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Revisiting aerodynamic admittance functions of bridge decks

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Key words: Aerodynamic admittance function; Bridge deck; Buffeting analysis; Wind tunnel test; Sensitivity analysis

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About admittance

❖Quasi-steady and strip assumption

Buffeting wind loads in the lift-direction

$$L_b(t) = \rho U B\left(C_L(\alpha)\chi_{Lu}u(t) + 1/2\left(C_L'(\alpha) + C_D(\alpha)\right)\chi_{Lw}w(t)\right)$$

$$S_L(\omega) = \rho^2 U^2 B^2\left(C_L^2(\alpha)\left|\chi_{Lu}\right|^2 S_u(\omega) + 1/4\left(C_L'(\alpha) + C_D(\alpha)\right)^2\left|\chi_{Lw}\right|^2 S_w(\omega)\right)$$

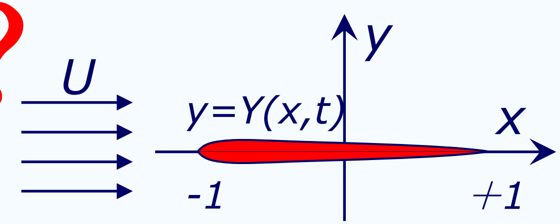
PSD of Lift force

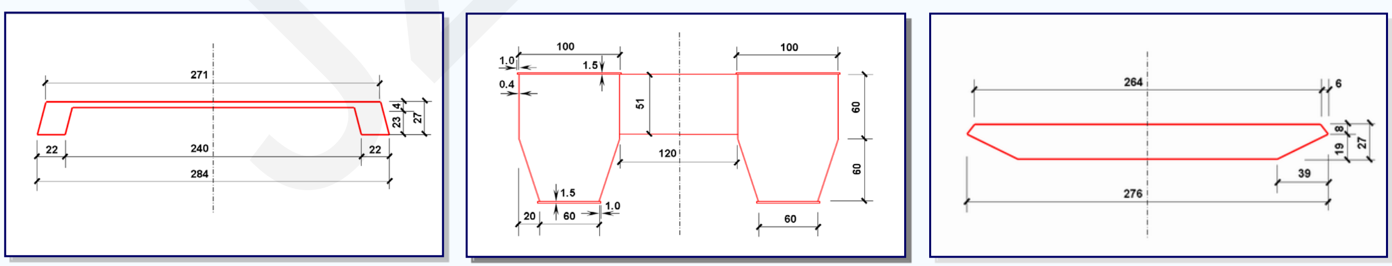
Admittance function

PSD of wind

Liepmann's approximation to Sears' function

$$\left|\phi_{sears}(K)\right|^2 = \frac{1}{1+\pi K} = \frac{1}{1+\pi\dfrac{\omega B}{U}}$$





Some typical blunt bridge sections

Identification of full para.

❖ Aerodynamic admittance identification (i.e. Lift admittance)

$$S_{Lu}(\omega)=\rho UB\left(C_L(\alpha)\chi_{Lu}(\omega)S_u(\omega)+1/2\left(C_L'(\alpha)+C_D(\alpha)\right)\chi_{Lw}(\omega)S_{wu}(\omega)\right)$$

$$S_{Lw}(\omega)=\rho UB\left(C_L(\alpha)\chi_{Lu}(\omega)S_{uw}(\omega)+1/2\left(C_L'(\alpha)+C_D(\alpha)\right)\chi_{Lw}(\omega)S_w(\omega)\right)$$

$$\chi_{Lu}(\omega)=\frac{S_w(\omega)S_{Lu}(\omega)-S_{wu}(\omega)S_{Lw}(\omega)}{\rho UBC_L(\alpha)\left[S_u(\omega)S_w(\omega)-S_{wu}(\omega)S_{uw}(\omega)\right]}$$

$$\chi_{Lw}(\omega)=\frac{S_u(\omega)S_{Lw}(\omega)-S_{uw}(\omega)S_{Lu}(\omega)}{1/2\,\rho UB\left[C_L'(\alpha)+C_D(\alpha)\right]\left[S_u(\omega)S_w(\omega)-S_{uw}(\omega)S_{wu}(\omega)\right]}$$

$$\left|\phi_{LL}(K)\right|^2=\frac{4C_L^2(\alpha)\left|\chi_{Lu}\right|^2S_u(K)+\left(C_L'(\alpha)+C_D(\alpha)\right)^2\left|\chi_{Lw}\right|^2S_w(K)}{\left(4C_L^2(\alpha)S_u(K)+\left(C_L'(\alpha)+C_D(\alpha)\right)^2S_w(K)\right)}$$

