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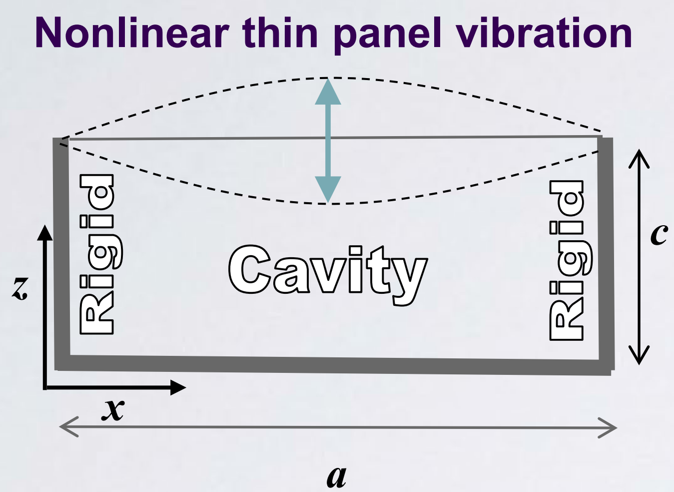
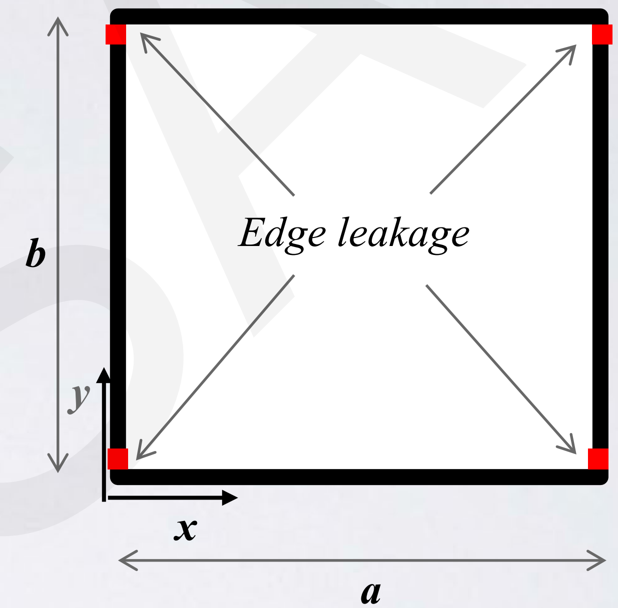
Large amplitude free vibration of a flexible panel coupled with a leaking cavity

Key words:
Large amplitude vibration, Elliptic integral method, Noise and vibration

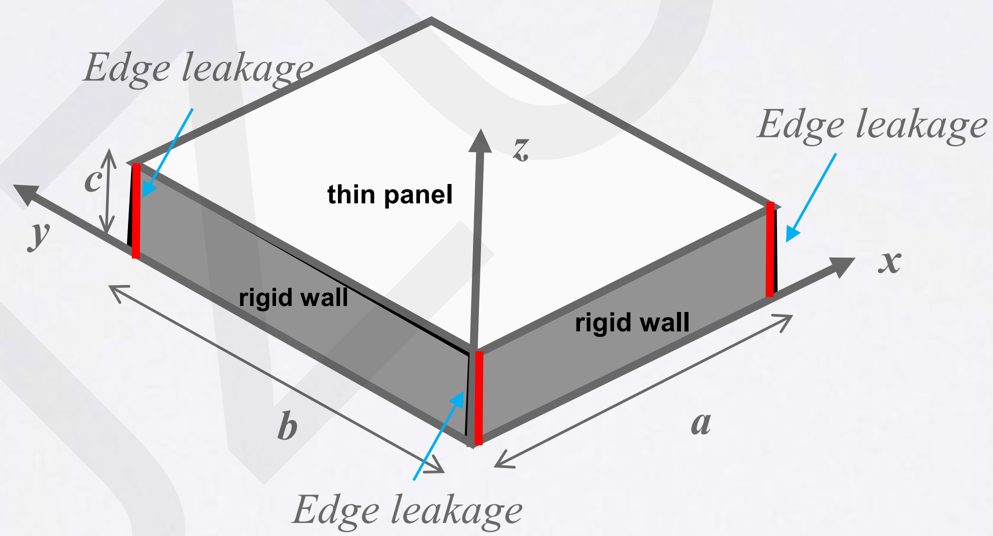
A panel backed by a cavity with edge leakages



