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A statistical method to identify main contributing tolerances in assemblability studies based on convex hull techniques

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Sensitivity Analysis for Assemblability
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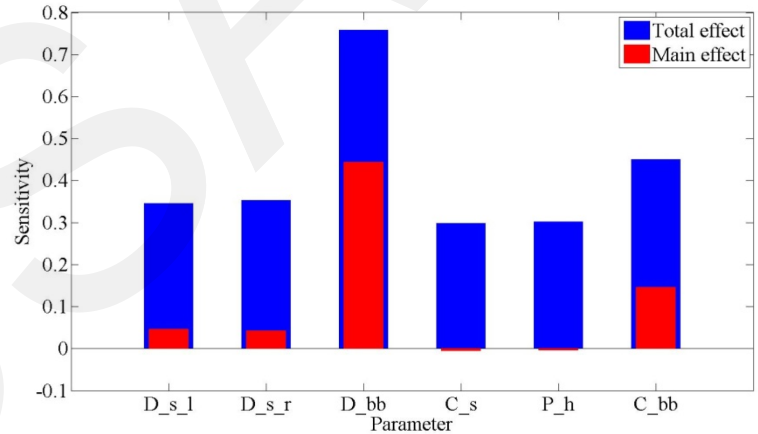

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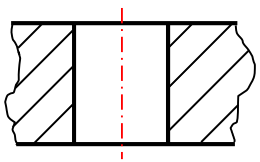
Tolerances

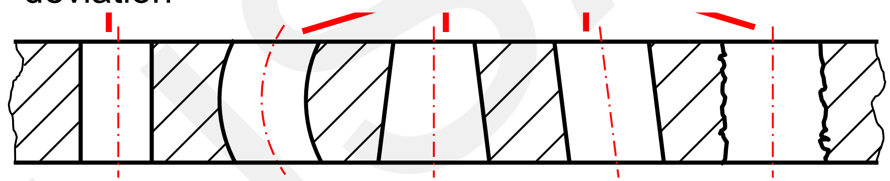
Axiom of manufacturing imprecision: All manufacturing processes are inherently imprecise and produce parts that vary.

Nominal drill hole



Dimensional deviation

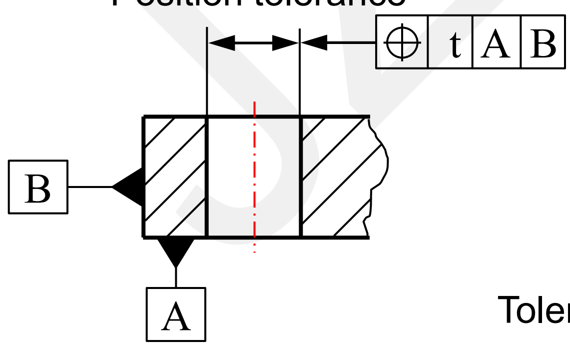
Geometric deviaions



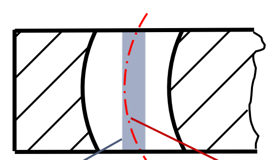
Drill hole with different kinds of deviations

Tolerances restrict the geometric deviations of parts with respect to the nominal geometry.

