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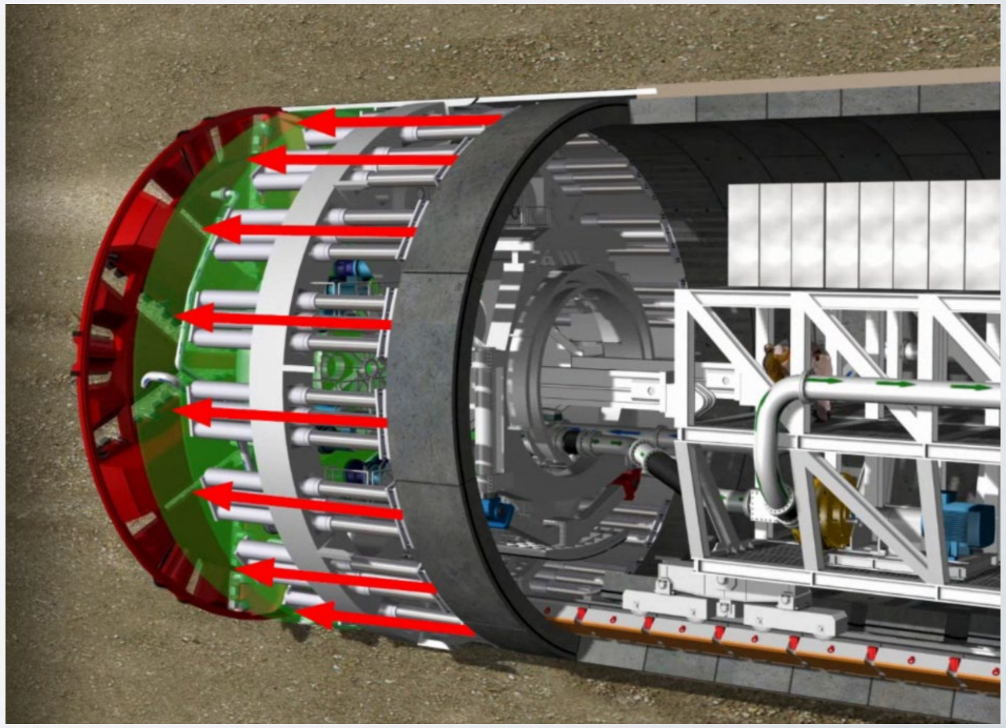
# Analytical solutions to ground settlement induced by ground loss and construction loadings during curved shield tunneling