Side view



Top view



ISO view

Highlights

- In many previous related works, the assumption of no leakage was adopted
- The leakage effect on a nonlinear panel cavity system is newly studied
- An elliptical integral method recently developed is adopted here
- A small leakage in a panel cavity system can seriously affect the vibration behaviors

Theory

Acoustic part

Wave equation

$$\nabla^2 P^h - \frac{1}{C_a^2}\frac{\partial^2 P^h}{\partial t^2} = 0$$

Acoustic mode equations

$$\varphi_{uv}(x,y) = \cos\left(\frac{(u+H(y))\pi}{a}x - H(y)\frac{\pi}{2}\right)\cos\left(\frac{v\pi}{b}y\right)$$

$$H(y) = 1 \text{ at } 0 \le y \le d$$

$$H(y) = 1 - \frac{1}{\Delta d}\left[(y-d) - \frac{\sin(\frac{2\pi}{\Delta d}(y-d))}{\frac{2\pi}{\Delta d}}\right] \text{ at } d < y \le d + \Delta d$$

$$H(y) = \frac{1}{\Delta d}\left[[y-(b-d-\Delta d)] - \frac{\sin(\frac{2\pi}{\Delta d}[y-(b-d-\Delta d)])}{\frac{2\pi}{\Delta d}}\right] \text{ at } b\text{-}d\text{-}\Delta d \le y < b\text{-}d$$

$$H(y) = 1 \text{ at } b\text{-}d \le y \le b$$

Acoustic pressure

$$P^h(x,y,z,t) = \rho_a (h\omega)^2 \sum_u^U \sum_v^V \frac{\cosh(\mu_{uv}^h z)}{\sinh(\mu_{uv}^h c)} \frac{A^h \alpha_{uv}}{\mu_{uv}^h \alpha_\varphi} \varphi_{uv}(x,y)\cos(h\omega t)$$

$$\bar{P}_c^h(t) = \rho_a (h\omega)^2 \sum_u^U \sum_v^V \frac{\coth(\mu_{uv}^h c)}{\mu_{uv}^h} \frac{A^h (\alpha_{uv})^2}{\alpha_\varphi \alpha_\phi} \cos(h\omega t)$$

Average acoustic pressure

Structural part

Duffing equation

$$\rho \frac{d^2 A}{dt^2} + (\rho\omega_o^2 + K_a)A + \beta A^3 + \sum_{h=1,3,5\ldots} \bar{P}_c^h(t) - K_a A = 0$$

Reduced Duffing equation

$$\rho \frac{d^2 \bar{A}}{dt^2} + (\rho\omega_o^2 + K_a)\bar{A} + \beta \bar{A}^3 = 0$$

