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Dynamics of a periodically driven chain of coupled nonlinear oscillators

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Abstract

- A 1D chain of coupled oscillators is considered, including the Duffing-type nonlinearity, viscous damping, and kinematic harmonic excitation
- The approximate equations for the vibrational amplitudes and phases are derived by means of the classical averaging method
- A simple analysis of the equations allows one to determine the conditions for the two basic synchronous steady-states of the system: the in-phase and anti-phase motion
- The validity of these predictions is examined by a series of numerical experiments
- The effect of the model parameters on the rate of synchronization is analyzed
- For the purpose of systematic numerical studies, the cross-correlation of time-series is used as a measure of the phase adjustment between particular oscillators

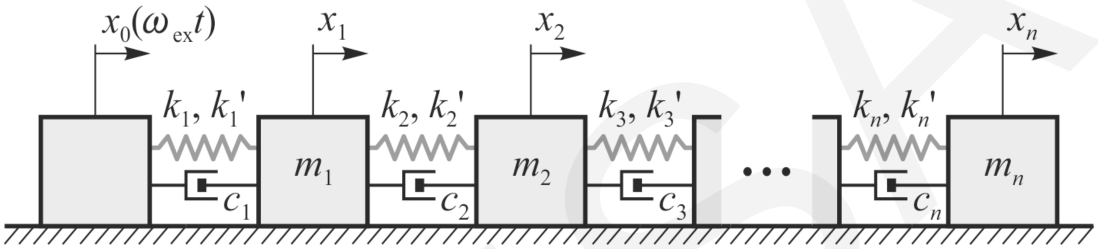

Fig. 1 The 1D chain-like mechanical system

The equations of motion of the system in a non-dimensional form:

$$\begin{aligned}\ddot{X}_i + \omega_i^2 X_i &= \alpha_{i,i-1}X_{i-1} + \alpha_{i,i+1}X_{i+1} \\ &\quad - \beta_{i,i-1}(X_i - X_{i-1})^3 + \beta_{i,i+1}(X_{i+1} - X_i)^3 \\ &\quad - \gamma_{i,i-1}(\dot{X}_i - \dot{X}_{i-1}) + \gamma_{i,i+1}(\dot{X}_{i+1} - \dot{X}_i),\end{aligned}$$

where $i = 1, 2, \ldots, n$, and the coefficients α, β, γ correspond to the linear elastic, nonlinear elastic and viscous interactions between particular members.

Analytical studies – Averaging procedure

According to the classical method of averaging the trial solutions are assumed in the form:

$$X_i(\tau) = a_i(\tau)\cos\psi_i(\tau)$$

$$\psi_i(\tau) = \Omega_{\mathrm{ex}}\tau + \phi_i(\tau)$$

where the amplitudes a_i and total phases ψ_i are regarded as slowly varying with time.

Treating a_i and ϕ_i as constants over the excitation period, the averaged equations describing the slow variations of amplitudes and phases are obtained.

Next, the approximate conditions for steady-state in-phase and anti-phase solutions can be determined.