Numerical check

❖Numerical validation of identification algorithm

- 100Hz sampling frequency and 60s sampling time interval
- Wind velocity: 5.0-10.0m/s, turbulent intensity: 10.0-30.0%
- Iterative point numbers of PSD: 512-1536, and total point: 4096
- Identification results of discrete points fitted with 3th double-logarithm polynomial

$$\log_{10}\left(F(\omega)\right)=\sum_{i=0}^{3}\left(a_i \log_{10}{}^{i}\left(\omega B/U\right)\right)$$

Precision assessment of admittance identification results

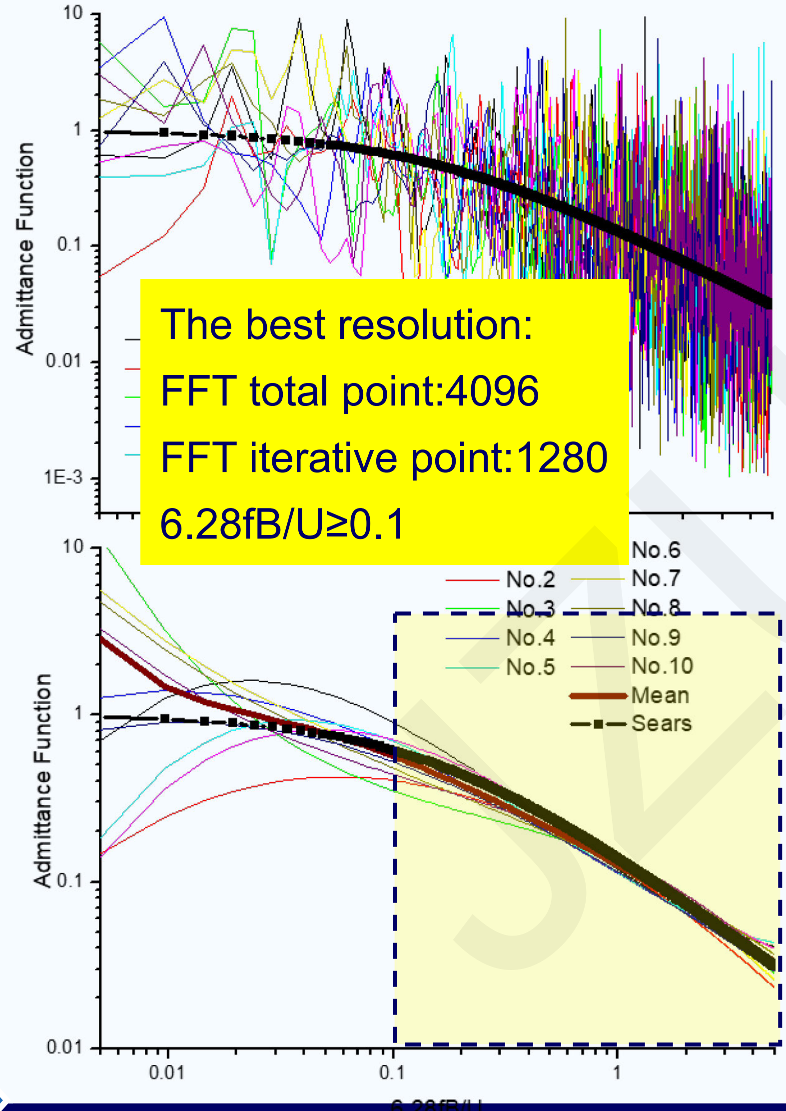
$$\delta_{\mathrm{Discrete}}=\underset{\mathrm{i=1,K=0}}{\overset{\mathrm{i=Times,K=Fre}}{\mathrm{COV}}}\left(\frac{F_{\mathrm{Discrete}}(i,K)}{F_{\mathrm{Fitting,mean}}(K)}\right) \qquad \delta_{\mathrm{Fitting}}=\underset{\mathrm{i=1,K=0}}{\overset{\mathrm{i=Times,K=Fre}}{\mathrm{COV}}}\left(\frac{F_{\mathrm{Fitting}}(i,K)}{F_{\mathrm{Fitting,mean}}(K)}\right)$$