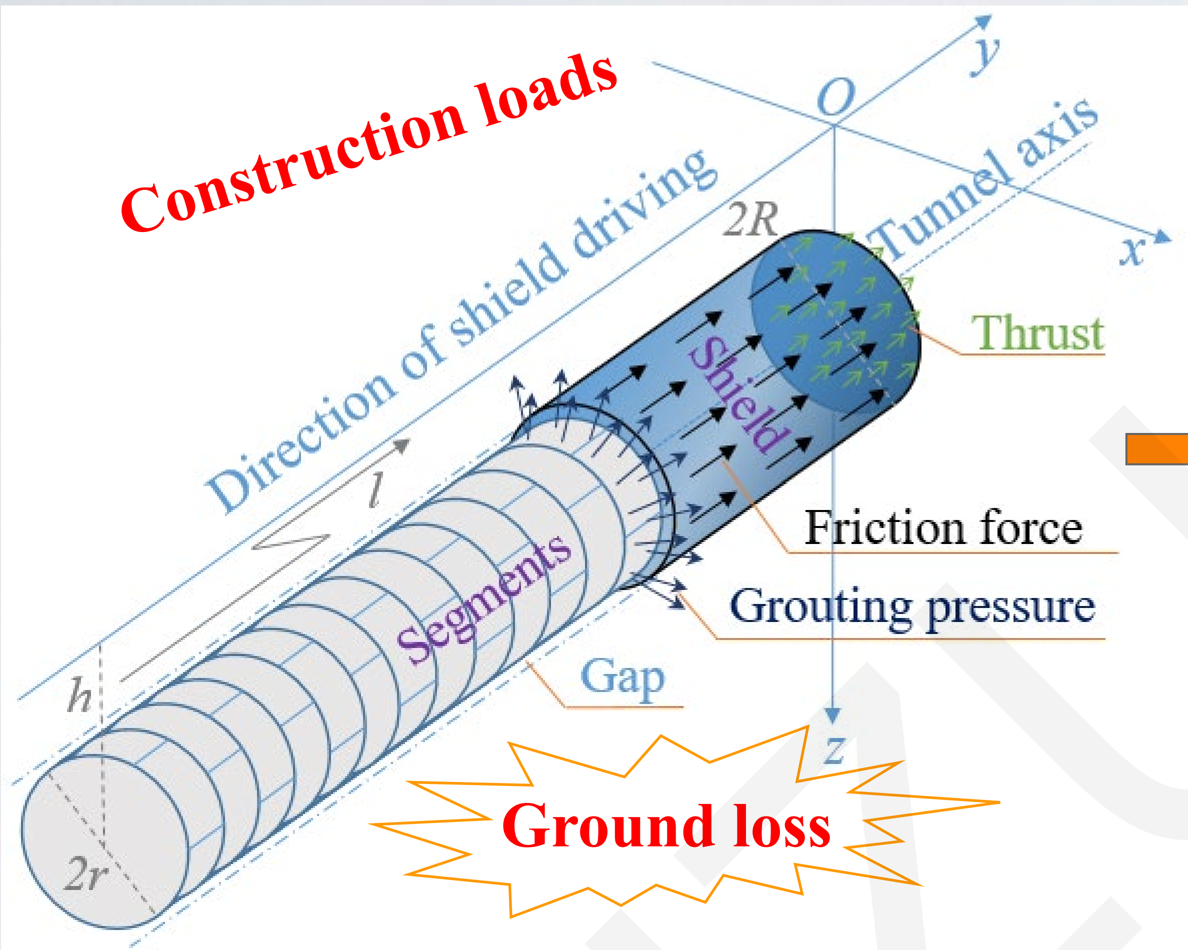
**Key words:**

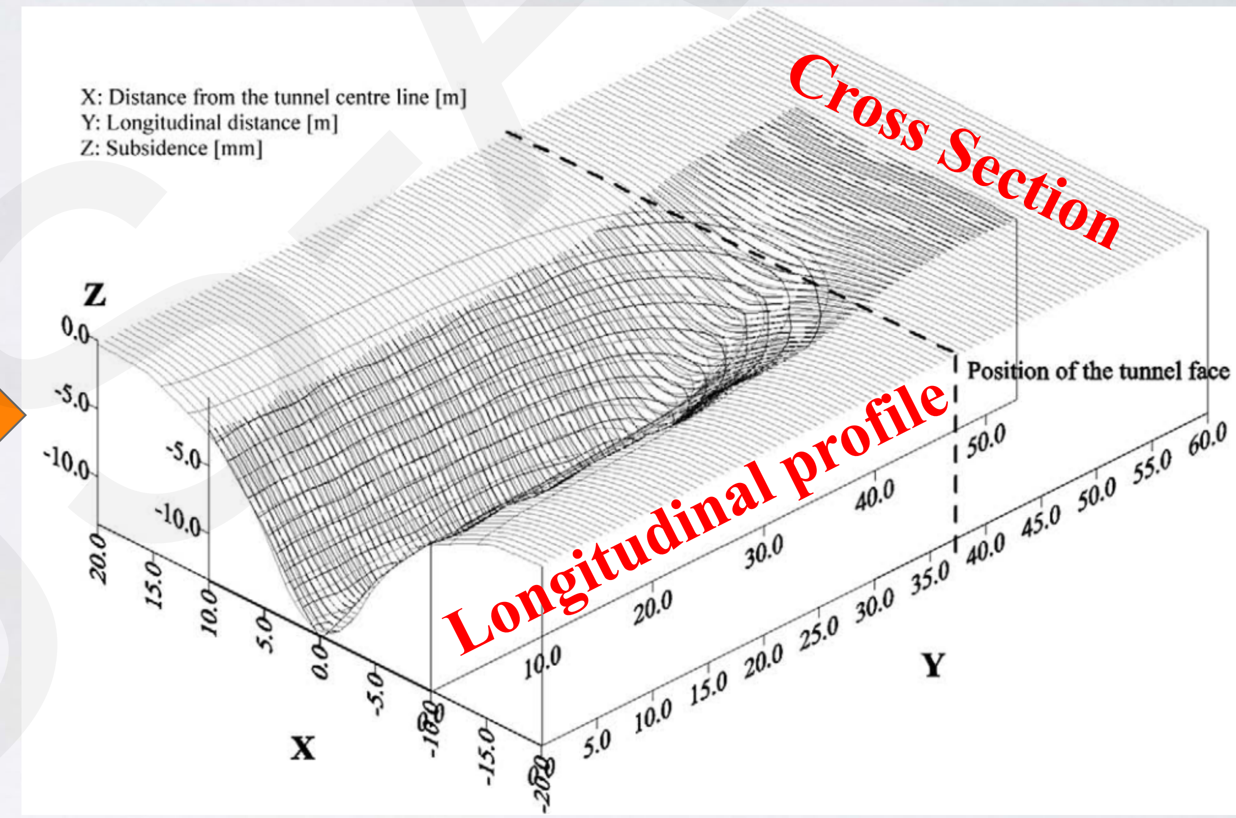
Curved tunnel; Image theory; Mindlin's solution; Ground loss; Ground settlement