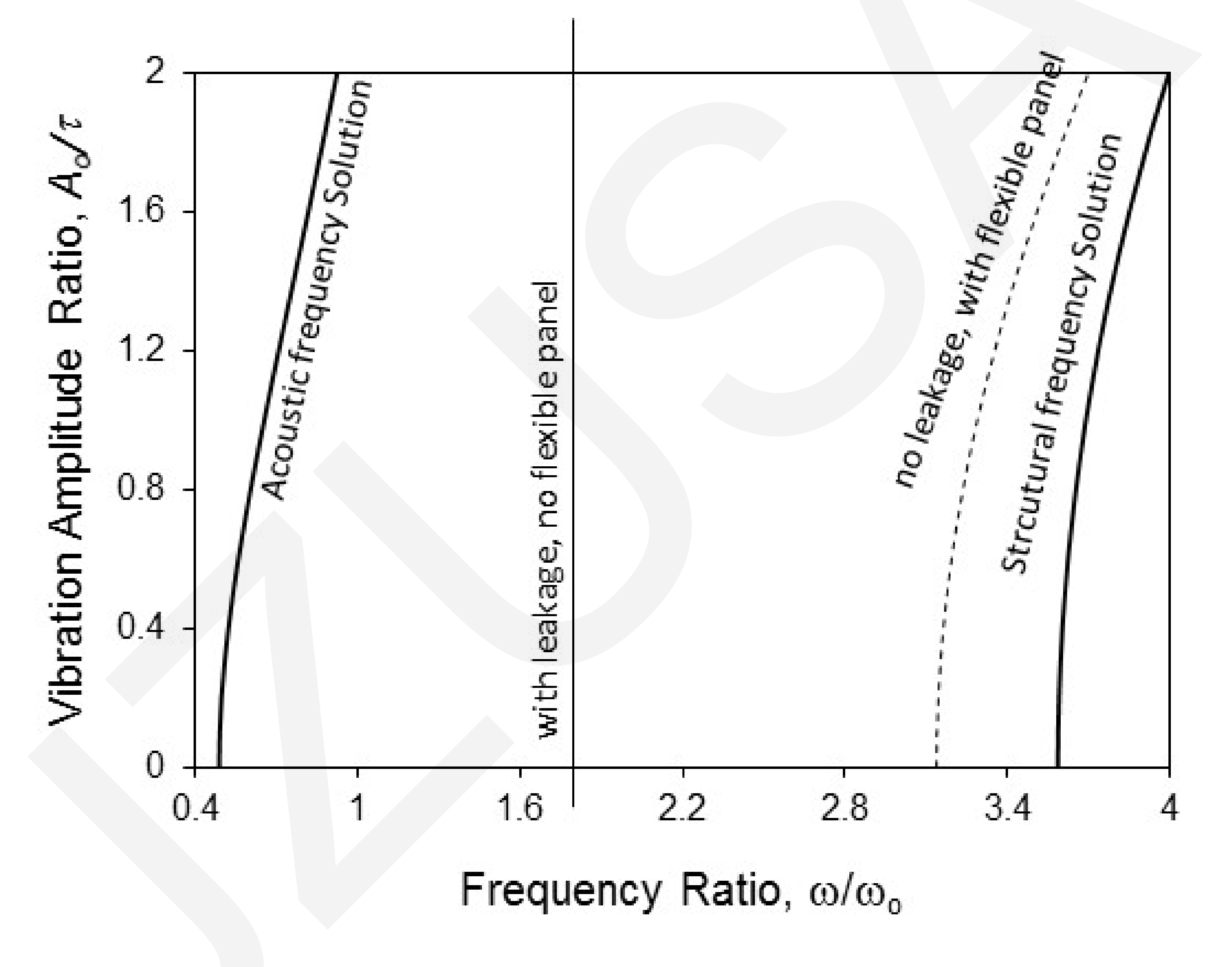
$\bar{A} = A_o cn(\mathbf{u}(\kappa))$ is the elliptical integral solution

$\mathbf{u}$ is the elliptic integral

$\kappa = \dfrac{\beta A_o^2}{2(\rho\omega_o^2 + K_a + \beta A_o^2)}$ is the modulus of $\mathbf{u}$

cn is the elliptic cosine

Selected Result
Vibration amplitude ratio versus frequency ratio



(panel dimensions = 0.3048m x 0,3048m x 0.6096mm, cavity depth = 0.1524m, leakage size = 1%)

Conclusions

- The edge leakages induce one more low frequency resonance
- The edge leakages can significantly affect the structural natural frequency in a panel cavity system and makes it higher
- Generally, when the leakage size is bigger, its effect on the acoustic and structural natural frequencies is also higher