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Normal gravity model for inertial navigation of a hypersonic boost-glide vehicle

Key words:

hypersonic boost-glide vehicle; inertial navigation system; normal gravity; gravity disturbance;



Fig. 1. The hypersonic vehicle flying in near space

Inertial Navigation System and Gravitation Model

- Inertial navigation system get the attitude, velocity and position by numerical integration.

Inertial measurement unit

- Specific force $\boldsymbol{f}^a$ is measured by accelerometer.
- Angular velocity $\boldsymbol{\omega}_{ab}^b$ is measured by gyroscope.

$$\boldsymbol{\Omega}_{ab}^b = (\boldsymbol{\omega}_{ab}^b \times)$$

$$\begin{bmatrix} \dot{\boldsymbol{V}}_a \\ \dot{\boldsymbol{P}}_a \\ \dot{\boldsymbol{R}}_b^a \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}^a + \boldsymbol{g}^a \\ \boldsymbol{V}_a \\ \boldsymbol{R}_b^a \boldsymbol{\Omega}_{ab}^b \end{bmatrix}$$

Gravitational model

- Gravitation is an importance part of the vehicle's.
- $\boldsymbol{g}^a$ cannot be observe by accelerometer ,but given by gravitation model.

EIGEN-6C4 gravity and normal gravity

- ENGEN-6C4 model takes the place of the real Earth's gravity in the navigation simulation in this paper.
- Normal gravity model is a simplified model usually used in inertial navigation.
- Difference between normal gravity and EIGEN-6C4 gravity is consider as the error of normal gravity.

