Nonparametric modeling on random uncertainty and reliability analysis of a dual-span rotor

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Research Background

- Most of the contributions on the multi-span rotor are based on a deterministic model, without taking uncertainty into account.
- In practice, the probability density function (PDF) of the data can not be precisely obtained and different data components are dependent on each other.
- Previous studies were concerned with the parameter uncertainties while the nonparametric approach can deal with both parameter and model uncertainties.
- The nonparametric approach is applied to the reliability analysis in this work, and complicated multivariate equations are avoided.

Formulation of Problem

- The finite element model of the deterministic dual-span rotor can be expressed as

$$\underline{M}\ddot{\mathbf{U}} + \underline{C}\dot{\mathbf{U}} + \underline{K}\mathbf{U} = \underline{F},$$

- $\underline{\mathrm{M}}$, $\underline{\mathrm{F}}$, and $\underline{\mathrm{C}}$ are all assumed to be accurately measured, i.e. the system is assumed to have no approximation nor simplification, and the material properties are also assumed to have no change during the running of the system.
- In reality, errors always exist between the design parameters and real working conditions. To represent the real operational state, uncertain factors must be taken into account, and consequently, the random matrix model is of more practical significance .

$$M_{\mathrm{r}}\ddot{Z} + C_{\mathrm{r}}\dot{Z} + K_{\mathrm{r}}Z = F_{\mathrm{r}},$$

Reliability analysis

- Makes full use of the advantage of the nonparametric method and the response spectral analysis

$$U(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\psi H(\omega)F(\omega)\mathrm{e}^{j\omega t}\,\mathrm{d}\omega$$

- While in the complex frequency domain, we have

$$R_z(\tau)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}H_{\mathrm{r}}^{*}(\omega)S_{\mathrm{r}}(\omega)H_{\mathrm{r}}(\omega)\mathrm{e}^{j\omega\tau}\mathrm{d}\omega,$$

- We focus on the deviation distance of the shaft and not the coordinate values of vibration, so the object variable

$$U^2=\frac{1}{2\pi}\psi\int_{-\infty}^{+\infty}H_{\mathrm{r}}^{*}(\omega)S_{\mathrm{r}}(\omega)H_{\mathrm{r}}(\omega)\,\mathrm{d}\,\omega\psi^{\mathrm{T}},$$

- The reliability degree R_{d} can be expressed by a normalization processing

$$R_{\mathrm{d}}=\rho(\beta)-\frac{\varsigma}{3!}\rho^{(3)}(\beta)-\frac{\zeta-3}{4!}\rho^{(4)}(\beta)-\frac{10\varsigma^{2}}{6!}\rho^{(6)}(\beta)$$

Reliability analysis

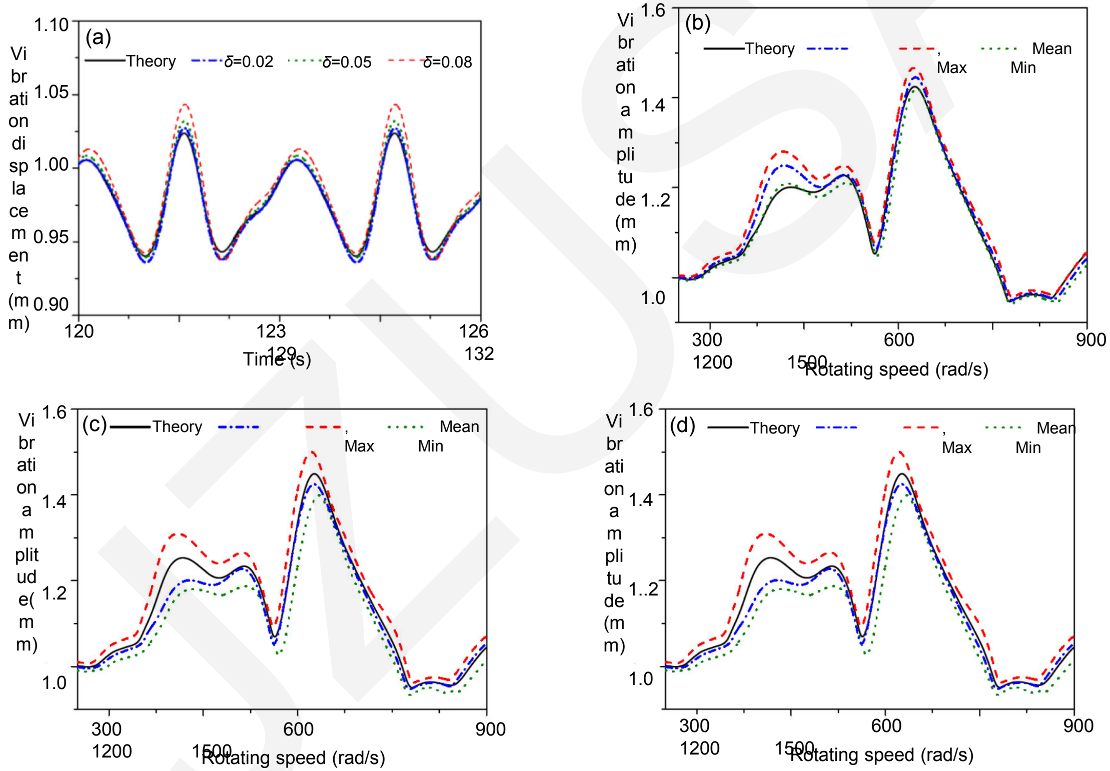
- To adequately illustrate the effect of uncertainties, the short-term predictability is adopted as a supplement and verification to the proposed procedure.
- The principle is: first, formulate a null hypothesis that the original time series and the measured data are produced by the same mechanism, then, determine the statistic of the distance during a short period of time. Generally, a larger value of $|Z^*|$ than 2.33 signifies that the null hypothesis does not hold at the 99% confidence level.