Mean square deviation of discrete and fitted curves

Numerical check

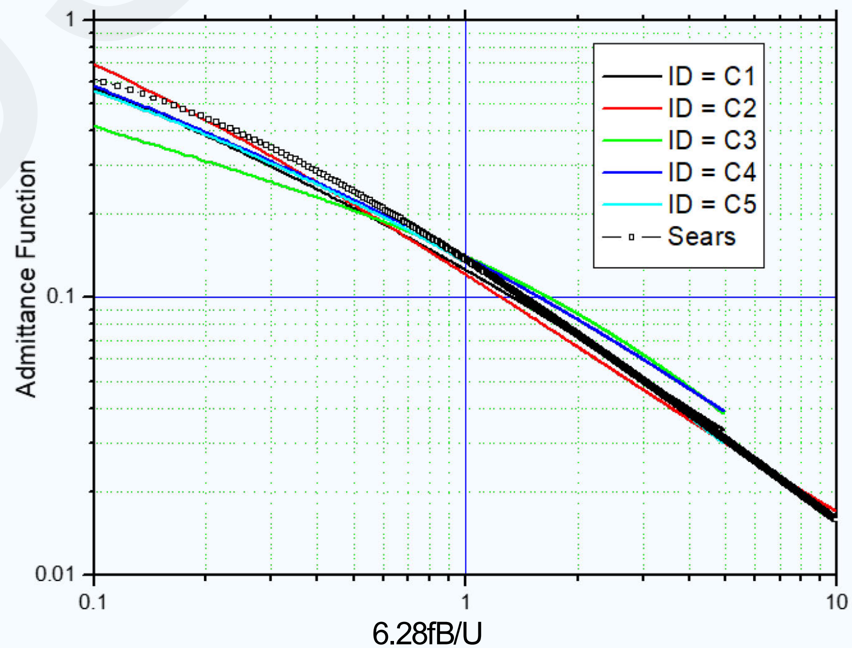
For the case: U=10.0m/s, Iu=10.2%, Code spectrum

The best resolution:
FFT total point:4096
FFT iterative point:1280
6.28fB/U≥0.1



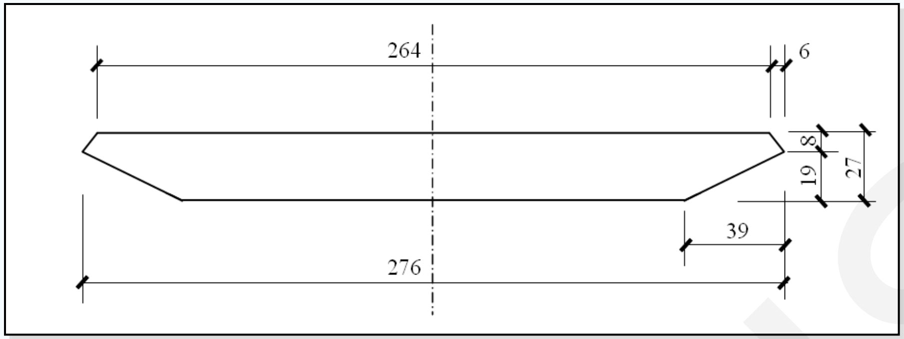
Numerical tests about identification method

ID	Mean Speed (m/s)	Turbulent density	Wind spectrum	FFT (total/iterative point)	δ_D	δ_F
C1	10.0	10.2%	Codes	4096/1280	1.72	0.11
C2	5.0	9.4%	Codes	4096/1280	1.83	0.12
C3	10.0	33.6%	Codes	4096/768	1.03	0.08
C4	10.0	22.2%	Codes	4096/1024	1.36	0.10
C5	10.0	14.3%	Measured	4096/1280	1.54	0.09