Position tolerance



Manufactured geometry



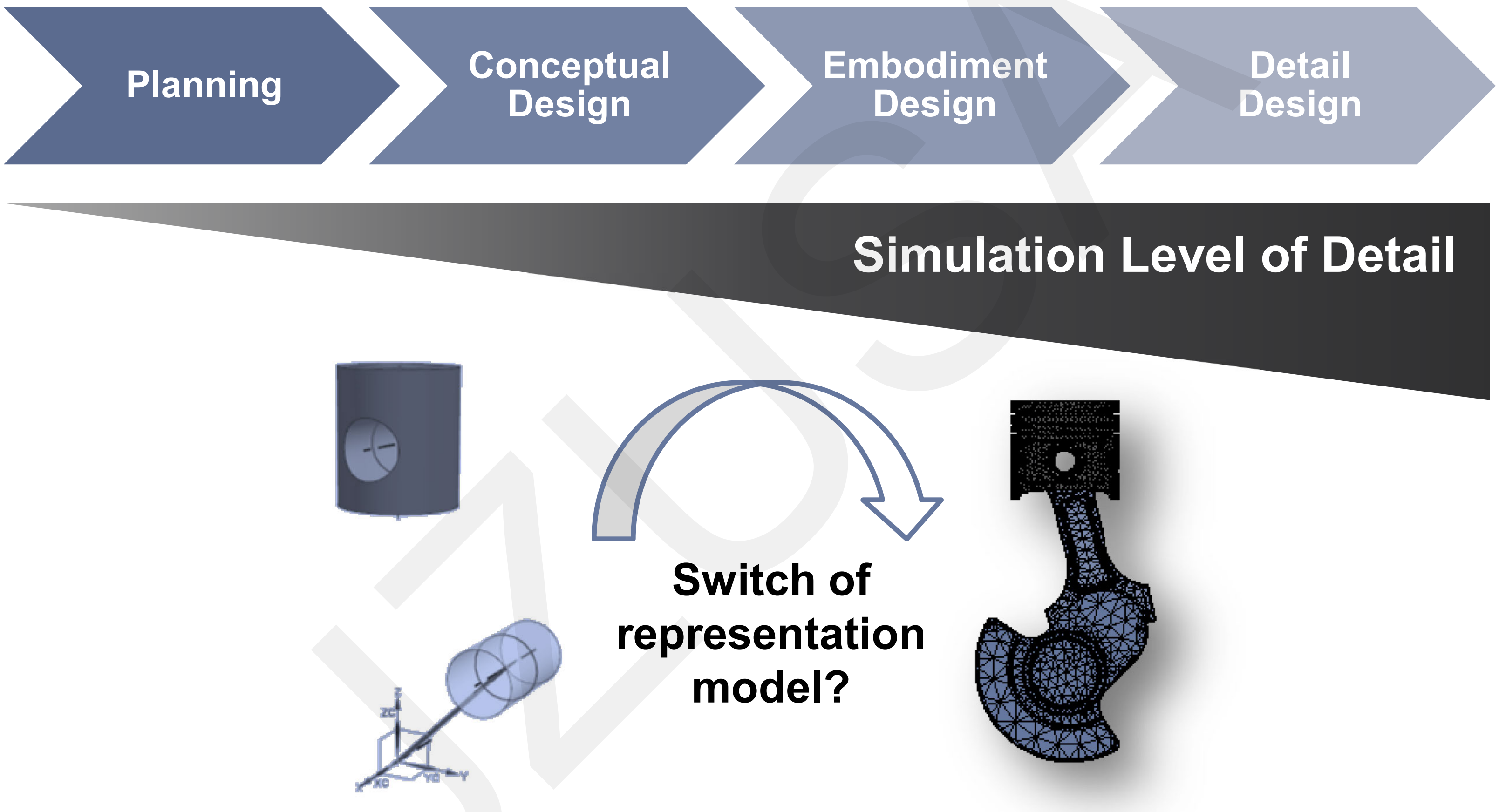
Deviating axis

Tolerance zone

References: Jorden (2012), Srinivasan (1999)

The need for model free SA in Tolerancing

Motivation



There is a need for model free sensitivity analysis in tolerancing!

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The need for model free SA in Tolerancing

Methods in Tolerancing

„Sensitivity analysis should be quantitative, global and **model free**“

SA Methods in Tolerancing	Quant.	Global	Model free*
High-Low-Median (HLM)	✓	✗	✗
Arithmetic Contributor	✓	✗	✗
Statistical Contributor	✓	✗	✗
Derivative-based (Wu)	✓	✓	✗
Variance-based (Stockinger/Walter)	✓	✓	✗
Variance/Density-based (Caniou)	✓	✓	✗

*For tolerance simulations, model free can be interpreted as „independent of mathematical deviation representation“

Reference: Saltelli and Tarantola (2002)

