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A theoretical insight into morphological operations in surface measurement by introducing the slope transform

Key words: morphological operations; slope transform; tangential dilation; linear convolution; surface metrology

Linear convolution VS Nonlinear morphological operation

- **Convolution**: $f(x) * g(x) = \int_{-\infty}^{+\infty} f(\tau) g(x-\tau) d\tau$

- **Convolution theorem**: the Fourier transform of a convolution is the product of Fourier transforms.

Step 1: $\mathcal{F}[f(x)] = F(\omega)$

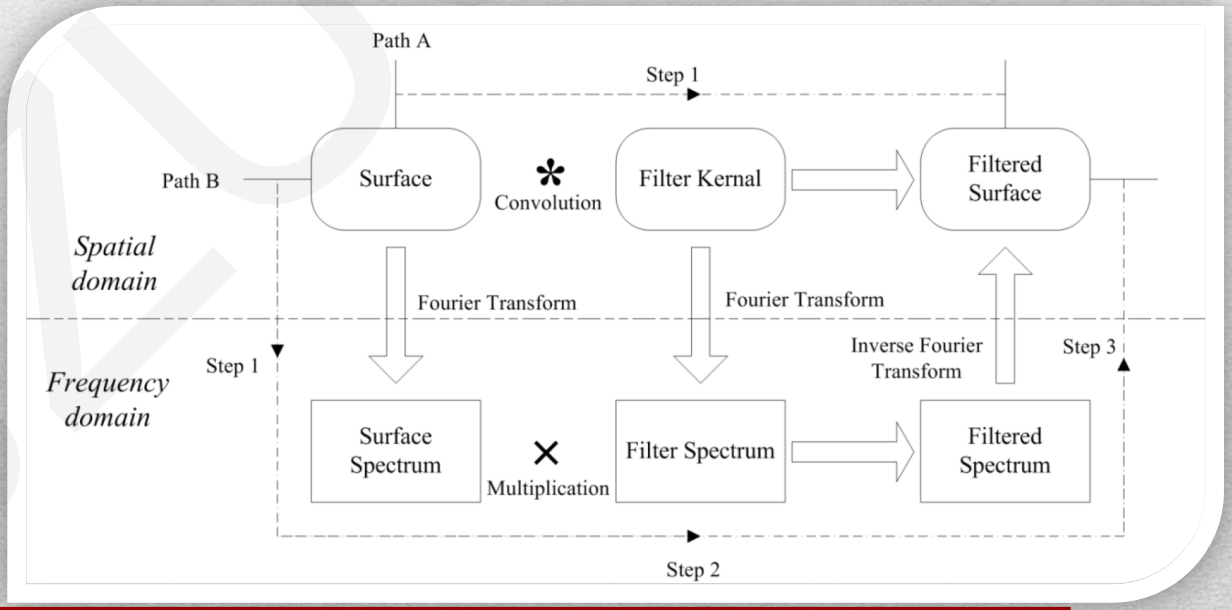
$\mathcal{F}[g(x)] = G(\omega)$

Step 2:

$\mathcal{F}[(f * g)(x)] = F(\omega) \times G(\omega)$

Step 3:

$(f * g)(x) = \mathcal{F}^{-1}[F(\omega) \times G(\omega)]$



Linear convolution VS Nonlinear morphological operation

- **Morphological operations**: dilation/erosion/opening/closing
- Morphological operation examples in surface metrology
  - ✓ Dilation: tactile probe scanning
  - ✓ Erosion: mechanical surface reconstruction
  - ✓ Opening/closing: surface filtration, functional surface simulation

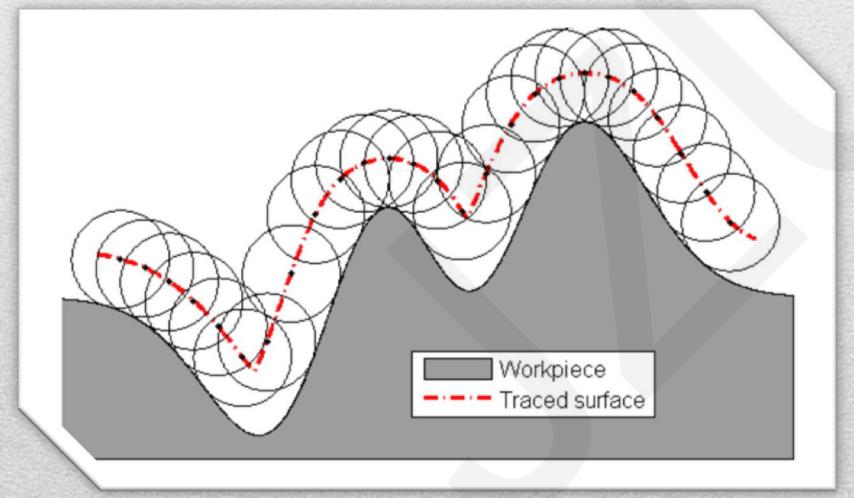


Fig. 3 Scanning of the workpiece surface by a probe.