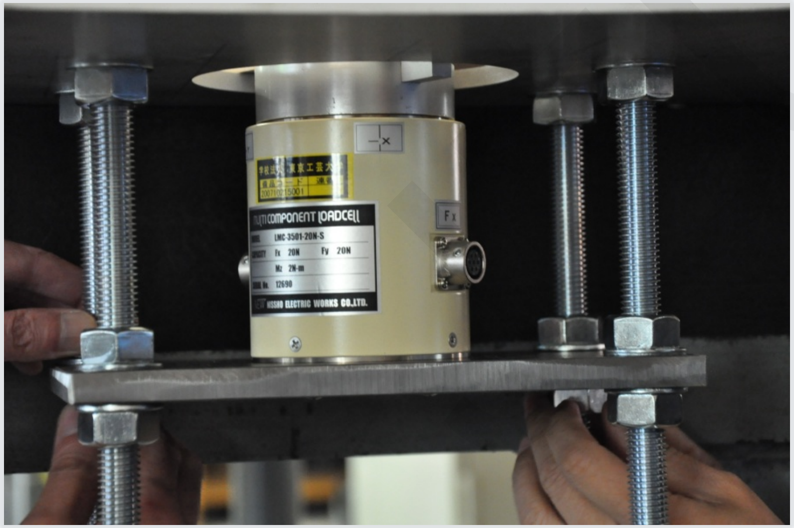


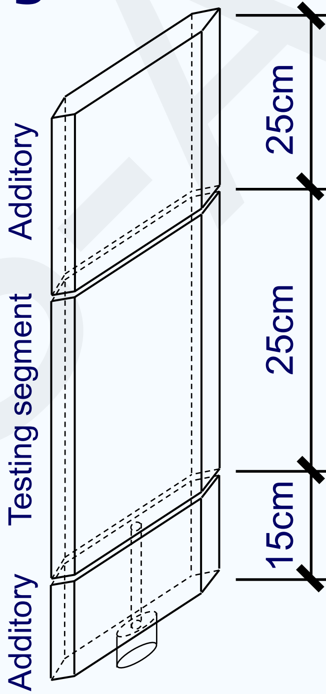
Testing check

Admittance identification using balance



Section model of streamlined box girder (unit: mm)





Parameters:

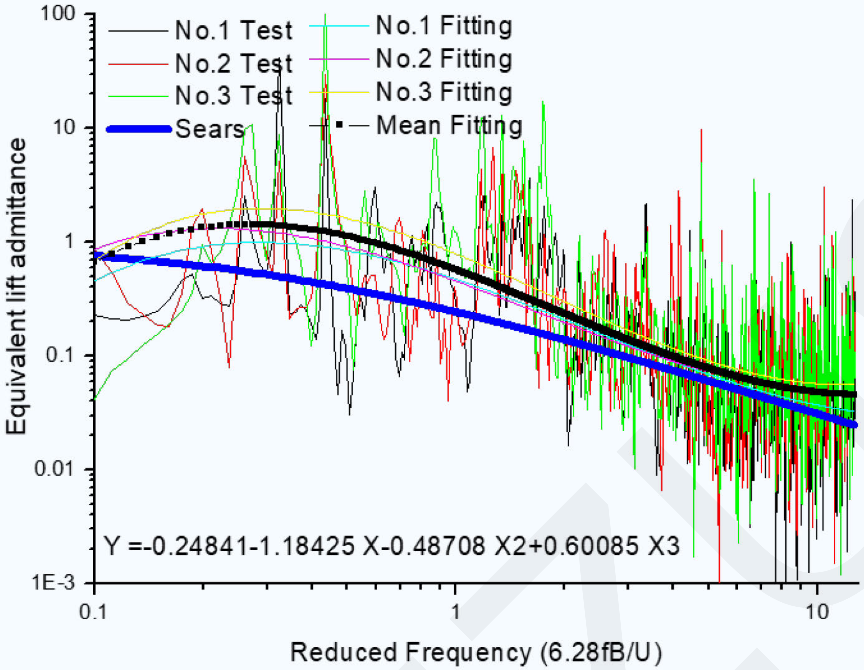
Fx, Fy ***20N*** **Mz** ***2N•m***

Precision: ***2‰***

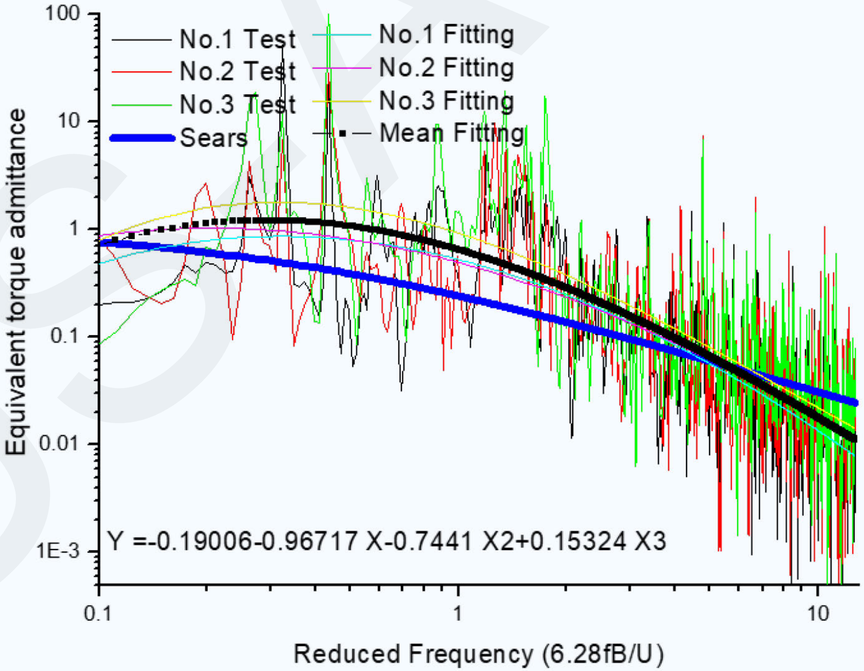
Sensitivity: **Fx, Fy** ***2g*** **Mz** ***20g•cm***

