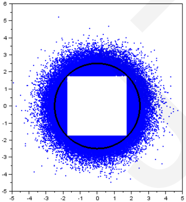
Illustration of the iterative algorithm



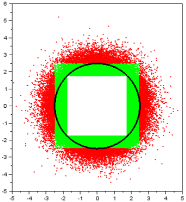
Step 1 : Define Monte Carlo population
$N = 500000$



Step 2 : full Monte Carlo with inner strategy
$N_d = 4$

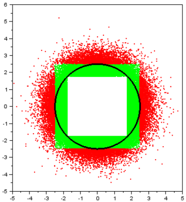


Step 3 : save non assembly points $N = 74276$

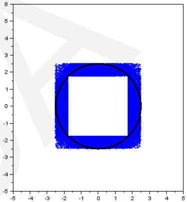


Step 4 : Monte Carlo with outer strategy
$N_d = 4$

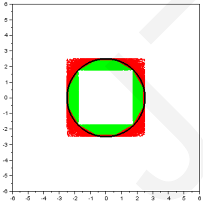
Illustration of the iterative algorithm



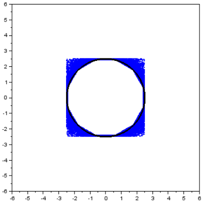
Step 4 : Monte Carlo with outer strategy
$N_d = 4$



Step 5 : save uncertain points $N = 61858$

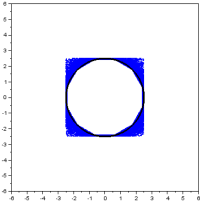


Step 6 : Monte Carlo with inner strategy
$N_d = 12$

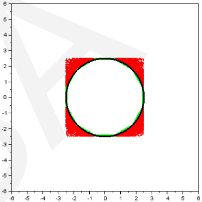


Step 7 : save non assembly points $N = 13052$

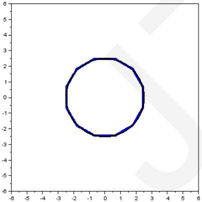
Illustration of the iterative algorithm



Step 7 : save non assembly points $N = 13052$



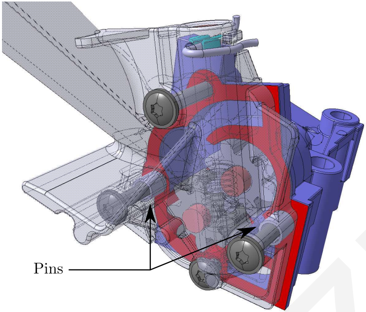
Step 8 : Monte Carlo with outer strategy $N_d = 12$

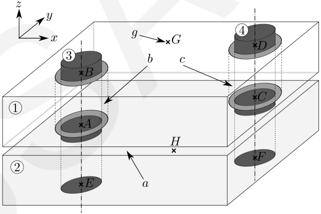


Step 9 : save uncertain points $N = 4949$

- The procedure stops when a stopping criterion $rCI = \frac{P_{inner} - P_{outer}}{P_{inner}}$ is small enough
- The number of required optimizations decreases at each step
- The number of calls to the most expensive models is smaller

Industrial application : planar joint with two pins





Characteristics

- 38 geometrical deviations (random variables) $\mathbf{x}$
- 15 gap variables $\mathbf{g}$
- 12 compatibility equations $C_c(\mathbf{x}, \mathbf{g}) = 0$
- $4 \times N_d$ quadratic interface constraints $C_i(\mathbf{x}, \mathbf{g}) \leq 0$

Industrial application : numerical comparison

- The number of samples (= optimization calls) of the Monte Carlo simulation is set to $14e6$, the coefficient of variation is about 5%.
- The iterative procedure starts with $N_d = 4$ and stops at $N_d = 108$.
- The stopping criterion is equal to $rCI = 1\%$.

Monte Carlo simulation

N_d	$P_{inner} \times 10^{-5}$	$P_{outer} \times 10^{-5}$	Time for one optim.	Total time
4	45.9	0.49	$1.1e^{-3}$ s	4.1 h
12	4.94	2.99	$1.8e^{-3}$ s	7.1 h
36	4.34	4.06	$5e^{-3}$ s	19.5 h
108	4.19	4.15	$22.5e^{-3}$ s	87.7 h

Iterative procedure

N_d	$P_{inner} \times 10^{-5}$	$P_{outer} \times 10^{-5}$	Optim. calls	Time
4	45.9	0.49	$14e6 + 6426$	4.16 h
12	4.94	2.99	$6357 + 622$	
36	4.34	4.06	$273 + 189$	
108	4.19	4.15	$38 + 19$	

Thank you for your attention

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