Mathematical deviation representation
Deviation Domain

Deviation Representation

Linearized homogeneous transformation matrices

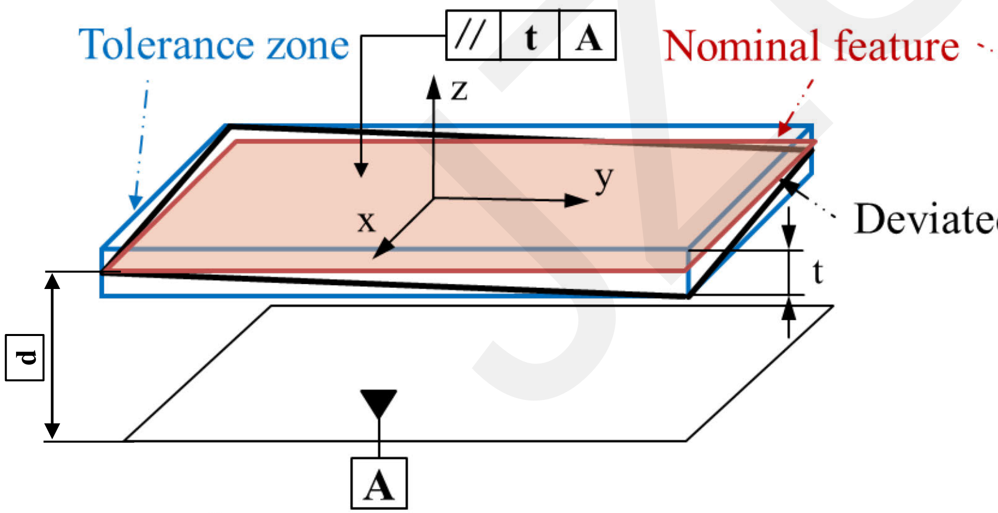
$$T^{lin}(\boldsymbol{x},\boldsymbol{\theta}) = \begin{bmatrix} 1 & -\theta_1 & \theta_2 & x_1 \\ \theta_1 & 1 & -\theta_3 & x_2 \\ -\theta_2 & \theta_3 & 1 & x_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Tolerance Representation

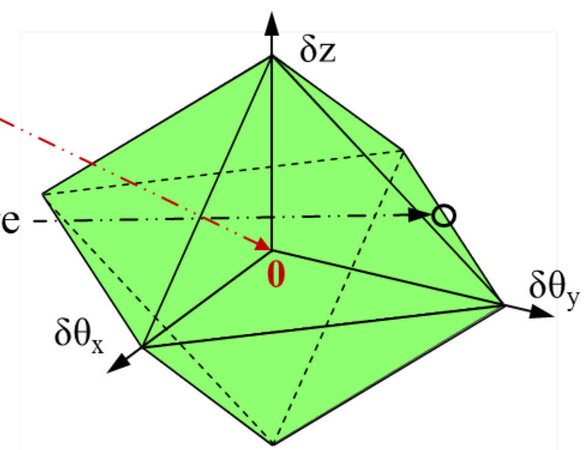
constraints for the transformation parameters $(\boldsymbol{x},\boldsymbol{\theta})$

$$\max_{\boldsymbol{u}\in f} \left\| \left(T^{lin}(\boldsymbol{x},\boldsymbol{\theta}) - I\right)\cdot \boldsymbol{u} \right\| \leq \frac{t}{2}$$

Feature with tolerance



Abstract deviation space



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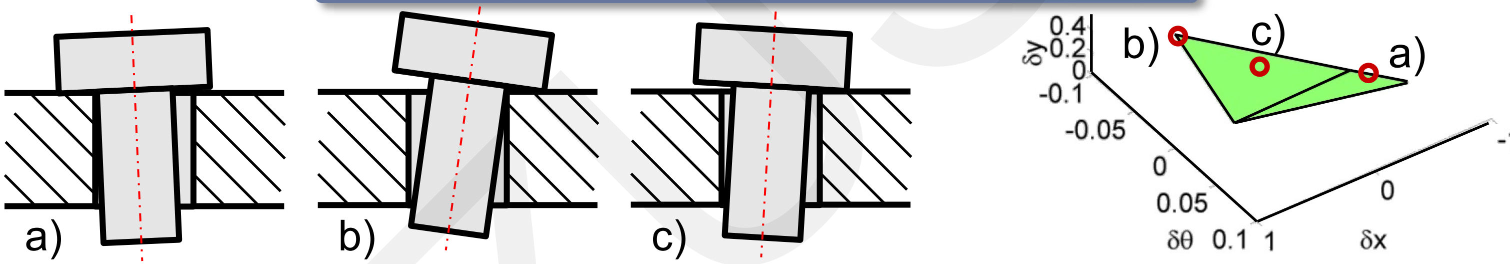
Mathematical deviation representation

Assemblability: Clearance Domain

Clearance Representation

The Clearance Domain $C_{pi,pj}^{dev}$ consists of all transformation parameters $(\boldsymbol{x}, \boldsymbol{\theta})$ for relative positions of two deviated parts considering assembly constraints.