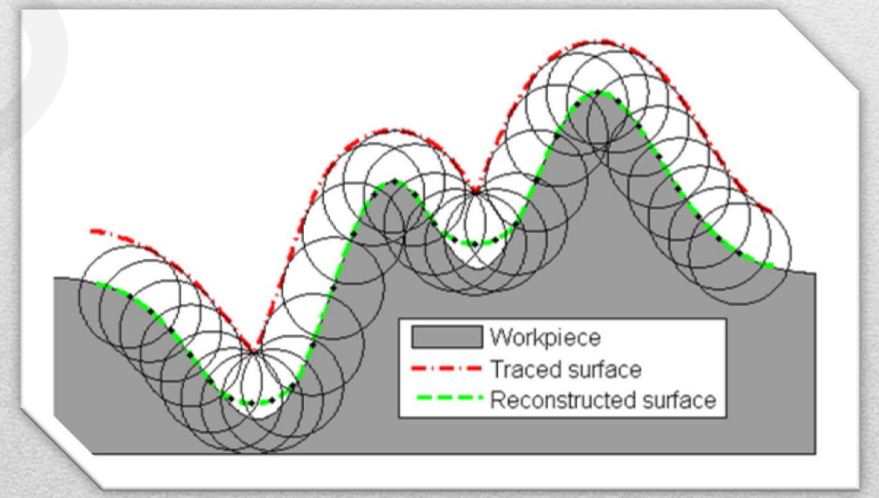


Fig. 4 Reconstruction of the workpiece surface.

Tangential dilation and classical dilation

- **Tangential dilation**: $\left(f \breve{\oplus} g\right)(x) = \underset{u}{\text{stat}}\left[f(u) + g(x-u)\right]$

  where $\underset{u}{\text{stat}}\, f(u) = \left\{f(\bar{u}) \,|\, \nabla f(\bar{u}) = 0\right\}$

- **Relationship** between tangential dilation and classical dilation:

$$\left(f \oplus g\right)(x) = \mathrm{V}\left(f \breve{\oplus} g\right)(x)$$

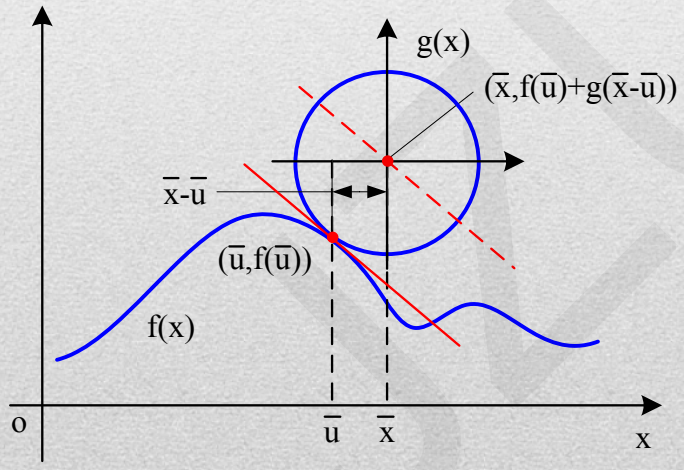


Fig. 5 Computation of the classical dilation and the tangential dilation.

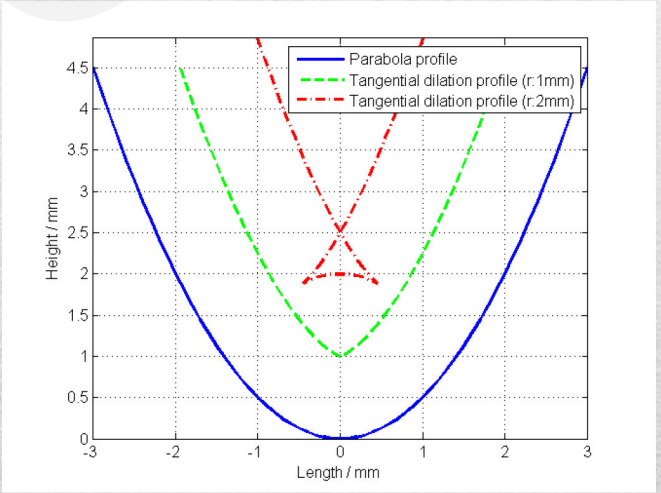


Fig. 6 Tangential dilation of a parabola with disk radii 1 mm and 2 mm.