In-phase and anti-phase vibrations

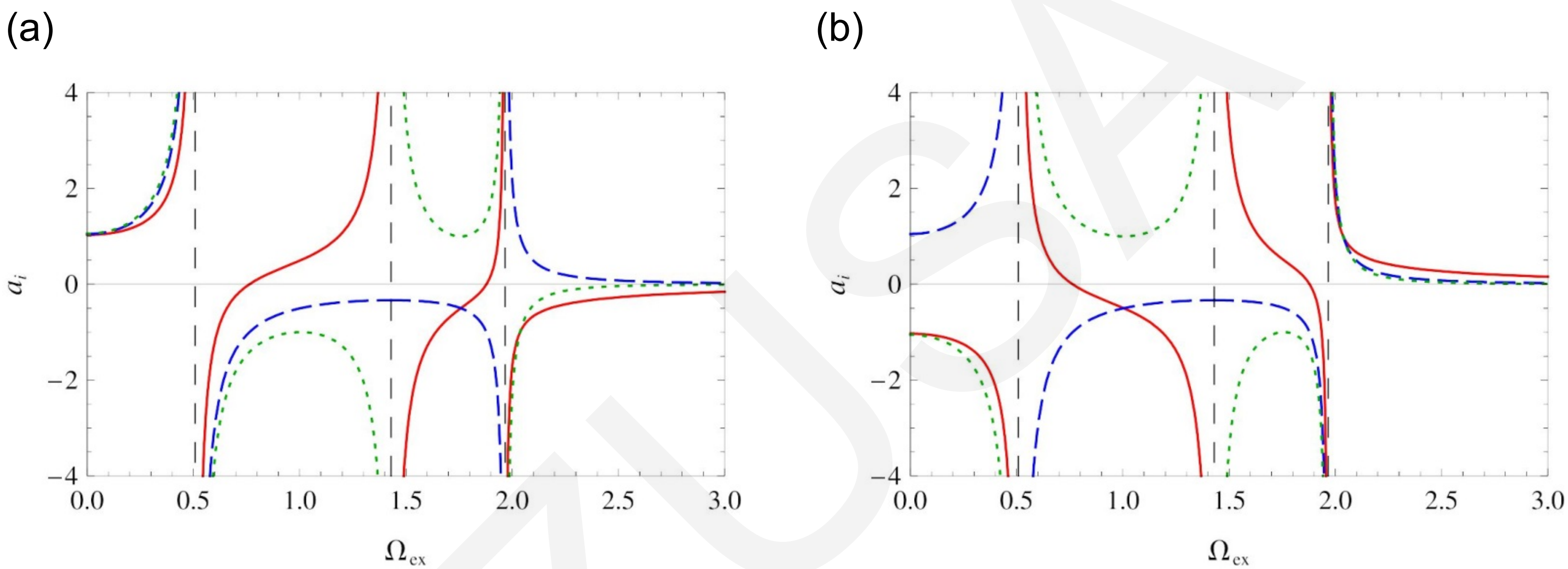

Fig. 2 Amplitudes of particular oscillators in the linear case for $n = 3$ with the assumption of in-phase (a) and anti-phase (b) vibrations: a_1 (red), a_2 (blue), and a_3 (green). The vertical asymptotes correspond to the natural frequencies of the system

Numerical studies

For the purpose of systematic numerical studies, the cross-correlation between time series X_i and X_j is applied:

$$r_{i,j}(\Delta\tau_J) = \sum_I X_i(\tau_I) X_j(\tau_I - \Delta\tau_J),$$

where the uppercase indices I, J refer to time points. If h denotes the step size, then

$$\tau_I = Ih, \quad \Delta\tau_J = Jh, \quad I \geq J.$$

The cross-correlation is computed over at least one period of steady-state response, after omitting the range of transient motion.

