Rocking of a rigid block freestanding on a flat pedestal

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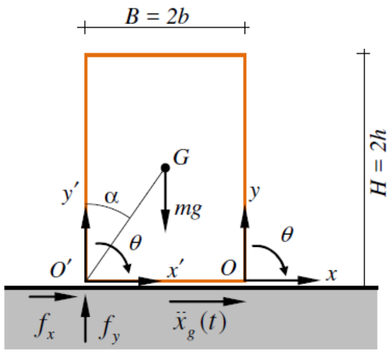
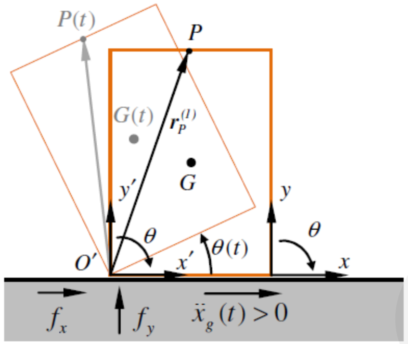
Single rocking block





$$r = 1 - \frac{3}{2}\, sin^2\alpha \quad \text{with} \quad 0 < r < 1$$

$$\dot{\theta}^{+}(t) = r\,\dot{\theta}^{-}(t)$$

Square root of the Housner restitution coefficient

$$\left(I_o - 2\,m\,r\,b\,sin\alpha\right)\dot{\theta}^{-}(t) = I_o\,\dot{\theta}^{+}(t)$$

Conservation of angular momentum

$$\left|f_x\right| \le \mu_s f_y$$

Friction force

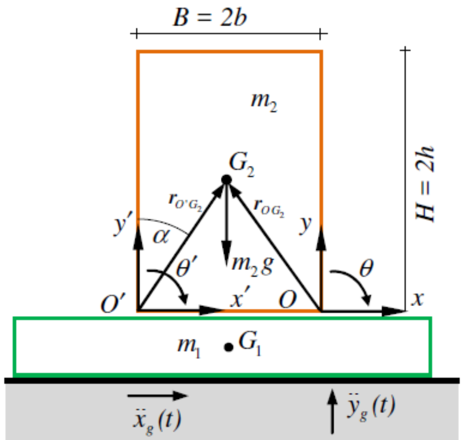
Equation of motion (clockwise positive)

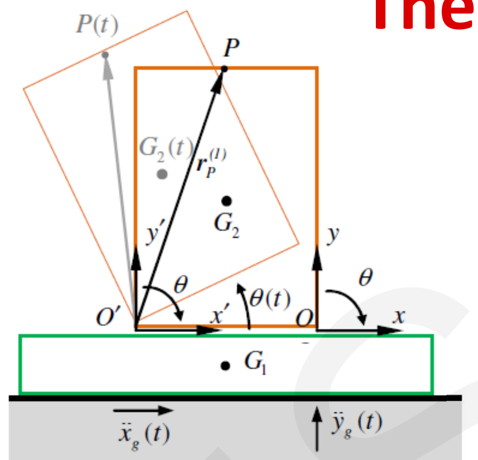
$$I_{O'}\,\ddot{\theta}(t) + m\,g\,R\,sin\left(-\alpha - \theta(t)\right) = -m\,\ddot{x}_g(t)\,R\,cos\left(-\alpha - \theta(t)\right) \quad , \quad \theta(t) < 0$$

$$I_O\,\ddot{\theta}(t) + m\,g\,R\,sin\left(\alpha - \theta(t)\right) = -m\,\ddot{x}_g(t)\,R\,cos\left(\alpha - \theta(t)\right) \quad , \quad \theta(t) > 0$$

$$\dot{\theta}^{+}(t) = r\,\dot{\theta}^{-}(t) \quad , \quad \theta(t) = 0$$

The double block problem





$$T_1(t) = \frac{1}{2} m_1 \dot{\boldsymbol{x}}_{G_1} \cdot \dot{\boldsymbol{x}}_{G_1}$$

$$T_2(t) = \frac{1}{2}[J_{G_2} \dot{\theta}^2(t) + m_2 \dot{\boldsymbol{x}}_{G_2}(t) \cdot \dot{\boldsymbol{x}}_{G_2}(t)]$$

$$V_1(t) = m_1 \, g \, \boldsymbol{x}_{G_1} \cdot \boldsymbol{j}$$

$$V_2(t) = m_2 \, g \, \boldsymbol{x}_{G_2} \cdot \boldsymbol{j}$$

$$T(t) = T_1(t) + T_2(t)$$

$$V(t) = V_1(t) + V_2(t)$$

$$F_{friction} = -\mu_k (m_1 + m_2)(g + \ddot{y}_g)\ddot{x}_g \, \mathrm{sgn}(\ddot{x}_g(t))$$

$$L(t) = T(t) - V(t)$$

$$q_1 = \theta(t) \;,\; q_2 = x_{G_1}$$

Lagrangian parameters

$$\frac{\partial^2 L(t)}{\partial t \partial \dot{q}_k} - \frac{\partial L(t)}{\partial q_k} = Q_k(t) \;,\; k = 1,2$$