Testing check

Comparison of identified results



Equivalent lift admittance function

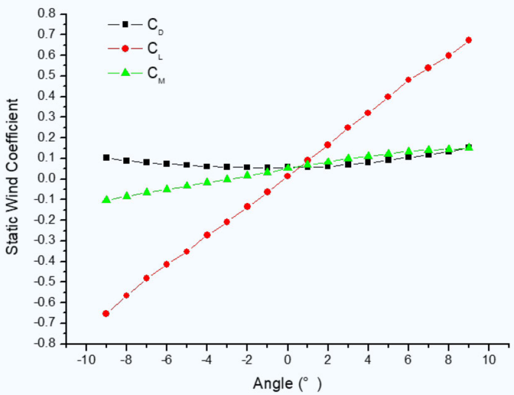


Equivalent torque admittance function

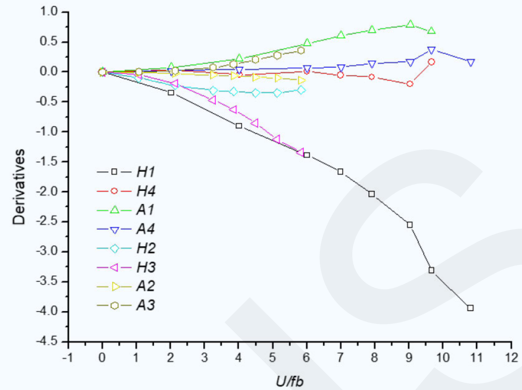
Reduced Fre./ Admittance Item	K=0.2		K=0.5		K=1.0		K=3.0	
	$\|\Phi_{LL}\|^2$	$\|\Phi_{MM}\|^2$	$\|\Phi_{LL}\|^2$	$\|\Phi_{MM}\|^2$	$\|\Phi_{LL}\|^2$	$\|\Phi_{MM}\|^2$	$\|\Phi_{LL}\|^2$	$\|\Phi_{MM}\|^2$
Experimental result	0.73	0.70	0.34	0.37	0.15	0.18	0.03	0.03
Analytic solution	0.61		0.38		0.24		0.09	