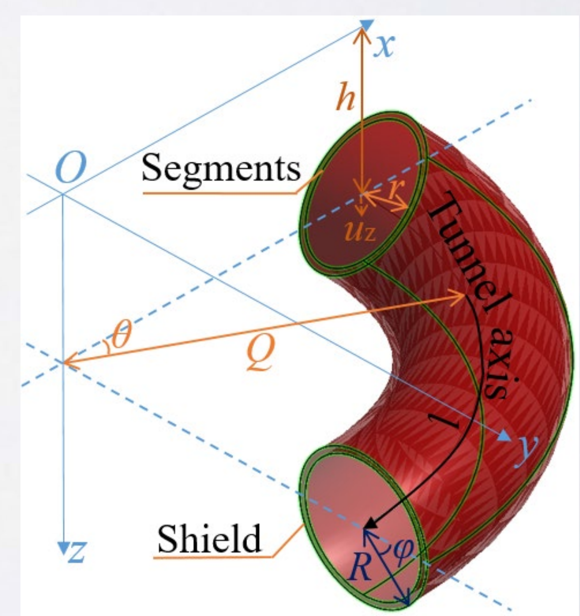
# Problem description

Ground settlement caused by shield construction




Research focuses on **curved shield tunnel** is rarely reported.



# Image theory and its application in curved tunnels

For a spherical micro gap ("point sink") with a unit radius of 1 at $F(x_0, y_0, z_0)$ in a Cartesian coordinate system, ground settlement at an arbitrary point $P(x, y, z)$ induced by the sink is:

**Step 1**

$$s_{z1} = -\frac{1}{3}\frac{z-z_0}{R_1^{3}}$$

The ground settlement produced at $P(x, y, z)$ due to the volume expansion (negative sink) of the same size at the image position point $F'(x_0, y_0, -z_0)$ is:

**Step 2**

$$s_{z2} = \frac{1}{3}\frac{z+z_0}{R_2^{3}}$$

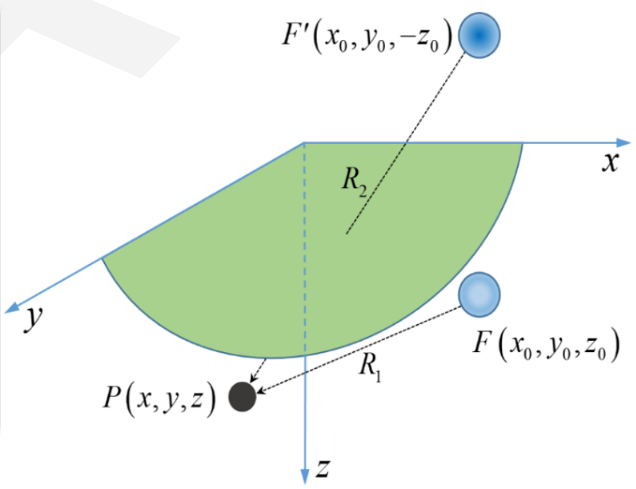
In the condition of infinite medium, the additional shear stress will generated in the first two steps.