Contribute of nonconservative forces

The double block problem: motion equations

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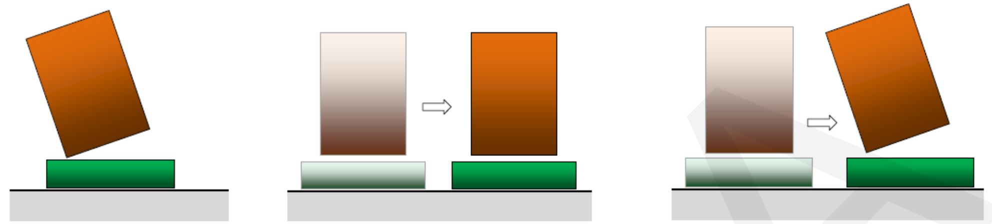


$$\begin{cases} J_O\ddot{\theta}(t) - m_2 R\cos[\alpha-|\theta|](\ddot{x}_g(t)+\ddot{x}_{G_1}(t)) + m_2 R\, g\, \mathrm{sgn}(\theta(t))\sin[\alpha-|\theta|]=0 & ,\theta(t)\neq 0\\ M(\ddot{x}_g(t)+\ddot{x}_{G_1}(t)) + \mathrm{sgn}(\theta(t))\{-m_2R[\sin(\alpha-|\theta|)\dot{\theta}^2(t)-\cos(\alpha-|\theta|)\,\ddot{\theta}(t)]+M\,\mu_k g\}=0 & ,\theta(t)\neq 0\\ \dot{\theta}^+(t)=r\,\dot{\theta}^-(t) & ,\theta(t)=0 \end{cases}$$

$$J_O\ddot{\theta}(t) - m_2 R\cos(\alpha-|\theta|)\,\ddot{x}_g(t) + m_2 R\, g\, \mathrm{sgn}(\theta(t))\sin(\alpha-|\theta|)=0$$

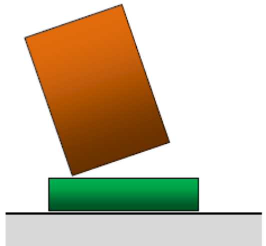
$$\ddot{x}_g(t)=A\cos\omega t$$

Double block problem

Single block problem
(upper block rocking)

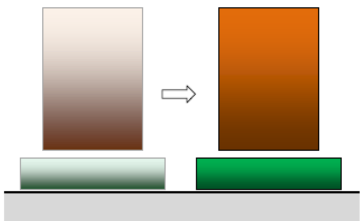
The double block problem: possible motions

(Mathematica©)



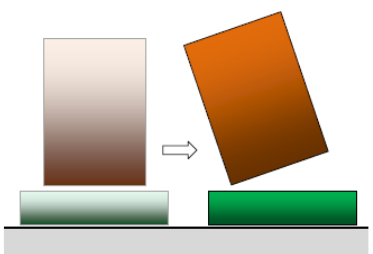
$$\begin{cases} M\,\ddot{x}_g(t) < \mathrm{sgn}(\dot{x}_g(t))\,M\,\mu_k\,g \\ \ddot{x}_{G_2}(t)\,h > g\,b\,. \end{cases}$$

$$t_a \;\Rightarrow\; \ddot{x}_{G_2}(t_a)\,h = g\,b$$



$$\begin{cases} M\,\ddot{x}_g(t) > \mathrm{sgn}(\dot{x}_g(t))\,M\,\mu_k\,g \\ \ddot{x}_{G_2}(t)\,h < g\,b\,. \end{cases}$$

$$t_b \;\Rightarrow\; M\,\ddot{x}_g(t_b) = \mathrm{sgn}(\dot{x}_g(t_b))\,M\,\mu_k\,g$$