Testing check

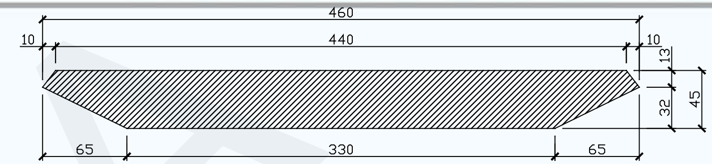
Buffeting tests of 2D model system

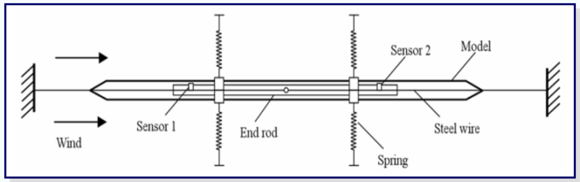


Static wind coefficient

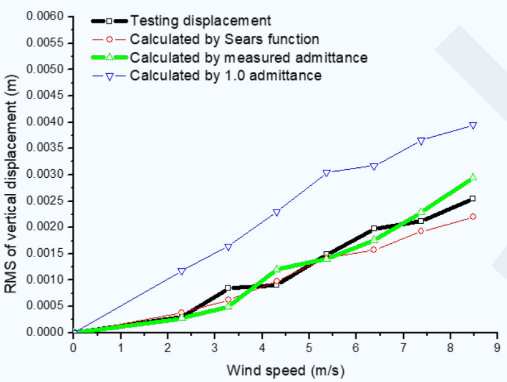


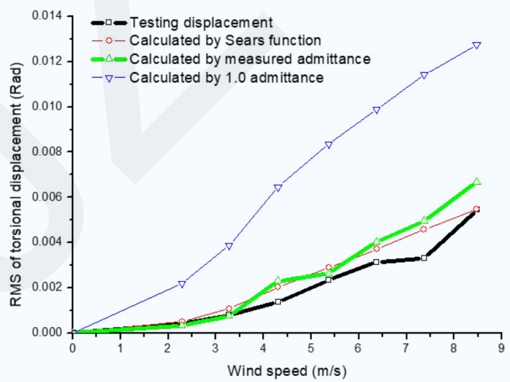
Flutter derivatives





Comparison of testing and calculated buffeting results





$$\mathbf{S}_{YY}(\omega) = \mathbf{H}^*(\omega)\mathbf{S}_{XX}(\omega)\mathbf{H}^T(\omega) = (-\omega^2\mathbf{M} - i\omega\mathbf{C} + \mathbf{K})^{-1}\mathbf{S}_{XX}(\omega)[(-\omega^2\mathbf{M} + i\omega\mathbf{C} + \mathbf{K})^{-1}]^T$$

$$\mathbf{S}_{XX}(\omega) = \begin{bmatrix} S_{LL}(\omega) & S_{LM}(\omega) \\ S_{ML}(\omega) & S_{MM}(\omega) \end{bmatrix} \qquad \mathbf{H}(\omega) = (-\omega^2\mathbf{M} + i\omega\mathbf{C} + \mathbf{K})^{-1}$$

$$\mathbf{K} = \begin{bmatrix} m\omega_h^2 - H_4 & -H_3 \\ -A_4 & I\omega_\alpha^2 - A_3 \end{bmatrix} \qquad \mathbf{C} = \begin{bmatrix} 2\xi_h m\omega_h - H_1 & -H_2 \\ -A_1 & 2\xi_\alpha I\omega_\alpha - A_2 \end{bmatrix}$$

About active turbulence

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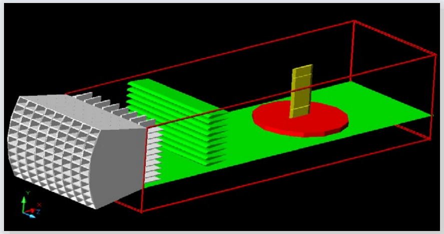
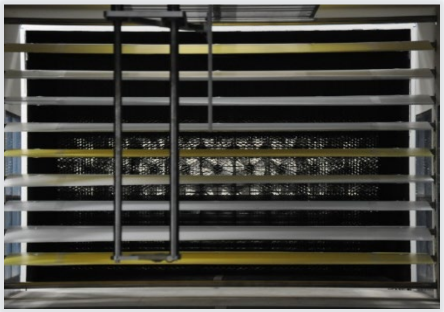
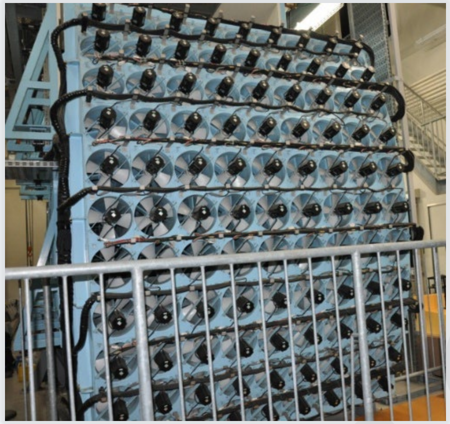
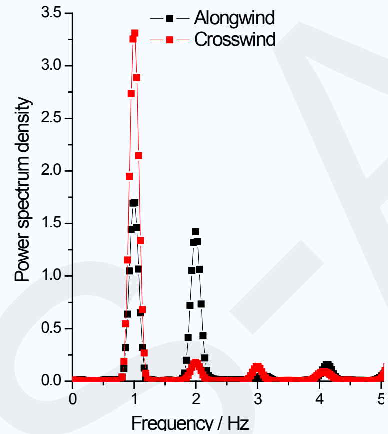
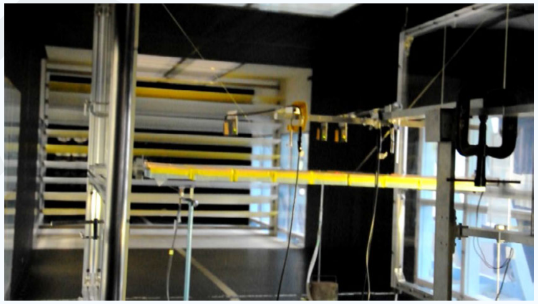






