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Direct reliability-based design optimization of uncertain structures with interval parameters

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Interval reliability-based design optimization model of an uncertain structure

With the uncertain factors described as interval variables and the interested mechanical properties described as the objective and constraint functions, the interval reliability-based design optimization model of an uncertain structure is described as

$$\begin{aligned}
&\min_{\boldsymbol{x}} f\left(\boldsymbol{x},\boldsymbol{U}\right)\\
&\text{s.t. } R_i\left[g_i\left(\boldsymbol{x},\boldsymbol{U}\right)\le B_i=\left[b_i^{\mathrm{L}},b_i^{\mathrm{R}}\right]\right]\ge\eta_i, i=1,2,\dots,p.\\
&\boldsymbol{x}=\left(x_1,x_2,\dots,x_n\right)\in\Omega^n\\
&\boldsymbol{U}=\left(U_1,U_2,\dots,U_m\right)
\end{aligned} \qquad (1)$$

where $\boldsymbol{x}$ is the n-dimensional design vector while $\boldsymbol{U}$ is the m-dimensional interval vector; $f(\boldsymbol{x},\boldsymbol{U})$ and $g_i(\boldsymbol{x},\boldsymbol{U})$ $(i=1,2,\dots,p)$ are the objective and constraint functions indicating the mechanical performance indices of the structure; B_i is the given interval constant of the ith constraint; R_i is the interval reliability of the ith constraint while η_i is the prescribed reliability requirement of the ith constraint.

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Unified formula for calculating interval reliability

The interval reliability $P(A \le B)$ for intervals A and B in any positional relationships can be calculated by:

$$R(A \le B) = P(A \le B)$$
$$= \max\left(\frac{\min\left(A^{R}-A^{L}, B^{R}-A^{L}\right)\times\left[\max\left(0, B^{R}-A^{L}\right)+\max\left(0, B^{R}-A^{R}\right)\right]}{2\,\mathrm{sign}\left[0.5+\mathrm{sign}\left(A^{L}-B^{L}\right)\right]\times\left(A^{R}-A^{L}\right)\times\left(B^{R}-B^{L}\right)}, \quad (2)$$
$$1-\frac{\min\left(A^{R}-B^{L}, B^{R}-B^{L}\right)\times\left[\max\left(0, A^{R}-B^{L}\right)+\max\left(0, A^{R}-B^{R}\right)\right]}{2\left(A^{R}-A^{L}\right)\times\left(B^{R}-B^{L}\right)}\right).$$

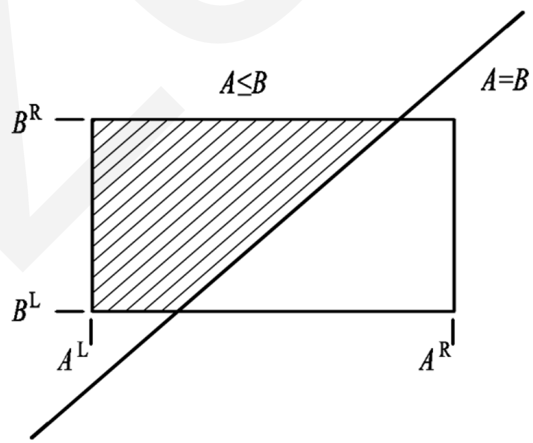

Fig.1 Sketch map of the graphical method.

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Degree of interval reliability violation (DIRV)

As far as the ith interval constraint $g_i(\boldsymbol{x},\boldsymbol{U}) \le B_i$ in Eq. (1) is concerned, the degree of interval reliability violation (DIRV) $V_i(\boldsymbol{x})$ corresponding to a design vector $\boldsymbol{x}$ is defined as

$$V_i(\boldsymbol{x}) = \max\left(0, \eta_i - R_i\left[g_i(\boldsymbol{x},\boldsymbol{U}) \le B_i = \left[b_i^{\mathrm{L}}, b_i^{\mathrm{R}}\right]\right]\right) \tag{3}$$

where R_i is calculated by the unified formula in Eq. (2).

After the DIRVs of all the constraint functions are calculated, the total DIRV (denoted as TDIRV for concision) corresponding to a design vector $\boldsymbol{x}$ can be obtained by

$$V_{\mathrm{T}}(\boldsymbol{x}) = \sum_{i=1}^{p} V_i(\boldsymbol{x}). \tag{4}$$

Then the design vector $\boldsymbol{x}$ is feasible when $V_{\mathrm{T}}(\boldsymbol{x}) = 0$ and it is infeasible when $V_{\mathrm{T}}(\boldsymbol{x}) > 0$.