Example: Nominal Pin & Hole (2D)



Assemblability Measure: Relative Clearance Domain Volume

$$\frac{|C_{pi,pj}^{dev}|}{|C_{pi,pj}^{nom}|}$$

Numerator: Measure for all relative positions of p_i and p_j for two deviated parts.

Denominator: Measure for all relative positions of p_i and p_j for the nominal geometry.

Sensitivity Analysis
Global Sensitivity Analysis

„Sensitivity analysis studies the relationships between information flowing in and out of the model“ *A. Saltelli*

Input Parameter → Model → Output Parameter

Local and Global Sensitivity Analysis in Tolerancing

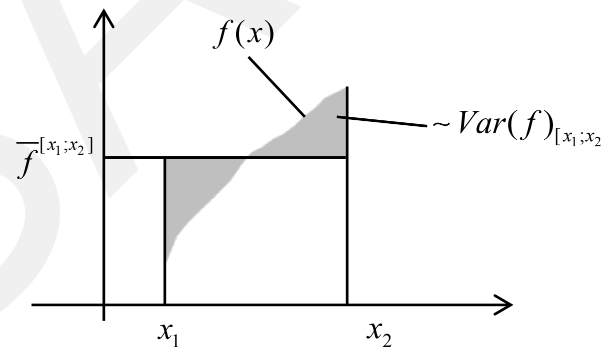
Local methods	Global methods
Consider an individual (deviating) feature	Consider a feature with tolerance (and deviation distribution)

Reference: Saltelli et al (2000)

Variance-based Sensitivity Analysis

Well established global method: variance-based SA

Let f be a generic model $f\colon (X_1, \dots, X_n) \to Y$ with random variables $X_i \in [0;1]$. The variance $V(Y)$ then decomposes to



$$V(Y) = E\big(V(Y|X_i)\big) + V\big(E(Y|X_i)\big),$$

where $E(Y|X_i)$ denotes the conditional expectation with respect to X_i and $V(Y|X_i)$ the conditional variance with respect to X_i. The main effect sensitivity S_i and total effect sensitivity S_{T_i} of the random variable X_i is defined as

$$S_i = \frac{V\big(E(Y|X_i)\big)}{V(Y)} \quad \text{and} \quad S_{T_i} = 1 - \frac{V\big(E(Y|\boldsymbol{X}_{\sim i})\big)}{V(Y)},$$

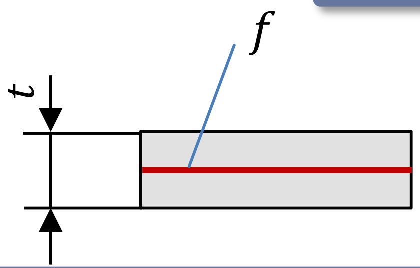
where $\boldsymbol{X}_{\sim i}$ denotes all input random variables aside from X_i.

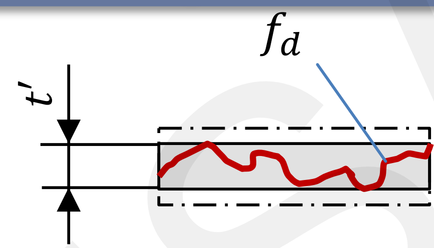
Reference: Saltelli et al (2008)

