Leibniz Universität Hannover

Journal of Zhejiang University–SCIENCE A (Applied Physics & Engineering)

Cite this as: Dung Nguyen Kien, Xiaoying Zhuang, 2021. A deep neural network-based algorithm for solving structural optimization. *Journal of Zhejiang University-SCIENCE A (Applied Physics & Engineering)*, 22(8):609-620. https://doi.org/10.1631/jzus.A2000380

A deep neural network-based algorithm for solving structural optimization

Dung Nguyen KIEN & Xiaoying ZHUANG

JZUSA

Keywords: Structural optimization, Deep learning, Artificial neural network, Sensitivity analysis.

Main Idea

Journal of Zhejiang University–SCIENCE A
(Applied Physics & Engineering)

- A so-called deep Lagrange method (DLM) is presented which is applied to sizing optimization and shape optimization inspired by neural networks.
- The method is based on the Lagrange duality and deep learning (feedforward neural networks).
- The input data are used for training the neural network until the output values resemble closely the predicted values.
- The minimum input values will be found when the min-max problem in Lagrange duality is solved following the interpolation from deep learning.
- Therefore, this deep learning-based method is an improvement when avoiding sensitivity analysis.
- It employs a large number of input data for the neural network. However, the proposed method is potentially applied to structural optimization problems that require a small number of design variables.
- Several test cases on sizing optimization and shape optimization are performed, and their results are then compared with analytical and numerical solutions.

JZUSA

Method

Journal of Zhejiang University–SCIENCE A
(Applied Physics & Engineering)

$$(\mathbb{P})\begin{cases}\min\limits_{\mathbf{x}} \hat{g}_0(\mathbf{x}) \\ \text{s.t.} \begin{cases}\hat{g}_j(\mathbf{x}) \le 0, \quad j=1,\ldots,m \\ \mathbf{x}\in\mathcal{X}\end{cases}\end{cases} \tag{1}$$

where,

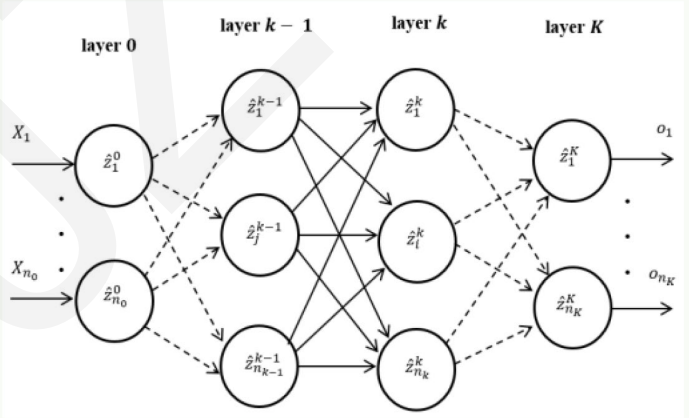
$$\mathcal{X}=\{\mathbf{x}\in\mathbb{R}^n : x_i^{min} \le x_i \le x_i^{max}, \quad i=1,\ldots,n\}$$

General optimization problem

$$\begin{aligned}\mathcal{L}(\mathbf{x},\boldsymbol{\lambda}) &= \hat{g}_0(\mathbf{x}) + \sum_{j=1}^{m}\lambda_j\hat{g}_j(\mathbf{x}) \\ &= \sum_{i=1}^{n}\hat{g}_{0i}(x_i) + \sum_{j=1}^{m}\lambda_j\left(\sum_{i=1}^{n}\hat{g}_{ji}(x_i)\right) \\ &= \sum_{i=1}^{n}\left(\hat{g}_{0i}(x_i) + \sum_{j=1}^{m}\lambda_j\hat{g}_{ji}(x_j)\right) \\ &= \sum_{i=1}^{n}\mathcal{L}_i(x_i,\boldsymbol{\lambda})\end{aligned}$$

The Lagrange function of optimization problem

The DLM method is supported by a proposed range-reduction technique that reduces the number of input data sets.



The architect of deep neural network that is utilized in the DLM method.

Algorithm The main idea of deep Lagrange method.

INPUT: Define input $\mathbf{X}=[\mathbf{x}]$, range of $\boldsymbol{\lambda}$, weight $\mathbf{w}$ initialization, neural network hyper-parameters configuration.

OUPUT: The minimum $\mathbf{x}^*$ and then find $\hat{g}_0^*$.

ALGORITHM:

i. Find the minimum of deep neural network output

for $i \leftarrow 1, I$ **do**

Get λ_i,

Obtain target $\mathbf{t} = \mathcal{L}(\mathbf{x},\lambda_i) = \hat{g}_0(\mathbf{x}) + \sum_{j=1}^{m}\lambda_{ji}\hat{g}_j(\mathbf{x})$, where $\hat{g}_0(\mathbf{x}) \le 0$

i.1. Deep neural network calculation.

i.2. Obtain the NN output from the input data set

i.3. Find the minimum of the NN output $\phi(\lambda_i) = \min \hat{\mathbf{z}}^K(\boldsymbol{\theta})$

end for

ii. Find $\phi_{max} = \max\limits_{\boldsymbol{\lambda}} \phi(\boldsymbol{\lambda})$

iii. Find the minimum $\mathbf{x}^*$ at ϕ_{max}

iv. Find the optimum $\hat{g}_0^*$

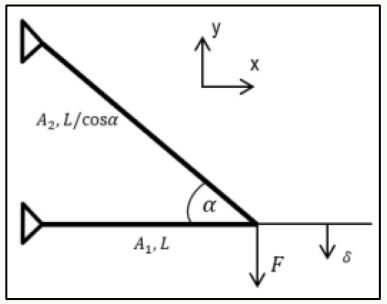
The brief main idea of the deep Lagrange method

JZUSA

Numerical Studies

Journal of Zhejiang University–SCIENCE A
(Applied Physics & Engineering)

Applications for structural optimization problems, sizing and shape optimization.
Several numerical test cases are applied. The results from DLM method are then compared with other analytical and numerical solutions. (The more detail will be shown in the paper.)

