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# Constitutive models of artificial muscles: a review

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# Schematics of a dielectric elastomer (DE)

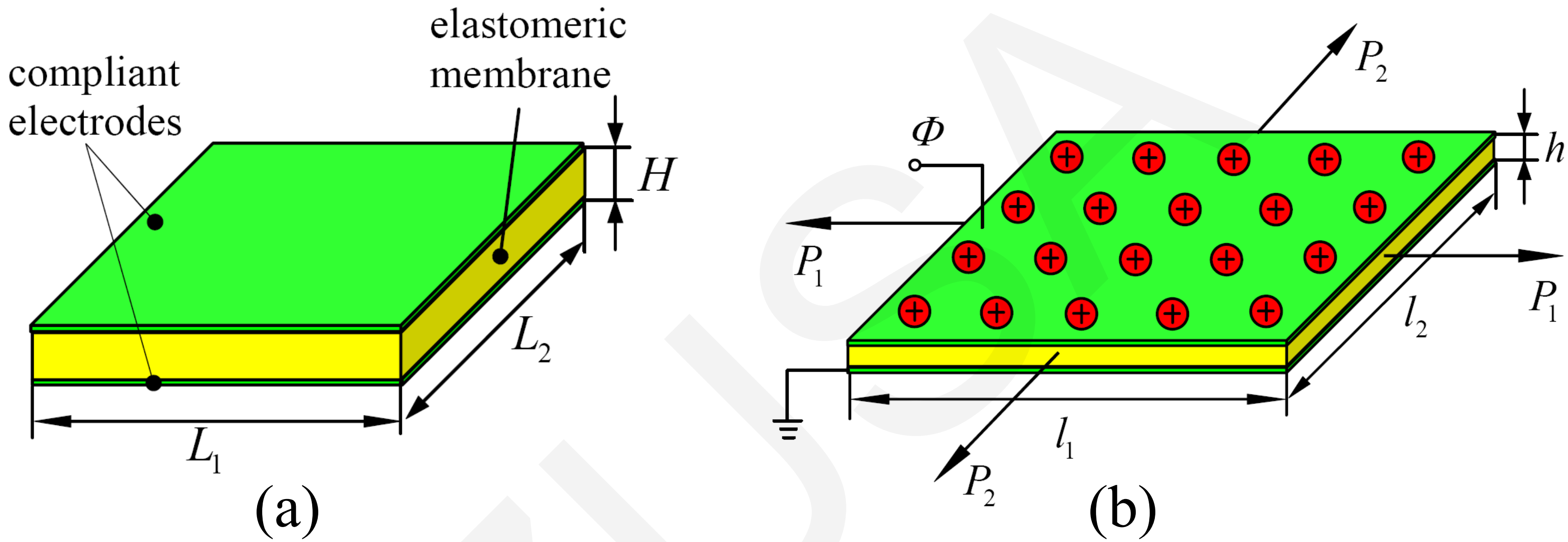
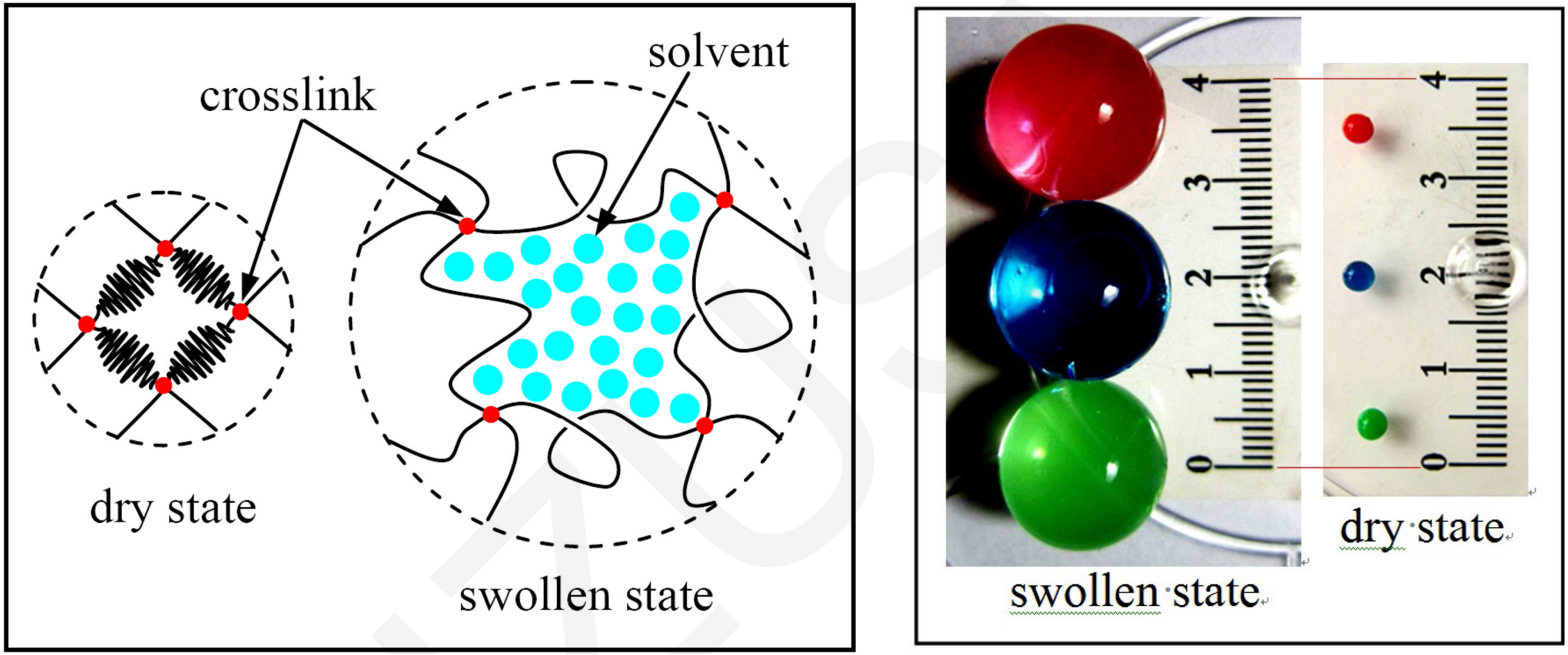