Sensitivity Analysis

Basis for model free SA: Deviation Quality Measure

Deviation Quality Measure λ





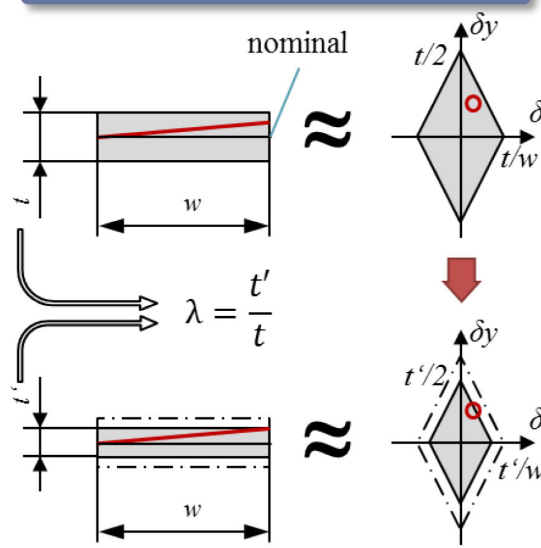
$$\lambda_t(f_d) = \frac{t'}{t}$$

Calculation for Deviation Domains

$$\lambda(\boldsymbol{x},\boldsymbol{\theta}) = \frac{2}{t}\max_{\boldsymbol{u}\in f}\left\|\left(T^{lin}(\boldsymbol{x},\boldsymbol{\theta}) - I\right)\cdot \boldsymbol{u}\right\|$$

- λ is homogeneous in $(\boldsymbol{x},\boldsymbol{\theta})$
- $\lambda(\mathbf{0},\mathbf{0}) = 0$
- $\lambda(\boldsymbol{x},\boldsymbol{\theta}) = 1 \Rightarrow (\boldsymbol{x},\boldsymbol{\theta}) \in \partial DD$

Example for an axis



Reference: Ziegler (2013)

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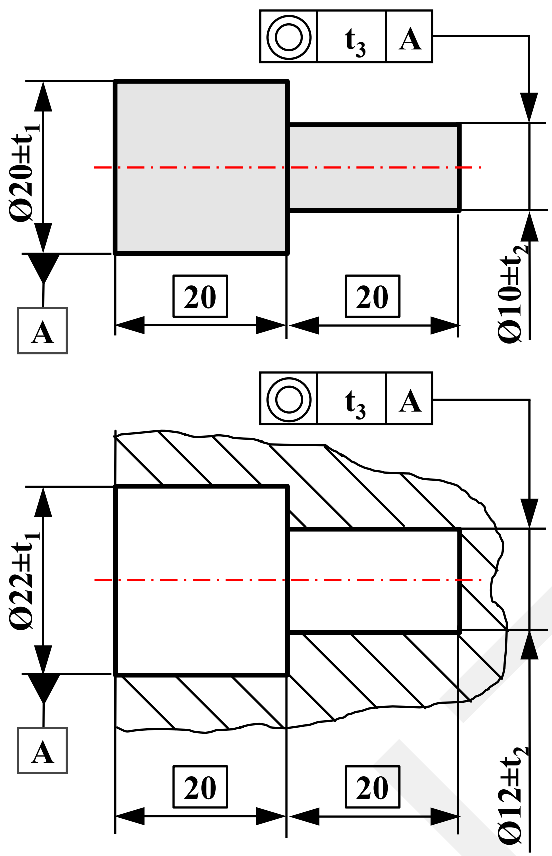
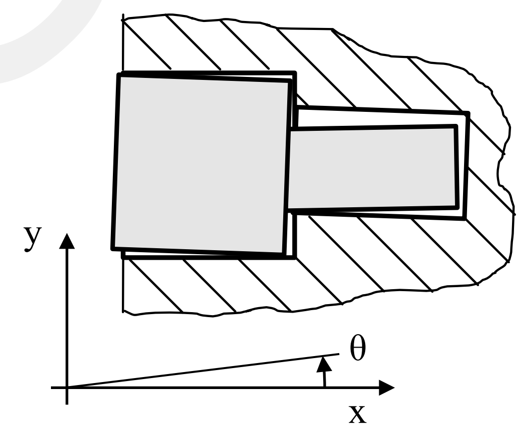
Application Example Pin-Hole



Tolerances

- 6 Tolerances, 4 dimensional and 2 geometric
- All tolerance values: 1 mm
- λ_i uniform distributed