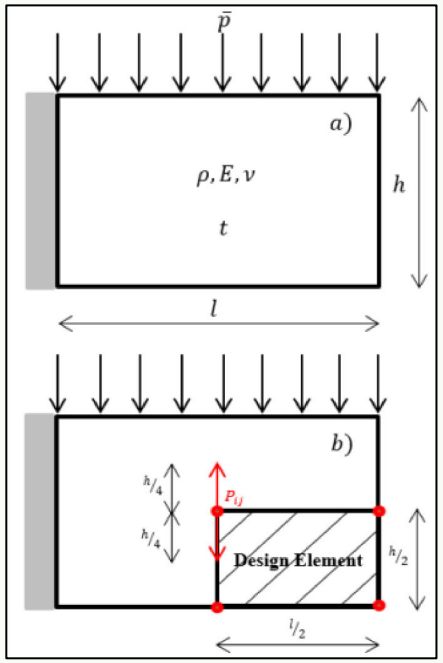


$$(\mathbb{SO})^1 \begin{cases} \min\limits_{x_1,\,x_2} \hat{g}_0(x_1,x_2) = \dfrac{4}{3x_1} + \dfrac{1}{x_2} \\ \text{s.t.} \begin{cases} \hat{g}_1(x_1,x_2) = \dfrac{4}{\sqrt{3}}x_1 + \sqrt{3}x_2 - 1 \le 0 \\ x_1 > 0, \quad x_2 > 0 \end{cases} \end{cases}$$

Table 1 Solution from the methods

	Exact	MMA	SLP	MPEA	DLM	RE*
Design variables						
x_1	0.2474	0.2474	0.2474	0.2474	0.2480	
x_2	0.2474	0.2474	0.2474	0.2474	0.2480	
$\min \hat{g}_0$	9.4300	9.4300	9.4300	9.4300	9.4086	0.2269

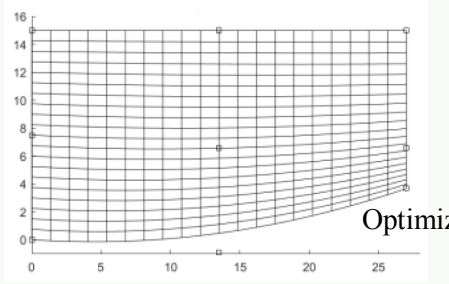
*The relative error of DLM (%).



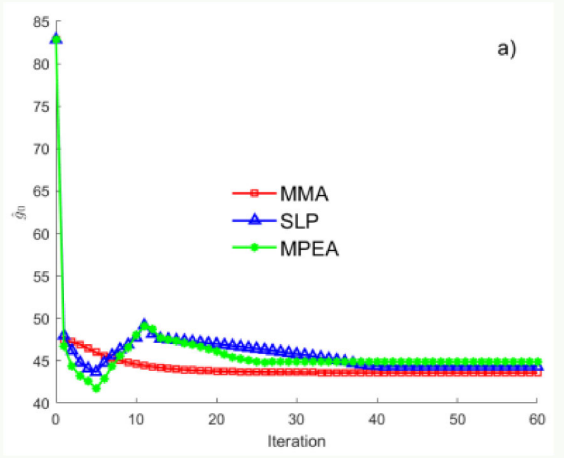
$$(\mathbb{SO})^4 \begin{cases} \min\limits_{\mathbf{x}} \hat{g}_0(\mathbf{x}) = g_0(\mathbf{x}, \mathbf{u}(\mathbf{x})) = \mathbf{F}^T\mathbf{u}(\mathbf{x}) \\ \text{s.t.} \begin{cases} \hat{g}_1(\mathbf{x}) = g_1(\mathbf{x}, \mathbf{u}(\mathbf{x})) \\ \quad = W_{sheet} - W_{max} \le 0 \\ \mathbf{x} \in \mathcal{X} = \{\mathbf{x} \in \mathbb{R}^n : x_i^{min} \le x_i \\ \quad \le x_i^{max}, i = 1, \ldots, n\} \end{cases} \end{cases}$$

Table 4 Solution from the methods.

	MMA	SLP	MPEA	DLM
Design variables				
α_1	0.3795	0.3795	0.3795	0.3700
α_2	0.3795	0.3820	0.3857	0.4000
α_3	0.9900	0.9900	0.9900	1.0000
α_4	0.3795	0.4414	0.4924	0.3200
$\min \hat{g}_0 = \mathbf{F}^T\mathbf{u}$	43.5700	44.2744	44.8585	42.8137
Total weight	379.9997	379.9988	379.9999	379.9609



Optimized shape



The history of $\hat{g}_0$ of the reference methods.

Conclusions

Journal of Zhejiang University–SCIENCE A
(Applied Physics & Engineering)

- The method can solve structural optimization problems.
- Advantage: it does not need a sensitivity analysis which can be sometimes difficult to perform.
- Disadvantage: when the input data is huge, it might be computationally expensive to utilize the method. So it limits the number of design variable inputs.
- The accuracy of the method depends on the interval size of the input. The smaller the interval input size, the larger the amount of the input dataset for the neural network is employed.
- Therefore, a more advanced technique for reducing the number of input datasets should be developed.

JZUSA