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DIRV-based preferential guidelines

The merit ranking of different design vectors can be determined by the following DIRV-based preferential guidelines:

(1) A feasible vector $\boldsymbol{x}_i$ is always superior to an infeasible vector $\boldsymbol{x}_j$. That is, vector $\boldsymbol{x}_i$ is superior to vector $\boldsymbol{x}_j$ when $V_{\mathrm{T}}(\boldsymbol{x}_i) = 0$ and $V_{\mathrm{T}}(\boldsymbol{x}_j) > 0$.

(2) As for two infeasible design vectors $\boldsymbol{x}_i$ and $\boldsymbol{x}_j$, the merit ranking can be determined by their corresponding TDIRV values. Namely, $\boldsymbol{x}_i$ is superior to $\boldsymbol{x}_j$ if $V_{\mathrm{T}}(\boldsymbol{x}_i) \leq V_{\mathrm{T}}(\boldsymbol{x}_j)$.

(3) As for two feasible design vectors $\boldsymbol{x}_i$ and $\boldsymbol{x}_j$, the merit ranking is determined by their corresponding objective values. Specifically, $\boldsymbol{x}_i$ is superior to $\boldsymbol{x}_j$ when $f^{\mathrm{C}}(\boldsymbol{x}_i) < f^{\mathrm{C}}(\boldsymbol{x}_j)$ or $f^{\mathrm{C}}(\boldsymbol{x}_i) = f^{\mathrm{C}}(\boldsymbol{x}_j)$ and $f^{\mathrm{W}}(\boldsymbol{x}_i) < f^{\mathrm{W}}(\boldsymbol{x}_j)$.

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Algorithm for direct interval reliability-based design optimization

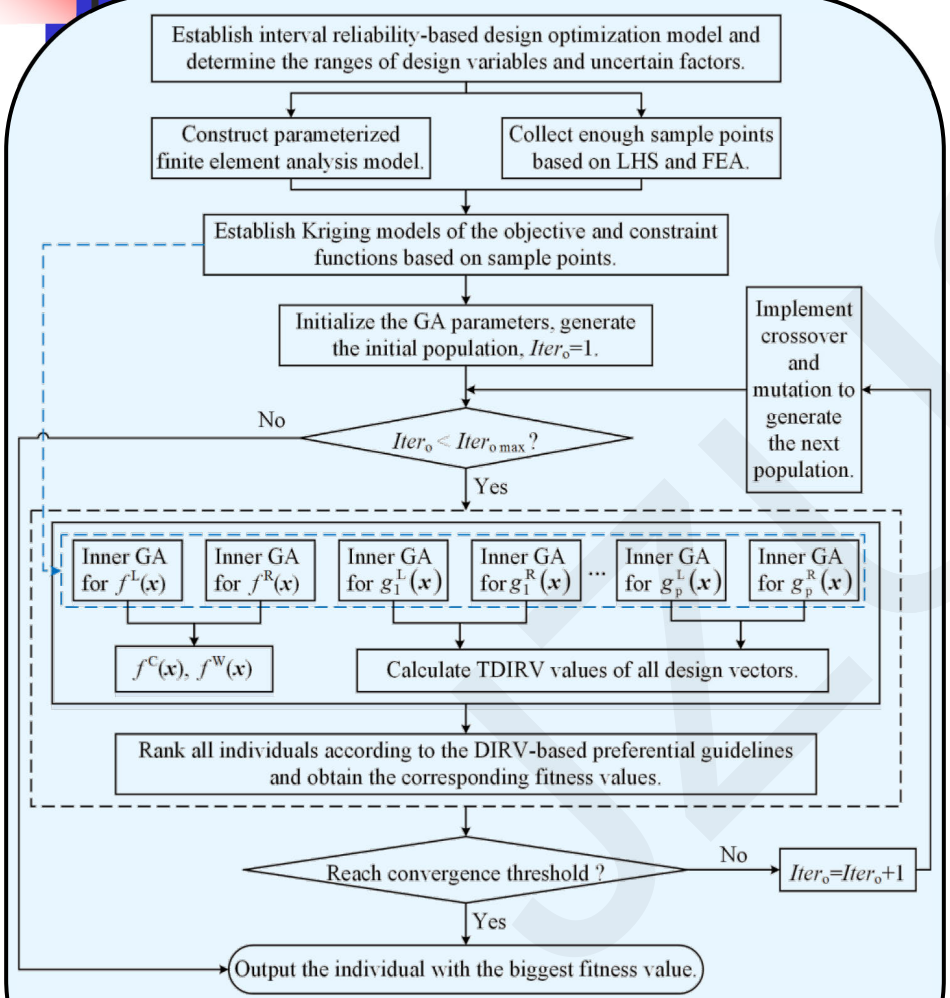

Fig.2 Flowchart of algorithm for solving the interval reliability-based design model