Slope transform and addition theorem

- **Slope transform**: $\mathcal{S}[f](\omega) = \underset{x}{\text{stat}}\left[f(x) - \langle \omega, x \rangle \right]$

  Inverse slope transform: $f(x) = \underset{\omega}{\text{stat}}\left[\mathcal{S}[f](\omega) + \langle \omega, x \rangle \right]$

- **Addition theorem**: the slope transform of the tangential dilation is the addition of the slope transforms.

Step 1: $\mathcal{S}[f(x)] = F(\omega)$

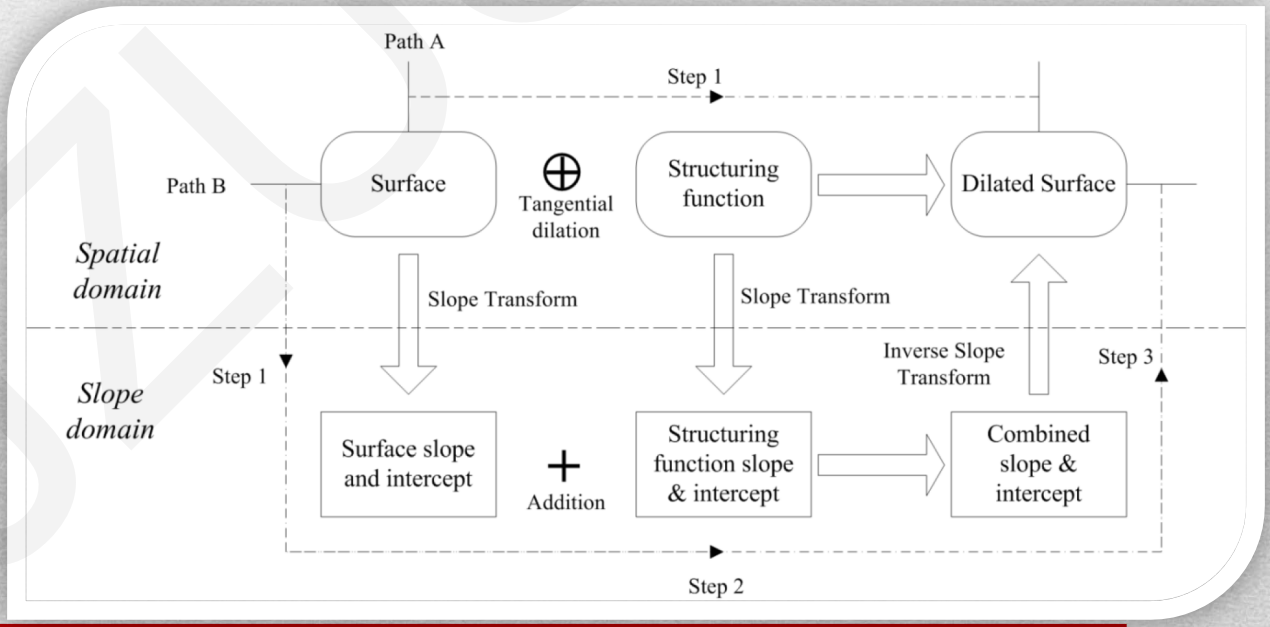
$\mathcal{S}[g(x)] = G(\omega)$

Step 2:

$$\mathcal{S}\left[\left(f \breve{\oplus} g \right)(x) \right] = F(\omega) + G(\omega)$$

Step 3:

$$\left(f \breve{\oplus} g \right)(x) = \mathcal{S}^{-1}\left[F(\omega) + G(\omega) \right]$$



Analytical example: a disk tangent-dilated by a disk

Spatial domain

Disk input: $f(x) = \sqrt{R_1^2 - x^2}$

Disk SE: $g(x) = \sqrt{R_2^2 - x^2}$

Slope transform

Slope domain

Disk input: $F(\omega) = R_1\sqrt{1+\omega^2}$

Disk SE: $G(\omega) = R_2\sqrt{1+\omega^2}$

Tangential dilation

addition

Slope domain

$$\left(f \breve{\oplus} g\right)(x) = R \cdot \sqrt{1 + \frac{x^2}{R^2 - x^2}} + \sqrt{\frac{1}{R^2 - x^2}}x^2$$

$$= \sqrt{R^2 - x^2}$$

$$(R = R_1 + R_2)$$

Inverse slope transform

Slope domain

$$F(\omega) + G(\omega) = \left(R_1 + R_2\right)\sqrt{1+\omega^2}$$

Result: the dilated disk is an expanded disk having a radius equivalent to the sum of those of the input disk and the structuring disk.

