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A deep energy method for functionally graded porous beams

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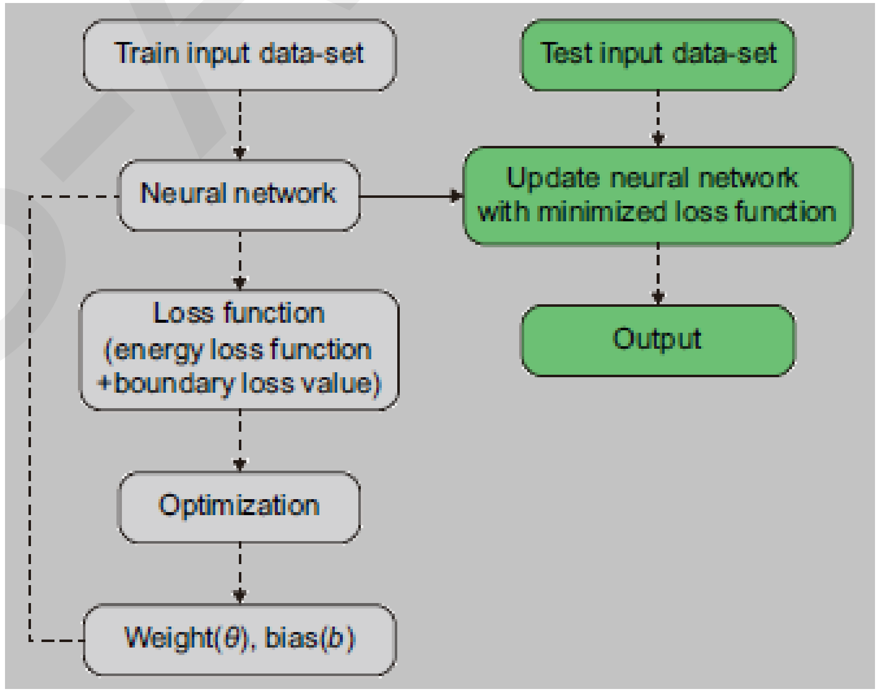
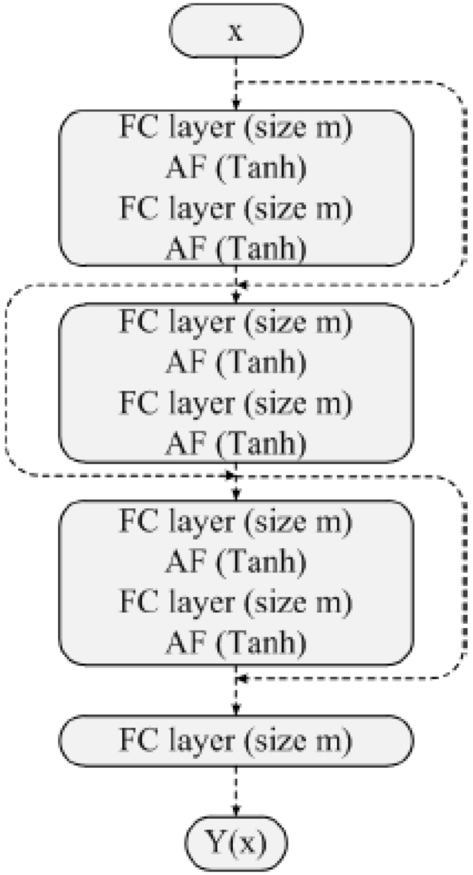
Deep energy method Algorithm

- Data of beam length
- multilayer perceptron (MLP) comprise an input layer, hidden layers, and an output
- a loss function, which is the potential energy (variational energy) of the system.
- variational energy as an objective function to optimize the neural network. objective function is minimized using the Adam optimizer.



Neural network architecture

- **Input Data (includes some point between both boundary conditions)**
- **Batching three blocks**
  a. Two fully connected (FC) linear transformations
  b. two nonlinear activation functions
  c. Residual connection
- **A Fully connected Layer**
- **Out put Layer (includes deflection of beam in each point)**



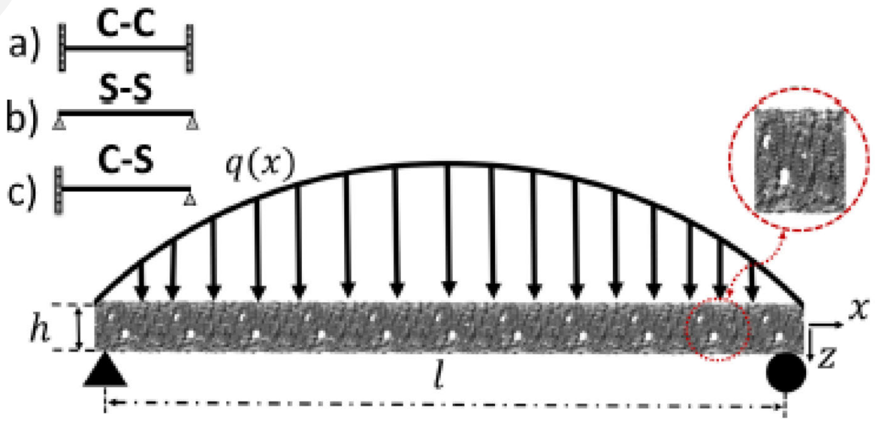
Energy method

The total potential energy (V) is the sum of strain energy (U) and the potential energy of the applied load (W):

$$V = U + W$$

$$U = \frac{1}{2}\int_v (\sigma_{xx}\varepsilon_{xx})dv$$

$$W = -\int_x q(x)w(x)dx,$$



Loss function and optimization

DEM is based on minimizing the potential energy

$$V = \int_x F\left(u, w, \frac{du}{dx}, \frac{dw}{dx}, \frac{d^2w}{dx^2}; x\right)dx + h(Y_L, Y_0)$$

Using the rectangle rule to evaluate the integrals

$$V = \frac{1}{n}\sum_{j=1}^{n} F_j(\theta, b) + \frac{\beta}{n_b}\sum_{i=1}^{n_b}(h_i(\theta, b) - h_i(Y_L, Y_0))^2$$

We used the Adam optimizer, which is based on the stochastic gradient descent method and readily available in PyTorch.

Results

The relative errors obtained from this network architecture and FC neural network are compared in the Table .

⋆	$R_e \times 10^{-2}$		
Layers	Fully connected		Residual connected
6	4.228	$S-S$	4.222
	3.520	$C-C$	3.496
	1.490	$C-S$	1.479
8	3.382	$S-S$	2.533
	2.816	$C-C$	2.098
	1.192	$C-S$	0.887
10	3.044	$S-S$	1.520
	2.534	$C-C$	1.260
	1.073	$C-S$	0.532
12	2.739	$S-S$	0.912
	2.280	$C-C$	0.756
	0.966	$C-S$	0.319

The Figure compares the solution of DEM with the exact solution for and simply-simply boundary conditions.