



**Cite this as:** Chun-xiao Zhan, Yi-hua Liu, 2015. Plane elasticity solutions for beams with fixed ends. *Journal of Zhejiang University-SCIENCE A (Applied Physics & Engineering)*, 16(10):805-819. [doi:10.1631/jzus.A1500043]

# Plane elasticity solutions for beams with fixed ends

## **Key words:**

Beam, Fixed end, Boundary condition, Plane stress, Elasticity solution



**The beam is a fundamental and important component in many engineering structures in civil engineering, mechanical engineering, and aerospace engineering.**





The plane stress problem of beams is a classical one in elasticity theory and the **Airy** stress function method is often employed to investigate the mechanical behaviors of beams.

The efficient stress conditions for the free and simply supported ends are adopted based on **Saint-Venant** principle.



## QUESTION

In many recent research of beams, the displacement boundary condition for the fixed end is treated as that proposed by **Timoshenko** and **Goodier** (1970), i.e.

$$u = 0, v = 0, \frac{\partial v}{\partial x} = 0 \quad \text{or} \quad u = 0, v = 0, \frac{\partial u}{\partial y} = 0$$

However, the elasticity solutions for beams with fixed end(s) are not in good agreement with the FEM results with the increasing thickness of beams.

**Dai and Ji, J Tongji Univ-Sci 36, 890 (2008, in Chinese).**



## METHOD

Based on the above two fixed boundary conditions, a new one is proposed as follows:

$$u = 0, v = 0, \frac{\partial v}{\partial x} - \beta \frac{\partial u}{\partial y} = 0$$

where  $\beta$  is a parameter to be determined by the minimization of the square sum of the longitudinal displacement over the total cross section of the fixed end.



## CONCLUSIONS

- 1. A new fixed boundary condition is proposed and the accuracy of the elasticity solution for beams with fixed end(s) is improved.**
- 2. Examples show that the elasticity solution obtained by the new fixed boundary condition best approaches the FEM result in comparison with the existed elasticity solutions with the increasing thickness of beams.**