$$d_{i,j} = \max_{0 \le s \le m_0 + m_\mathrm{d} - 1} \left| x_{i+s} - x_{j+s} \right|,$$

$$Z^* = \frac{U_\varepsilon - N_A N_B / 2}{\sqrt{\frac{1}{12} N_A N_B (N_A + N_B + 1)}}.$$

Numerical Example

- The levels of the uncertainty are usually difficult to determine. Here, we choose three different values of the divergence control parameter i.e. δ=0.02, 0.05, and 0.08.



- Fig. 2 Response of disc 1 of the dual-span rotor system with random uncertainties: (a) time series of vibration; (b), (c), and (d): vibration amplitude fluctuations with δ=0.02, 0.05, and 0.08, respectively.

Numerical Example

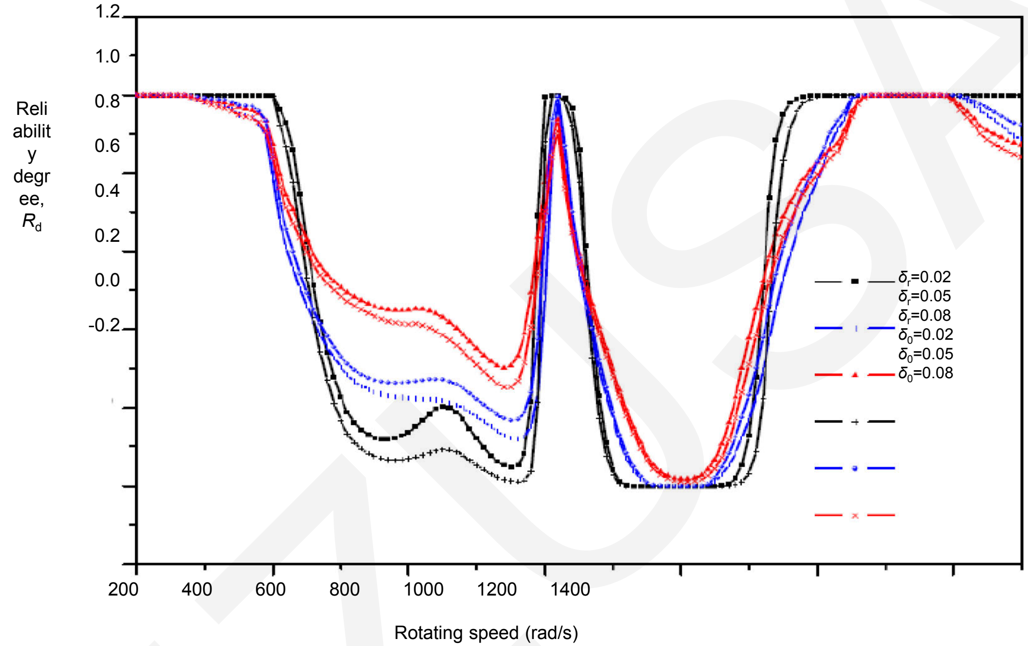
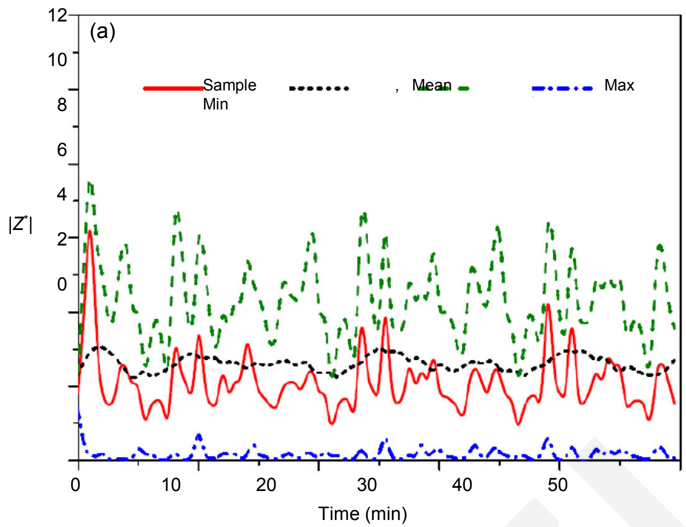
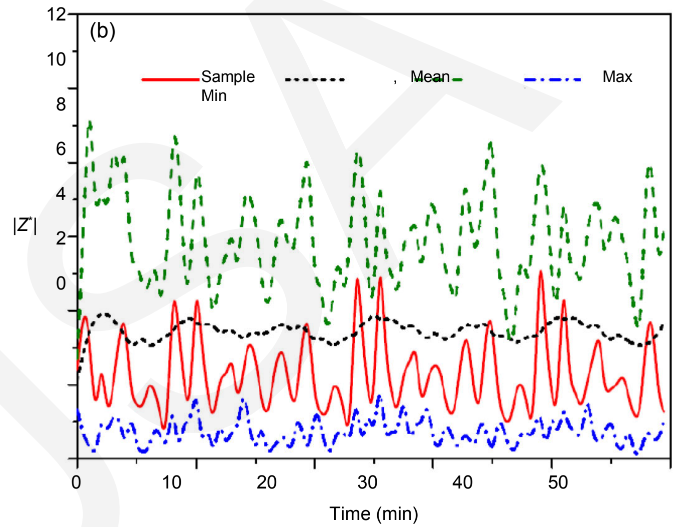