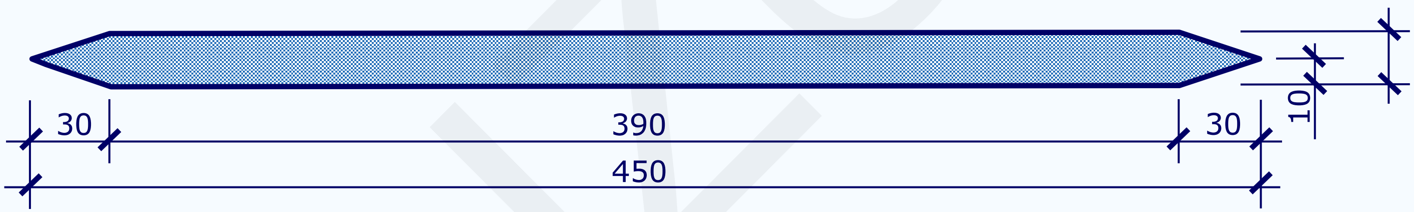


Case: u and w

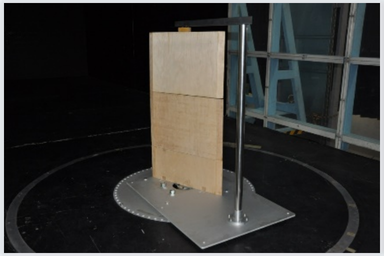


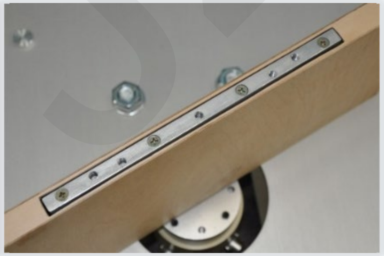
U=5.58m/s; Iu=11.8%; Fre. = 1.0Hz; Iw = **0.5%** or **6.8%**

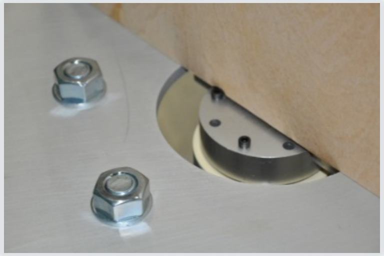
❖ Thin Plate Sectional Model Testing

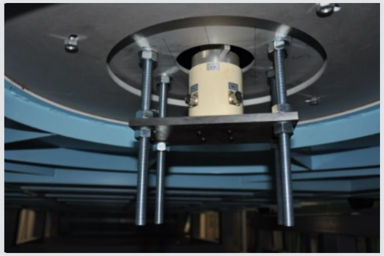


- Model weight : **360g**
- Weak-axis fre. : **24Hz**
- Strong-axis fre. : **44Hz**