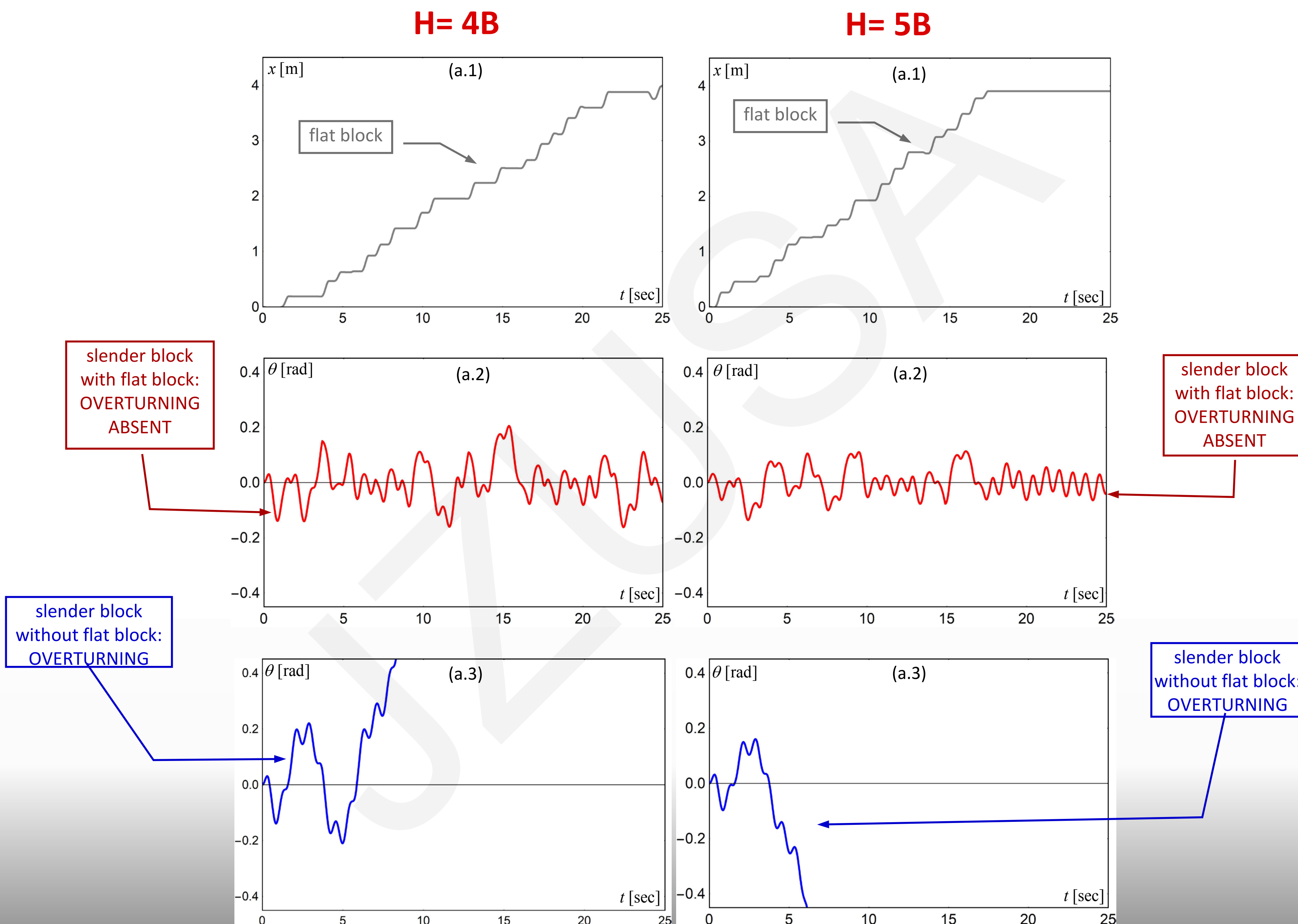
$$\begin{cases} M\,\ddot{x}_g(t) > \mathrm{sgn}(\dot{x}_g(t))\,M\,\mu_k\,g \\ \ddot{x}_{G_2}(t)\,h > g\,b\,. \end{cases}$$