Numeric example

$$
\begin{cases}
\min\limits_{x} f(\boldsymbol{x},\boldsymbol{U}) = 130.0 - U_1^2(x_1+2) - U_2x_2^2 - U_3^2x_3^2 \\
s.t. \\
R_{g1}\left[g_1(\boldsymbol{x},\boldsymbol{U}) = U_1x_1^2 - U_2^2x_2 + U_3x_3 \le [8.0, 10.0]\right] \ge R_{s1} = 0.80; \\
R_{g2}\left[g_2(\boldsymbol{x},\boldsymbol{U}) = U_1x_1 + U_2x_2 + U_3^2x_3^2 + 1.0 \ge [75.0, 90.0]\right] \ge R_{s2} = 0.85; \\
\quad x_1 \in [-1.0, 5.0], x_2 \in [-3.0, 6.0], x_3 \in [-2.0, 7.0]; \\
\quad U_1 = [1.0, 1.3], U_2 = [0.9, 1.1], U_3 = [1.2, 1.4].
\end{cases}
$$

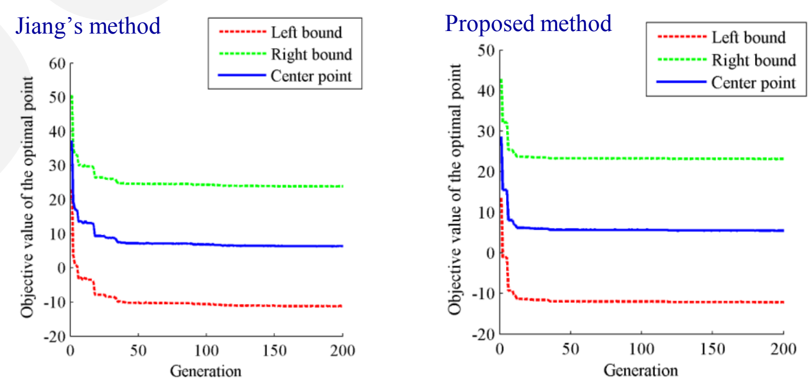

Fig.3 GA evolution of the objective value

Comparison of the optimization results obtained by different algorithms

Algorithm	Optimal solution	Constraint g_1	Constraint g_2	(R_1, R_2)	Objective f
Proposed	(1.97, 5.99, 7.00)	[5.02, 9.97]	[78.90, 106.15]	(0.80, 0.85)	<5.51, 17.68>
Jiang's	(1.89, 5.93, 7.00)	[4.80, 9.63]	[78.85, 105.93]	(0.86, 0.85)	<6.35, 17.57>

The optimal solution obtained by proposed algorithm is better than that of Jiang's since its corresponding objective's mid-point value is much smaller than that of Jiang's.

Application in engineering

The interval reliability-based design model of the upper beam in a high-speed press is

$$\min_{x} d(\boldsymbol{x},\boldsymbol{U}) = \min_{x} d(h_1,h_2,l_1,l_2,l_3,\rho,E)$$

s.t.

$$R_1\left[w(\boldsymbol{x},U_1) = w(\boldsymbol{x},\rho) \le [4990,5010]\text{kg}\right] \ge \eta_1 = 0.95;$$

$$R_2\left[\delta(\boldsymbol{x},\boldsymbol{U}) \le [44,45]\text{MPa}\right] \ge \eta_2 = 0.98.$$

$$\boldsymbol{x} = (h_1,h_2,l_1,l_2,l_3), \boldsymbol{U} = (U_1,U_2);$$

$$210\text{mm} \le h_1 \le 250\text{mm}, 250\text{mm} \le h_2 \le 300\text{mm},$$

$$80\text{mm} \le l_1 \le 120\text{mm}, 25\text{mm} \le l_2 \le 55\text{mm},$$

$$330\text{mm} \le l_3 \le 390\text{mm};$$

$$U_1 = \rho = [7280,7320]\text{kg}\cdot\text{m}^{-3},$$

$$U_2 = E = [126,154]\text{GPa}.$$

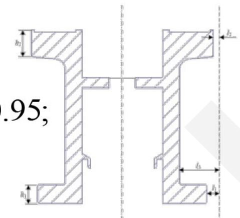

Fig.4 Cross section of the upper beam

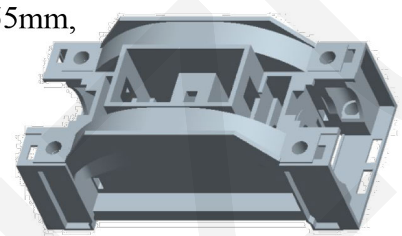

Fig.5 3D model of the upper beam

where $\boldsymbol{x} = (h_1,h_2,l_1,l_2,l_3)$ is the design vector;