Numerical studies

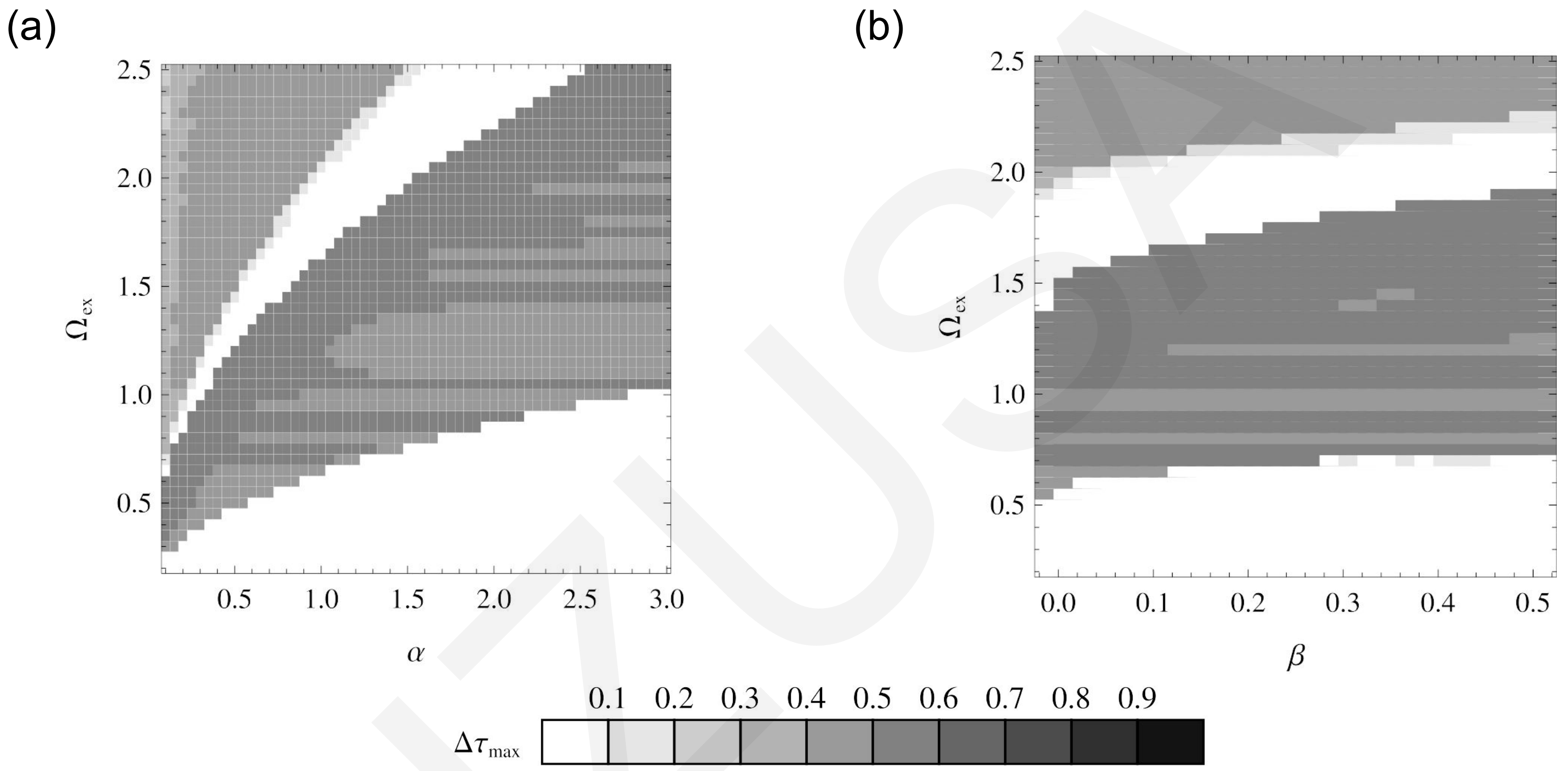

Fig. 3 The effect of parameters α, β, and Ω_{ex} on the cross-correlation between X_3 and X_0: time shift $\Delta\tau_{max}$ on the parameter planes (α, Ω_{ex}) (a) and (β, Ω_{ex}) (b).
A system of $n = 3$ oscillators

Conclusions and remarks

- The analysis based on the averaging method has enabled the determination of two synchronous regimes: the in-phase state (for low excitation frequency) and the anti-phase state (for high excitation frequency)
- The prediction related to the in-phase regime is quite trivial, and works well for multi-degree-of-freedom chains, however, the excitation frequency has to be considerably lowered for a large n
- The prediction of the anti-phase motion is valid in the case of short chains ($n < 14$); the phase shift between the consecutive oscillators response and the excitation increases linearly with growing n
- In the case of the system with friction, the considerations become much more complicated; it is highly important to determine the minimal excitation amplitude which is sufficient to generate vibrations of all members