**Fig. 1 Schematics of a dielectric elastomer in different states**

(a) reference state: an elastomeric membrane of sides $L_1$ and $L_2$ and thickness $H$ is sandwiched between two compliant electrodes. (b) deformed state: the dielectric elastomer deforms to sides $l_1$ and $l_2$ and thickness $h$ when subjected to the mechanical loadings $P_1$ and $P_2$ and a high voltage $\Phi$.

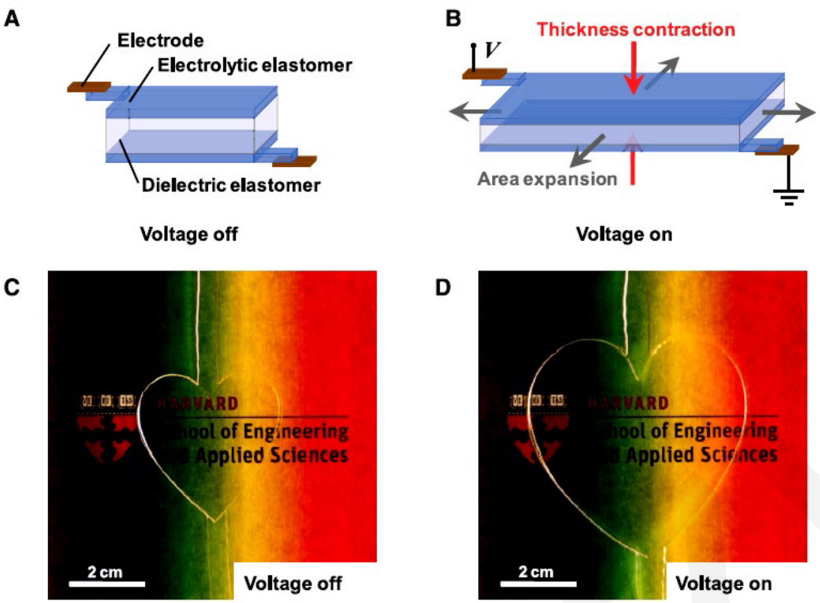
# Schematics of a responsive gel (RG)




(a) (b)