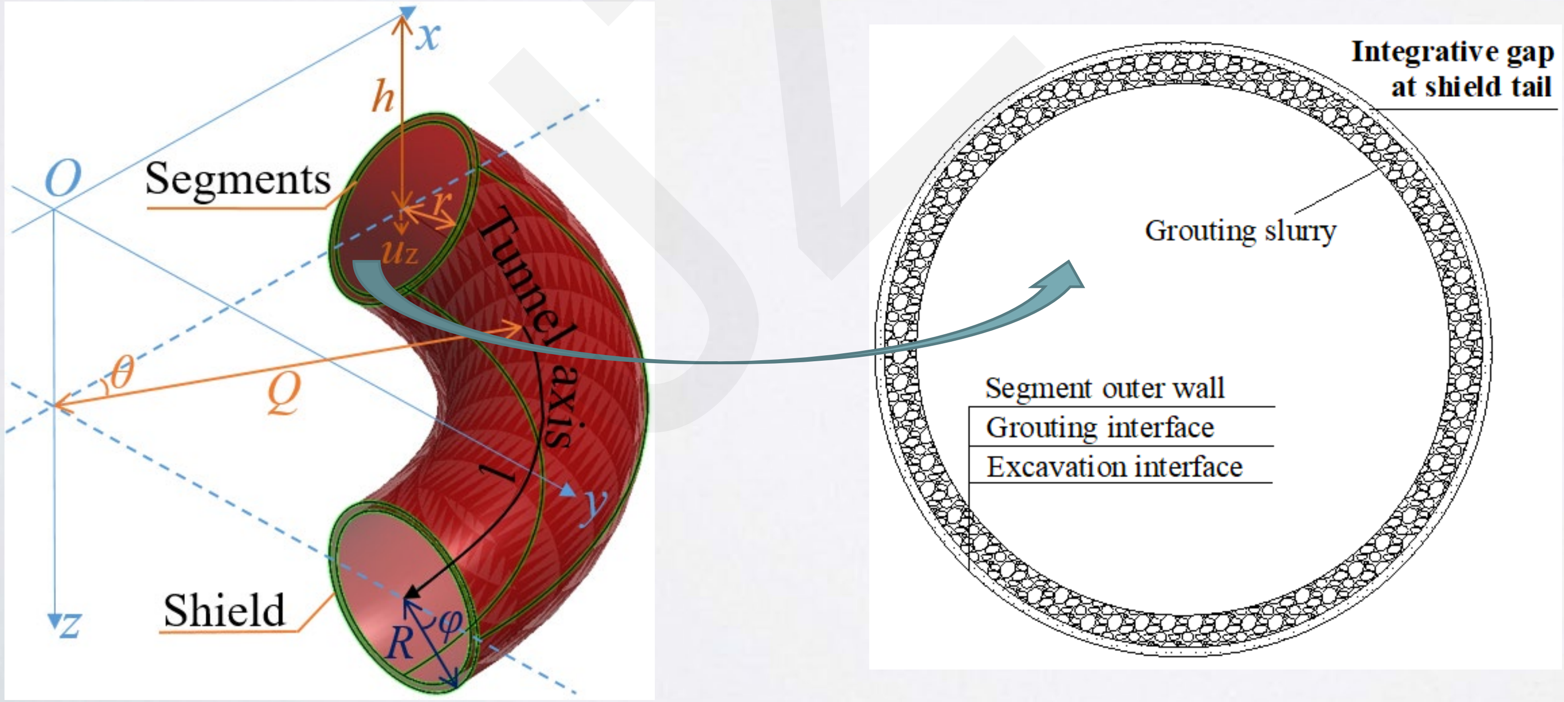
The additional shear stress at the ground surface ought to be exerted on the same position in the opposite direction, which will bring forth ground settlement:

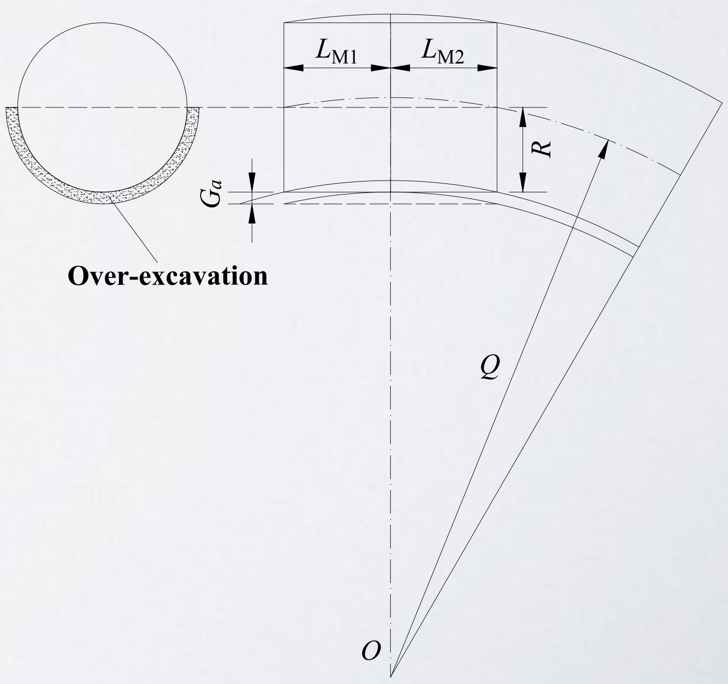
**Step 3**

$$s_{z3} = \lim_{b\to\infty}\lim_{c\to\infty}\int_{y_0-b}^{y_0+b}\int_{x_0-c}^{x_0+c} -\frac{1}{\pi}\left[\frac{z}{R_3^{3}} + \frac{1-2\mu}{R_3(R_3+z)}\right]\times\frac{z_0(u-x_0)(x-u)}{\left[(u-x_0)^2+(t-y_0)^2+z_0^{2}\right]^{5/2}}\,dudt$$
$$+\lim_{b\to\infty}\lim_{c\to\infty}\int_{y_0-b}^{y_0+b}\int_{x_0-c}^{x_0+c} -\frac{1}{\pi}\left[\frac{z}{R_3^{3}} + \frac{1-2\mu}{R_3(R_3+z)}\right]\times\frac{z_0(t-y_0)(y-t)}{\left[(u-x_0)^2+(t-y_0)^2+z_0^{2}\right]^{5/2}}\,dudt$$

In order to coincide with the condition of the actual boundary



# Rewritten and application of Mindlin solutions in curved tunnels

The **parametric equation** of spatial surface:

$$\left.\begin{aligned} x_0 &= x_0(u,v), \\ y_0 &= y_0(u,v), \\ z_0 &= z_0(u,v), \end{aligned}\right\}(u,v)\in D_{uv}$$

The ground settlement at an arbitrary point due to the concentrated force acting on the area of a **micro element** on the surface:

$$w_x f_x \sqrt{E_u E_v - E^2}\,\mathrm{d}u\mathrm{d}v$$

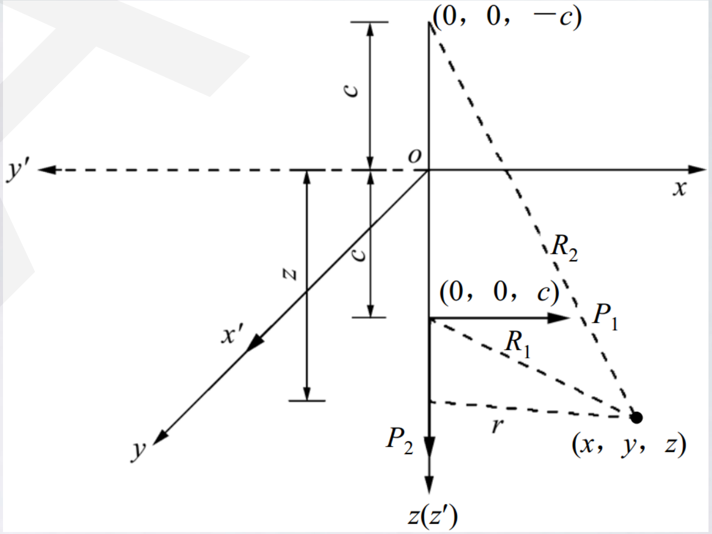
$$\left.\begin{aligned} E_u &= x_{0u}'^2 + y_{0u}'^2 + z_{0u}'^2, \\ E &= x_{0u}'x_{0v}' + y_{0u}'y_{0v}' + z_{0u}'z_{0v}', \\ E_v &= x_{0v}'^2 + y_{0v}'^2 + z_{0v}'^2, \end{aligned}\right\}$$

The ground settlement due to force per unit area acting on a **spatial surface** are given by:

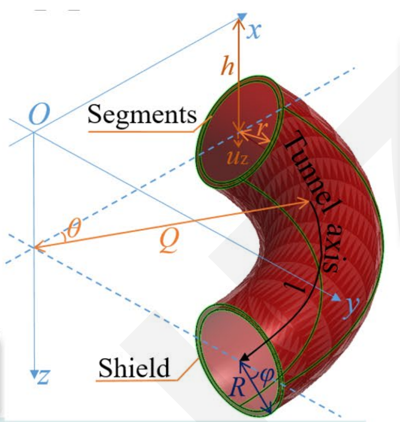
$$W_x = \iint_{D_{uv}} f_x w_x \sqrt{E_u E_v - E^2}\,\mathrm{d}u\mathrm{d}v$$

$$W_y = \iint_{D_{uv}} f_y w_y \sqrt{E_u E_v - E^2}\,\mathrm{d}u\mathrm{d}v$$

$$W_z = \iint_{D_{uv}} f_z w_z \sqrt{E_u E_v - E^2}\,\mathrm{d}u\mathrm{d}v$$




Mindlin solutions




The parametric equation of outer surfaces of shield:

$$\left.\begin{aligned} x_0 &= (Q + R\cos v)\cos u, \\ y_0 &= (Q + R\cos v)\sin u, \\ z_0 &= h + R\sin v, \end{aligned}\right\}$$

## $W_1$ For additional thrust

$u=u;\ v=\varphi$

$$W_1 = \int_0^{2\pi}\int_0^R (-f_1) w_x u\,\mathrm{d}u\,\mathrm{d}\varphi$$

## $W_2$ For friction force

$u=\theta;\ v=\varphi$

$$f_{2x} = f_2\cos(\theta+\pi/2)$$
$$f_{2y} = f_2\sin(\theta+\pi/2)$$

$$W_{2x} = \int_{\frac{\pi}{2}-\frac{L}{Q}}^{\frac{\pi}{2}}\int_0^{2\pi} f_{2x} w_x R(Q+R\cos\theta)\,\mathrm{d}\varphi\,\mathrm{d}\theta$$

$+$

$$W_{2y} = \int_{\frac{\pi}{2}-\frac{L}{Q}}^{\frac{\pi}{2}}\int_0^{2\pi} f_{2y} w_y R(Q+R\cos\theta)\,\mathrm{d}\varphi\,\mathrm{d}\theta$$

## $W_3$ For grouting pressures

$u=\theta;\ v=\varphi$

$$f_{3x} = f_3\cos\varphi\cos\theta$$
$$f_{3y} = f_3\cos\varphi\sin\theta$$
$$f_{3z} = f_3\sin\varphi$$

$$W_{3x} = \int_{\frac{\pi}{2}-\frac{L+b_1}{Q}}^{\frac{\pi}{2}-\frac{L}{Q}}\int_0^{2\pi} f_{3x} w_x R(Q+R\cos\theta)\,\mathrm{d}\varphi\,\mathrm{d}\theta$$