Clearance Estimation

8 Control Points (vertices of pin cylinders) for collision detection
→ 8 constraints for transformation parameters (y,θ)

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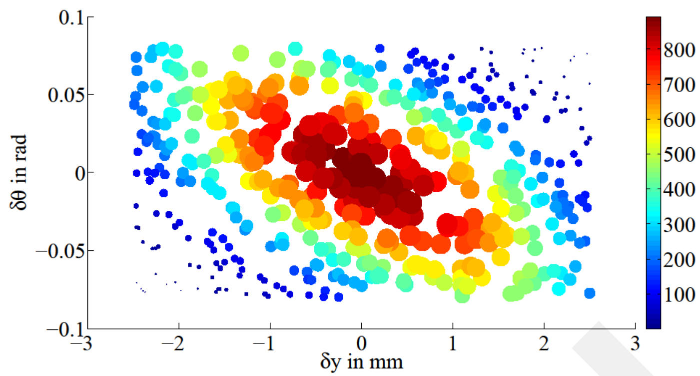
Application Example Pin-Hole

Results

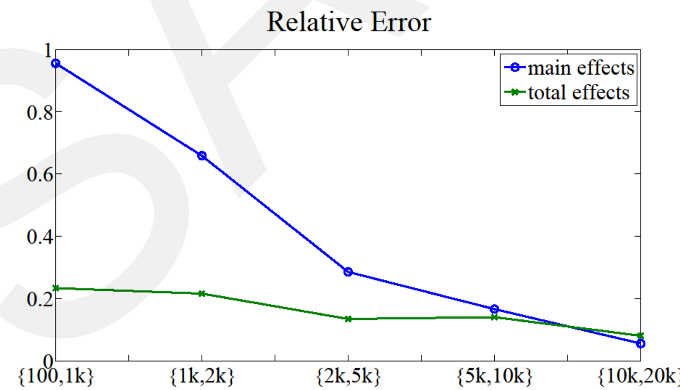
Clearance estimation

Few not mountable systems (≤ 10 %)

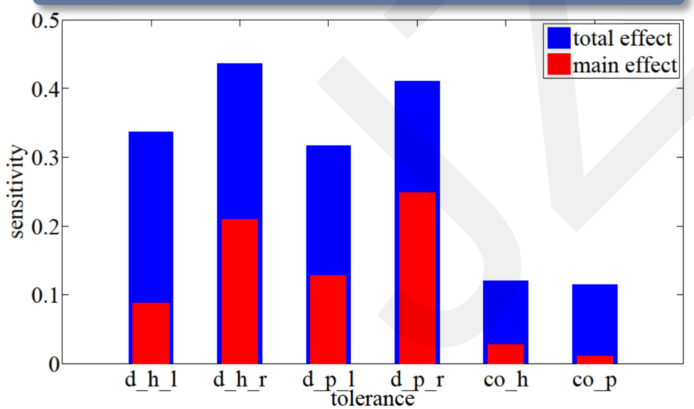


Monte Carlo Sample Size

Good results for 10,000+ SA-Samples



Sensitivity results



Arguments for the method

- The strong interactions between tolerances make a global SA method necessary for a realistic tolerance ranking
- As usually just few systems cannot be assembled, the upper left condition is mostly fulfilled

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Summary & Outlook

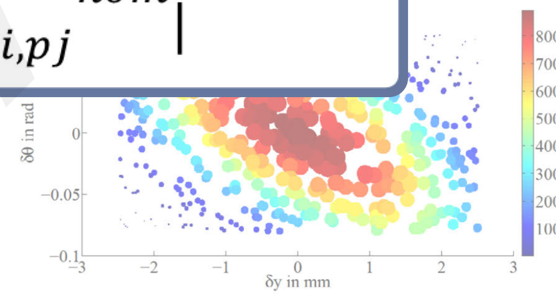
Sensitivity Analysis

Requirements: Quantitative,

Global and **Model free**

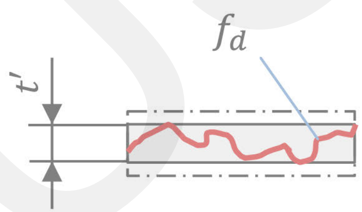
Assemblability

$$\frac{|C_{pi,pj}^{dev}|}{|C_{pi,pj}^{nom}|}$$



Deviation Characteristic

$$\lambda_t(f_d) = \frac{t'}{t}$$



Outlook

Other global SA methods and

Deviation Representations