**Fig. 2 Schematics (a) and photos (b) of a responsive gel**

# Representative applications of DE



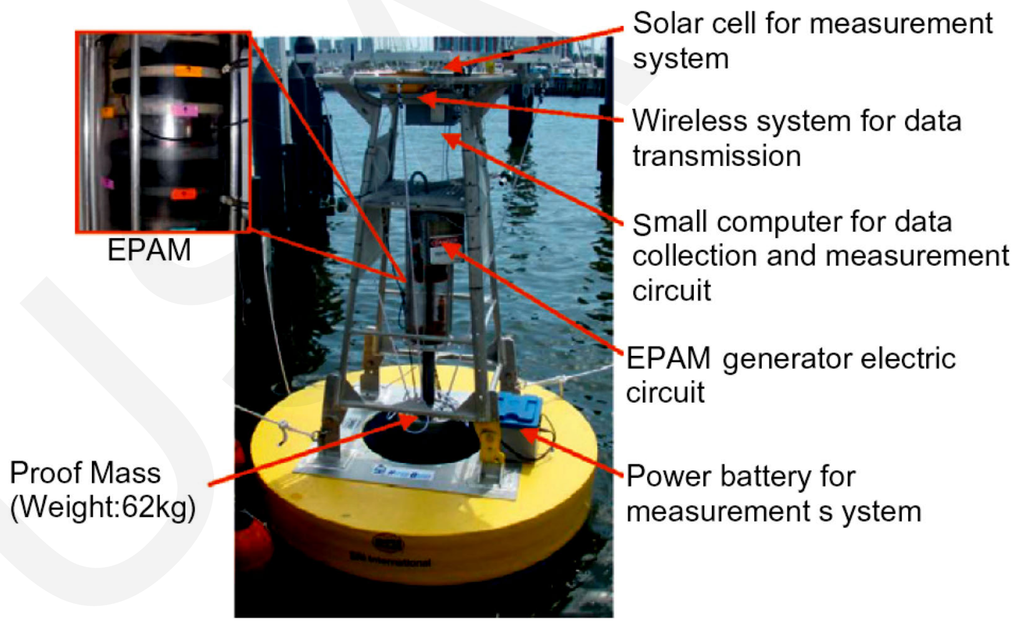




**Fig. 3 DE transparent actuator**

**Fig. 4 DE energy harvester**

C. Keplinger et al., Science, 2013, 341: 984-987.

Chiba S. et al., Applied Energy, 2013, 104: 497-502.