$+$

$$W_{3y} = \int_{\frac{\pi}{2}-\frac{L+b_1}{Q}}^{\frac{\pi}{2}-\frac{L}{Q}}\int_0^{2\pi} f_{3y} w_y R(Q+R\cos\theta)\,\mathrm{d}\varphi\,\mathrm{d}\theta$$

$+$

$$W_{3z} = \int_{\frac{\pi}{2}-\frac{L+b_1}{Q}}^{\frac{\pi}{2}-\frac{L}{Q}}\int_0^{2\pi} f_{3z} w_z R(Q+R\cos\theta)\,\mathrm{d}\varphi\,\mathrm{d}\theta$$

# Project overview

- **Case study object**

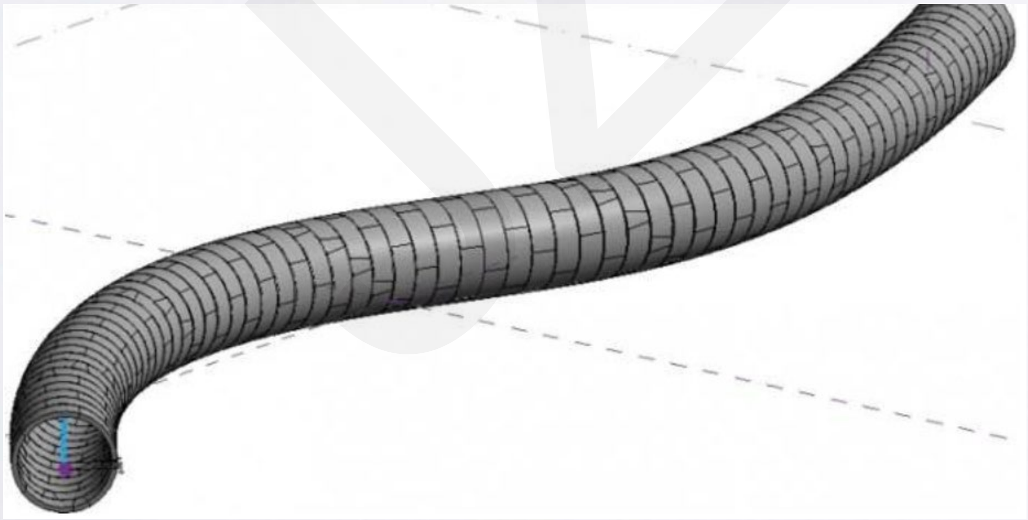
  **Taking the case of a shield tunnel with small curvature radius as a study.**

- **Parameters for calculation**

  $Q$=300 m， $h$=21.34 m， $R$=3.34 m， $r$=3.2 m， $b$=1.2 m， $l$=100 m， $L$=8 m， $G_t$=6 cm

**Mechanical parameters**

| Parameters | Thicknesses (m) | Density (kN/m$^3$) | Young's Modulus (MPa) | Poisson ratio (–) |
|---|---|---|---|---|
| TBM-shield | 0.12 | 78 | 200 000 | 0.3 |
| Segmental lining | 0.3 | 25 | 24 150 | 0.2 |
| Filling grout | 0.25 | 22 | 20 | 0.32 |



# Conclusions

- Considering the parameters of “integrative gap at shield tail” (IGST) and overcutting gap in a curved tunnel, a calculation model of ground settlement induced by ground loss is established based on the three-dimensional image theory.
- Analytical solutions of ground settlement due to construction loadings are proposed by the rewritten of Mindlin solutions.
- Transversal ground settlement troughs induced by those factors mentioned above are nonsymmetric about the tunnel axis for curved tunnels. Ground settlement at the inner side is larger than that at the outer side.
- On account of need for over-excavation during the tunneling of curved tunnels, the maximum ground settlements, offset distance and settlement trough widths under the coaction of the various factors are larger than that of the straight-line tunnel.