Lower block sliding stops when:

$$\dot{q}_1(t) = \dot{x}_{G_1}(t) = 0$$

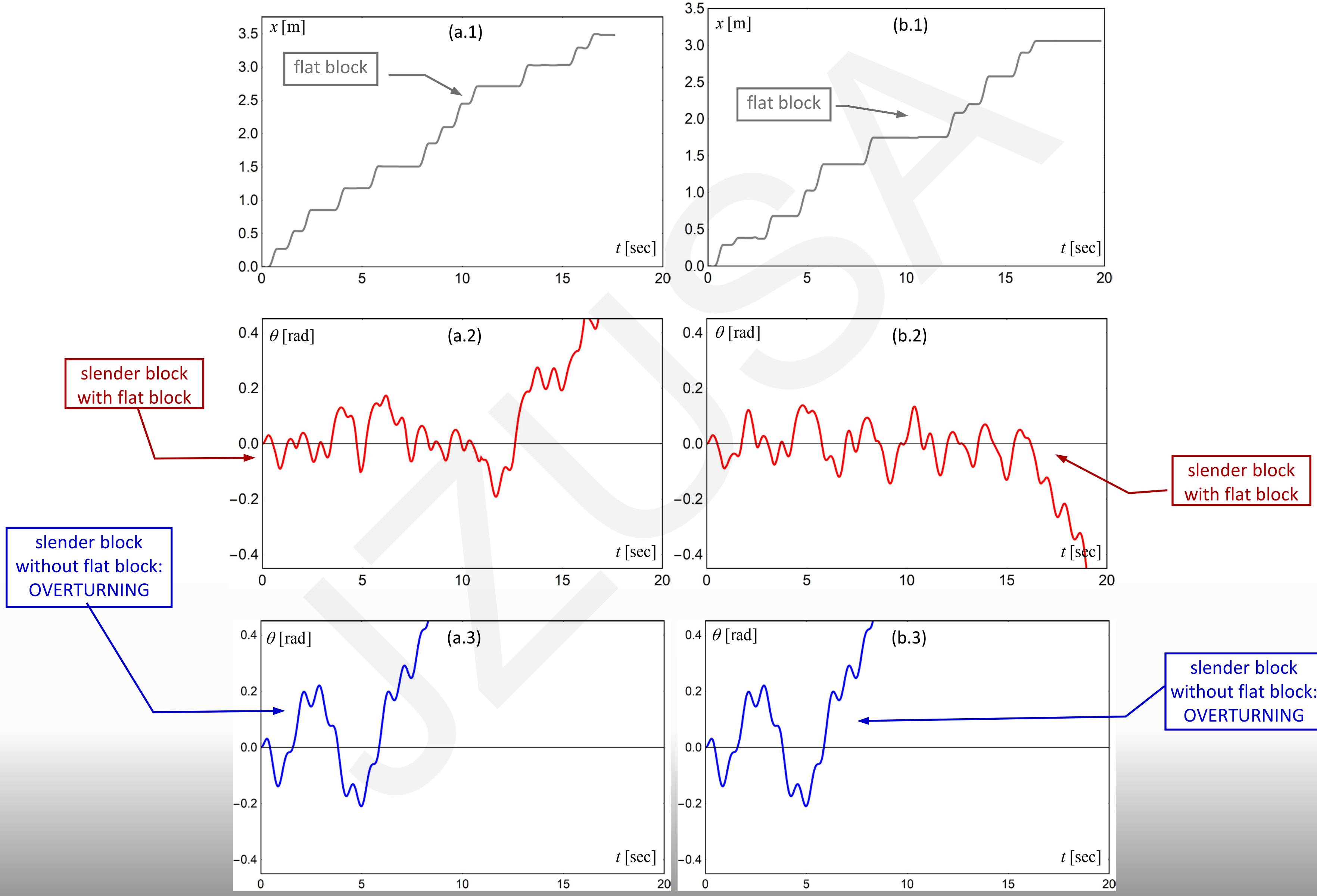
$$M\,\ddot{x}_g(t) - m_2 R\,\mathrm{sgn}(\theta(t))\,\{\sin(\alpha - |\theta(t)|)\,\dot{\theta}^2(t) - \cos(\alpha - |\theta(t)|)\,\ddot{\theta}(t)\} \geq \mathrm{sgn}(\theta(t))\,M\,\mu_s g$$

The double block problem: top block slenderness variation



The double block problem: mass variation

$m_2 = 4m_1$ | $m_2 = 6m_1$



Conclusions

- The real situation of a marble statue placed on a squat rigid base freestanding on a moving floor is presented in this paper.

- The motion patterns examined in this paper involve only the sliding motion for the lower flat block and rocking as the only possible motion for the stacked slender block.

- The solution of the differential equations system has been shown that the presence of a rigid surface delays and in some cases avoids the overturning of a slender rigid artifact.

- This is true especially for slender rocking blocks, and for increasing mass of the upper block. The numerical analysis can be a tool in the choice of the optimal design of the simple isolation system analyzed.