# Representative applications of RG

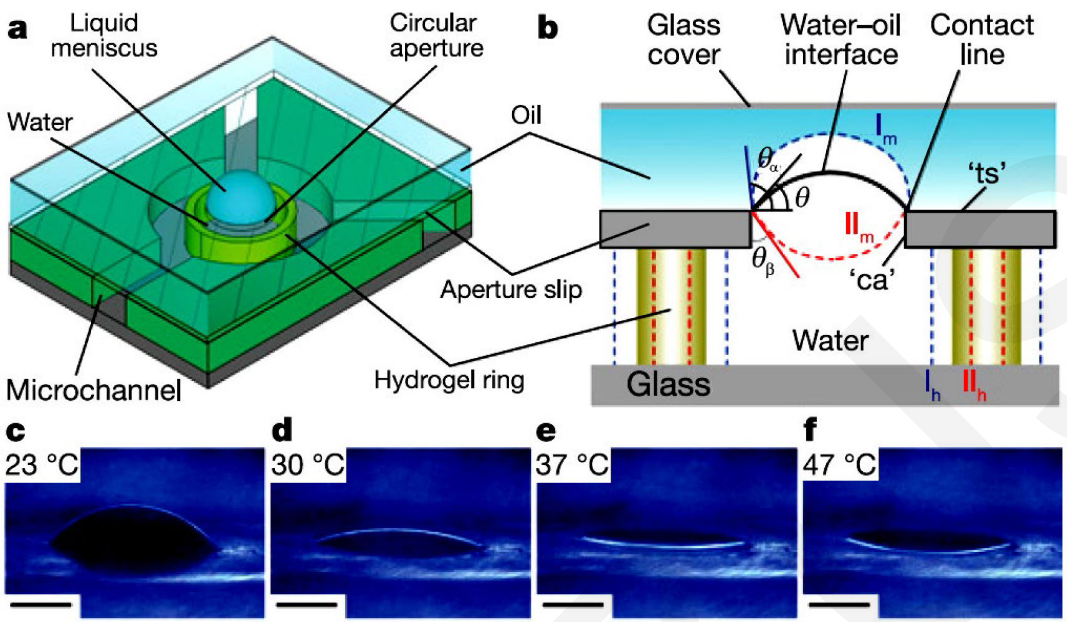



Fig. 5 microlens

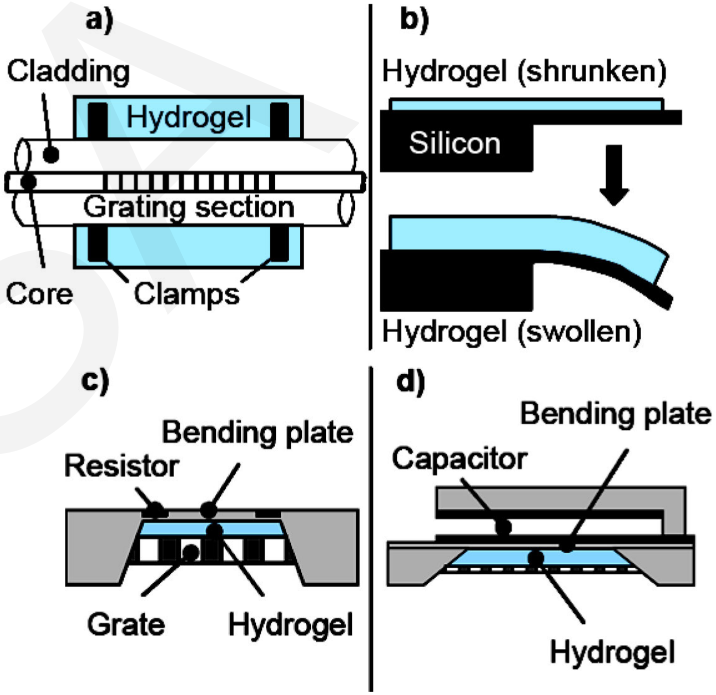



Fig. 6 Hydrogel sensors

L. Dong et al., Nature, 2006, 442: 551-554.

A. Richter, et al., Sensors, 2008, 8: 561-581.

## Table 1 Some frequently-used stretching energy functions for incompressible dielectric elastomers

| Model | Expression of $W_s$ | WLCE | NMP | Parameter | Type |
|---|---|---|---|---|---|
| neo-Hookean (Treloar, 1943) | $W_s = \frac{\mu}{2}(I_1 - 3)$ | No | 1 | $\mu$ | A |
| Mooney-Rivlin (Mooney, 1940; Rivlin, 1948) | $W_s = C_1(I_1 - 3) + C_2(I_2 - 3)$ | No | 2 | $C_1$, $C_2$ | B |
| Ogden (Ogden, 1972) | $W_s = \sum_{i=1}^{N} \frac{\mu_i}{\alpha_i}\left(\lambda_1^{\alpha_i} + \lambda_2^{\alpha_i} + \lambda_3^{\alpha_i} - 3\right)$ | Yes | $2N$ | $\mu_i$, $\alpha_i$ ($i$=1, 2, …, $N$) | B |
| Yeoh (Yeoh, 1990) | $W_s = \sum_{i=1}^{3} C_i (I_1 - 3)^i$ | No | 3 | $C_1$, $C_2$, $C_3$ | B |
| Arruda-Boyce (Arruda and Boyce, 1993) | $W_s = \mu n\left[\frac{\beta_{ch}\lambda_{ch}}{\sqrt{n}} + \ln\left(\frac{\beta_{ch}}{\sinh \beta_{ch}}\right)\right]$, $\lambda_{ch} = \sqrt{\frac{I_1}{3}} = \sqrt{n}\left(\frac{1}{\tanh \beta_{ch}} - \frac{1}{\beta_{ch}}\right)$ | Yes | 2 | $\mu$, $n$ | A |
| Gent (Gent, 1996) | $W_s = -\frac{\mu}{2} J_{\lim} \ln\left(1 - \frac{I_1 - 3}{J_{\lim}}\right)$ | Yes | 2 | $\mu$, $J_{\lim}$ | B |

* Note: type A: classic statistical mechanics model; type B: phenomenological model; NMP: number of material parameters; WLCE: with limiting chain extensibility; $I_1$ and $I_2$ are the first and second invariants of the right Cauchy-Green deformation tensor

# Conclusions

The constitutive models are an important foundation for predicting and improving the performance of artificial muscles.

Based on equilibrium and non-equilibrium thermodynamics, the constitutive models of dielectric elastomers and responsive gels can be effectively constructed by using free energy functions. In the models presented, the material parameters can be determined by fitting to the experimental data.

The constitutive models of artificial muscles have been widely used in analyzing the electromechanical or chemomechanical behaviors of dielectric elastomer and gel based actuators, sensors, energy harvesters, soft robots and soft tissues, etc.

There are a number of constitutive models for rubber-like materials. It is important to select a suitable model for theoretical analysis of soft materials.