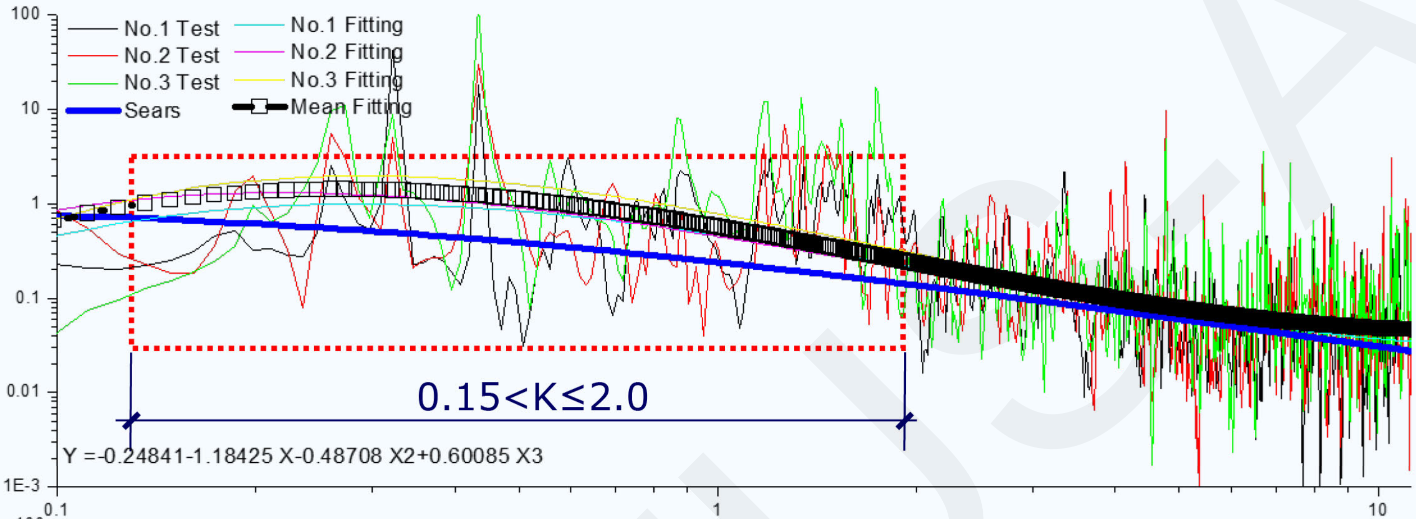






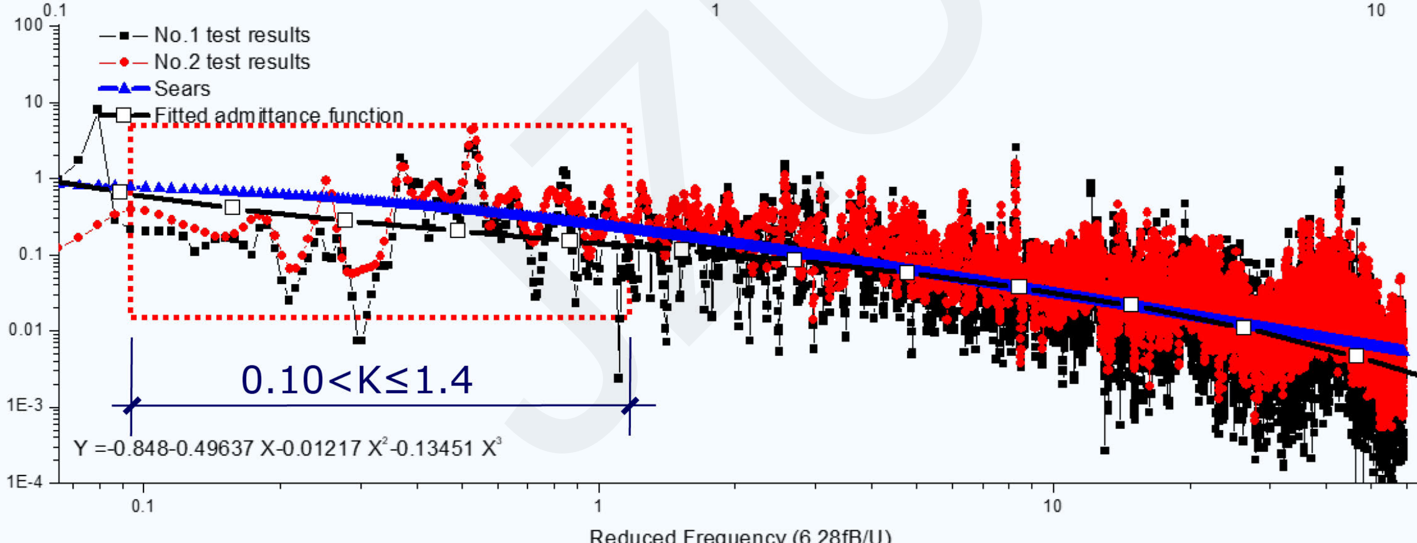
About active turbulence

Comparison of Aerodynamic Admittances of Thin Plate



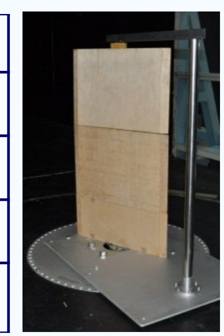
TJU wind tunnel

U	5.6m/s
I_u	11.5%
I_w	5.5%
L_{ux}	**0.25m**
L_{wx}	0.12m



Active wind tunnel

U	7.2m/s
I_u	9.2%
I_w	5.0%
L_{ux}	**0.63m**
L_{wx}	0.06m



K=6.28fB/U