Fig. 3 Reliability degrees Rd of the two methods for the three different dispersion parameters, i.e. δ=0.02, 0.05, and 0.08

Numerical Example





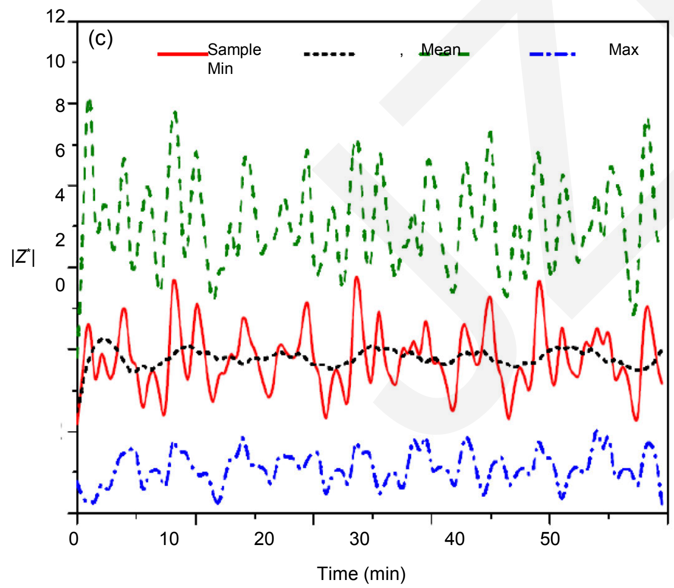


Fig. 4 Statistical indices in three cases: (a) δ=0.02, (b) δ=0.05, and (c) δ=0.08

Conclusions

- A reliability estimation procedure combining a nonparametric model with response spectrum analysis is proposed in this study.
- This validates the fact that the reliability calculation can be carried out without knowing discrete movements. Also, a large amount of information needed for determining the PDF is avoided in the reliability analysis.
- Solving tedious and consuming equations is avoided using this method. From the simulation results, the safe or failure ranges of the dual-span rotor system can be clearly found from the reliability curves.