Practical example: tangential dilation of a sine wave by disks

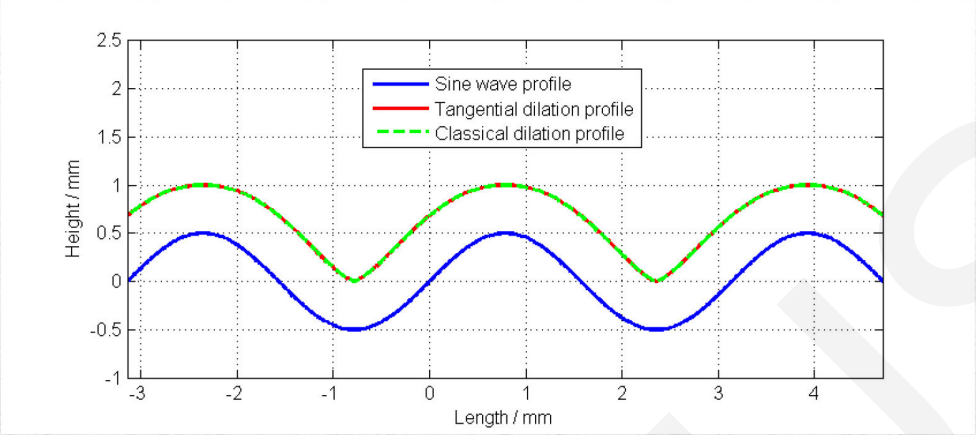


Fig. 8 Tangential dilation of the sine wave by a 0.5 mm disk.

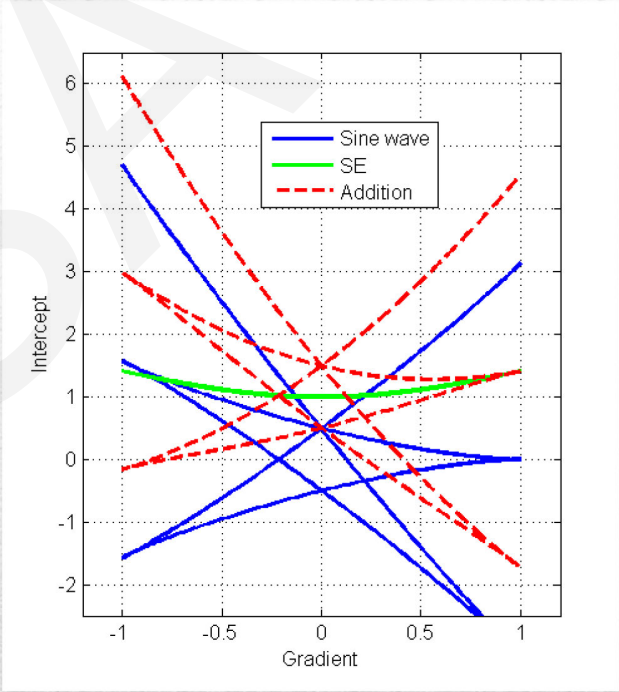
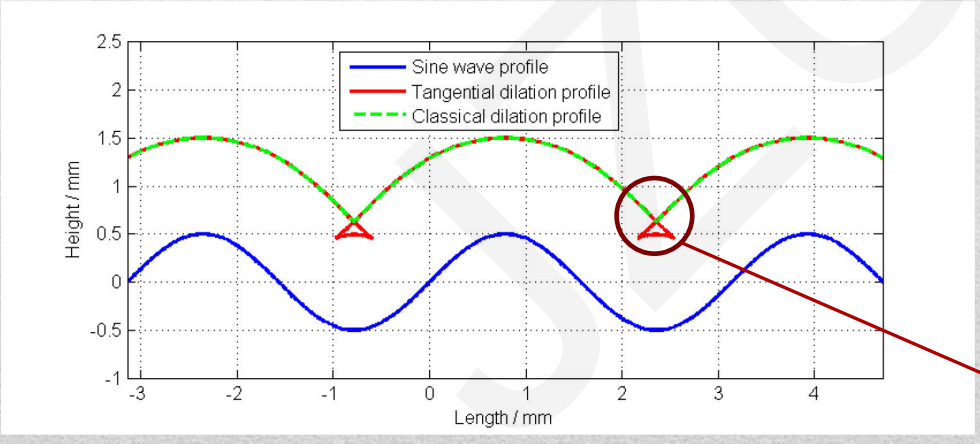


Fig. 9 Addition of the slope transform of the sine wave and the disk.



Overlapping region

Fig. 10 Tangential dilation of the sine wave by a 1 mm disk.

Conclusion

- As the Fourier transform switches the convolution into the multiplication in the frequency domain, the slope transform converts the tangential dilation into the addition in the slope domain. As such, the slope and curvature changes caused by the structuring element are revealed.
- The supremum of the tangential dilation is identical to the classical dilation. However the tangential dilation tends to produce the overlapping regions, which is undesired in practice.

Future work

- Feasibility of the slope transform in real practice.
- Areal extension of the tangential dilation and the slope transform.