$\boldsymbol{U} = (\rho,E)$ is the uncertain vector;

$d(\boldsymbol{x},\boldsymbol{U})$ is the maximum deformation of the upper beam;

$w(\boldsymbol{x},U_1)$ and $\delta(\boldsymbol{x},\boldsymbol{U})$ are the weight and equivalent stress;

R_1 and R_2 are the interval reliabilities of constraint functions $w(x, U_1)$ and $\delta(x, U)$ while η_1 and η_2 are their corresponding desired reliabilities.

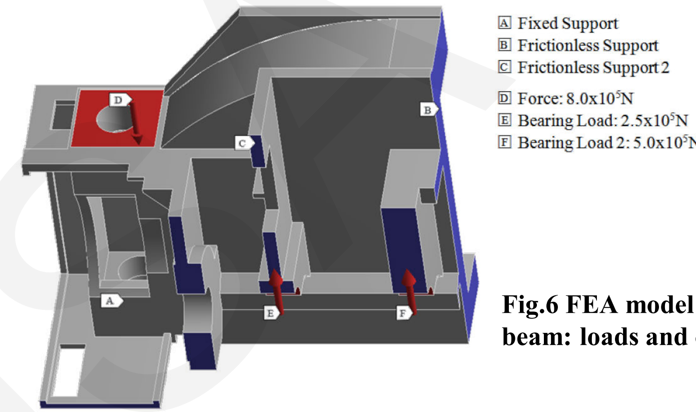

Fig.6 FEA model of 1/4 upper beam: loads and constraints

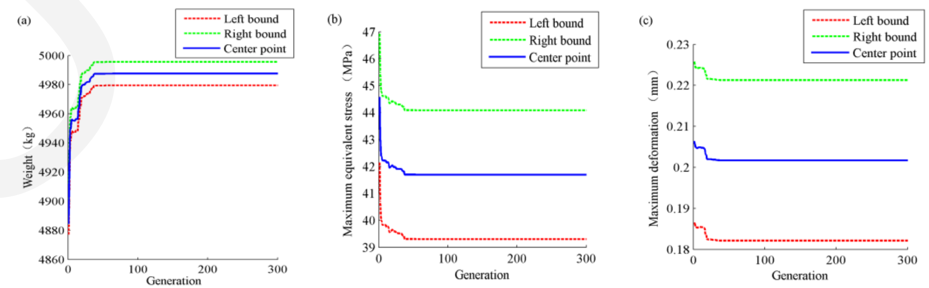

Fig.7 GA evolution curves of the mechanical performance indices obtained by proposed algorithm

Comparison of the optimization results of the upper beam obtained by proposed and indirect algorithms

Algorithm	Optimal solution (h_1,h_2,l_1,l_2,l_3) mm	Performance indices $[w^L, w^R]$ kg	$[\delta^L, \delta^R]$ MPa	$d(x, U)$ mm	Interval reliabilities (R_1, R_2)
Proposed	(249.89,262.32,80.36,36.88,388.40)	[4979.6, 4995.7]	[39.30, 44.08]	<0.2017,0.0196>	(0.95, 1.00)
Jiang's	(243.34,251.46,80.15,32.24,389.39)	[4979.6, 4995.5]	[39.52, 44.30]	<0.2162,0.0196>	(0.95, 0.99)

The optimal solution obtained by the proposed algorithm has the smaller deformation and larger interval reliability of constraint $\delta(x, U)$ than that obtained by indirect one.

Conclusions

- An interval reliability-based design optimization model is constructed to enhance the reliability of an uncertain structure and reduce its chance of function failure under potentially critical conditions.
- With the introduction of a unified formula for efficiently computing interval reliability, a new concept of the degree of interval reliability violation (DIRV) and the DIRV-based preferential guidelines are put forward for directly ranking various design vectors.
- A direct interval optimization algorithm integrating nested GA and Kriging technique is proposed for solving the interval reliability-based design optimization model.
- Both numeric and engineering examples demonstrate the feasibility and effectiveness of the proposed direct interval reliability-based design optimization method as well as its superiority to the conventional indirect one.

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