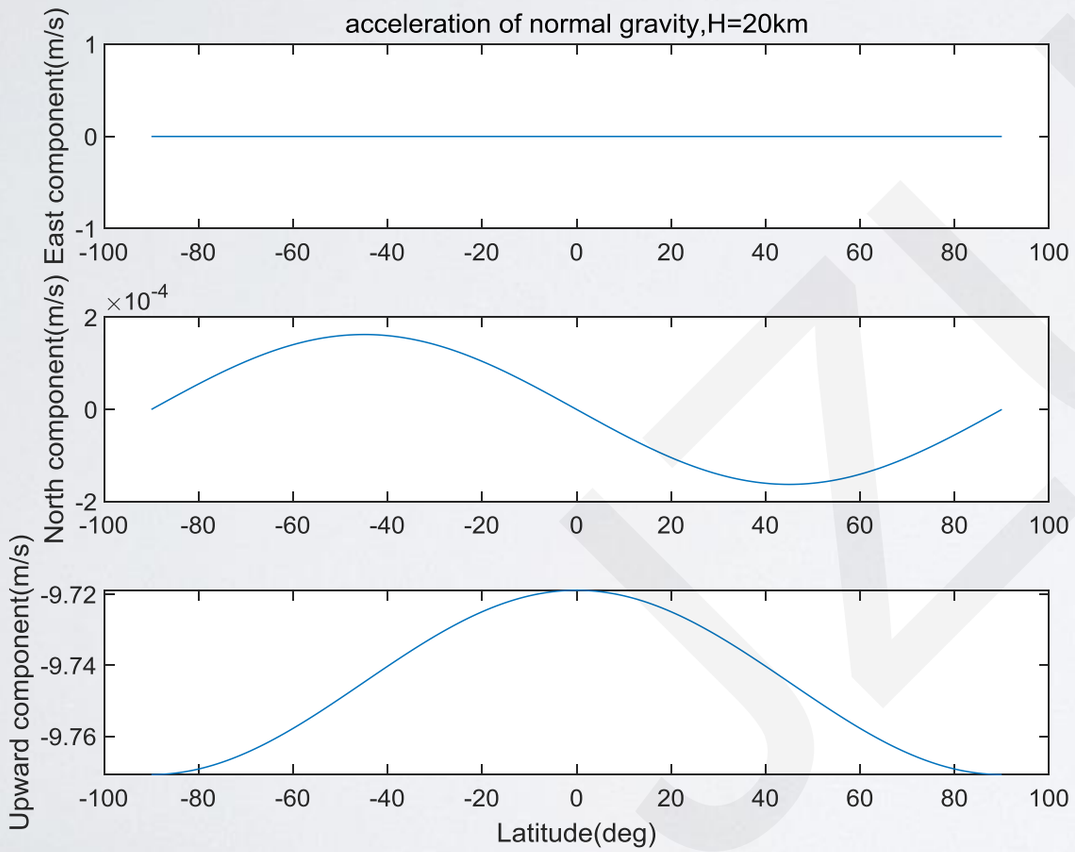

Fig. 2. Three component of normal gravity

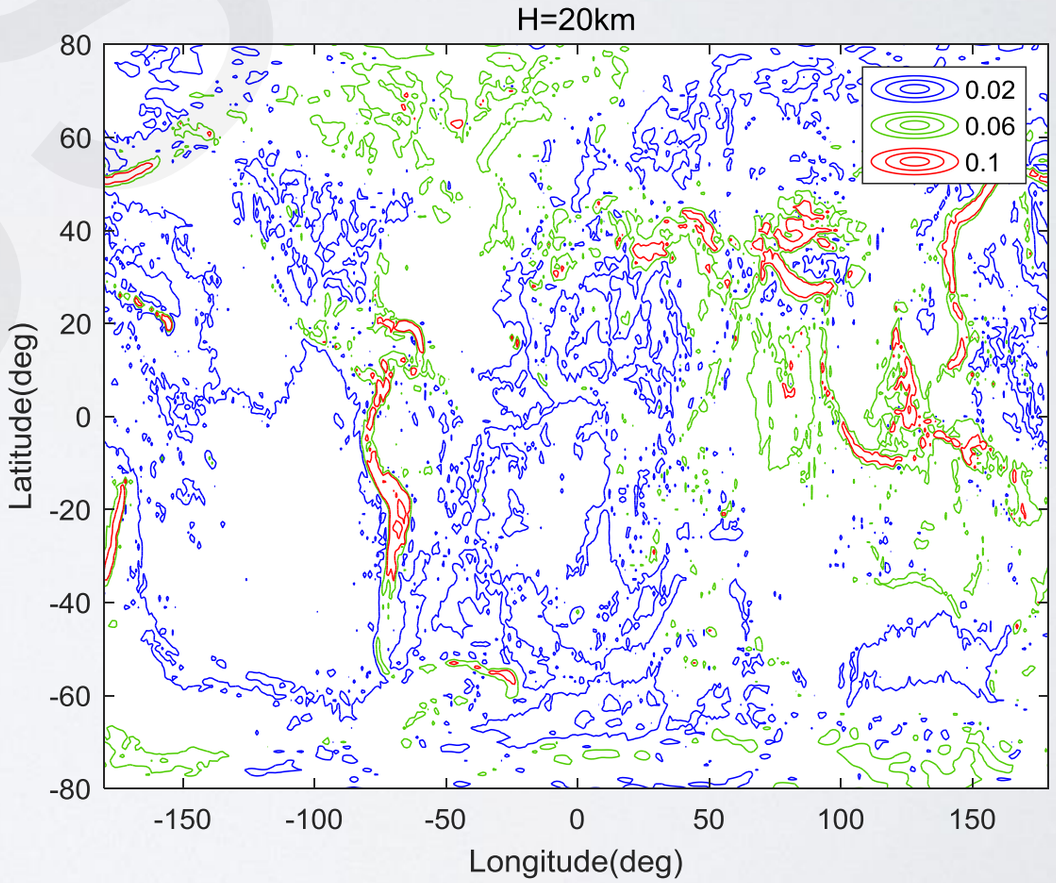

Fig. 3. Difference between normal gravity and EIGEN-6C4 gravity

Simulation trajectory and condition

- A boost-glide flight trajectory is adopted as simulated trajectory.
- Inertial navigation system word independently of 1100s in the simulations.
- EIGEN-6C4 model (considered as real gravity) is used in generation the trajectory.

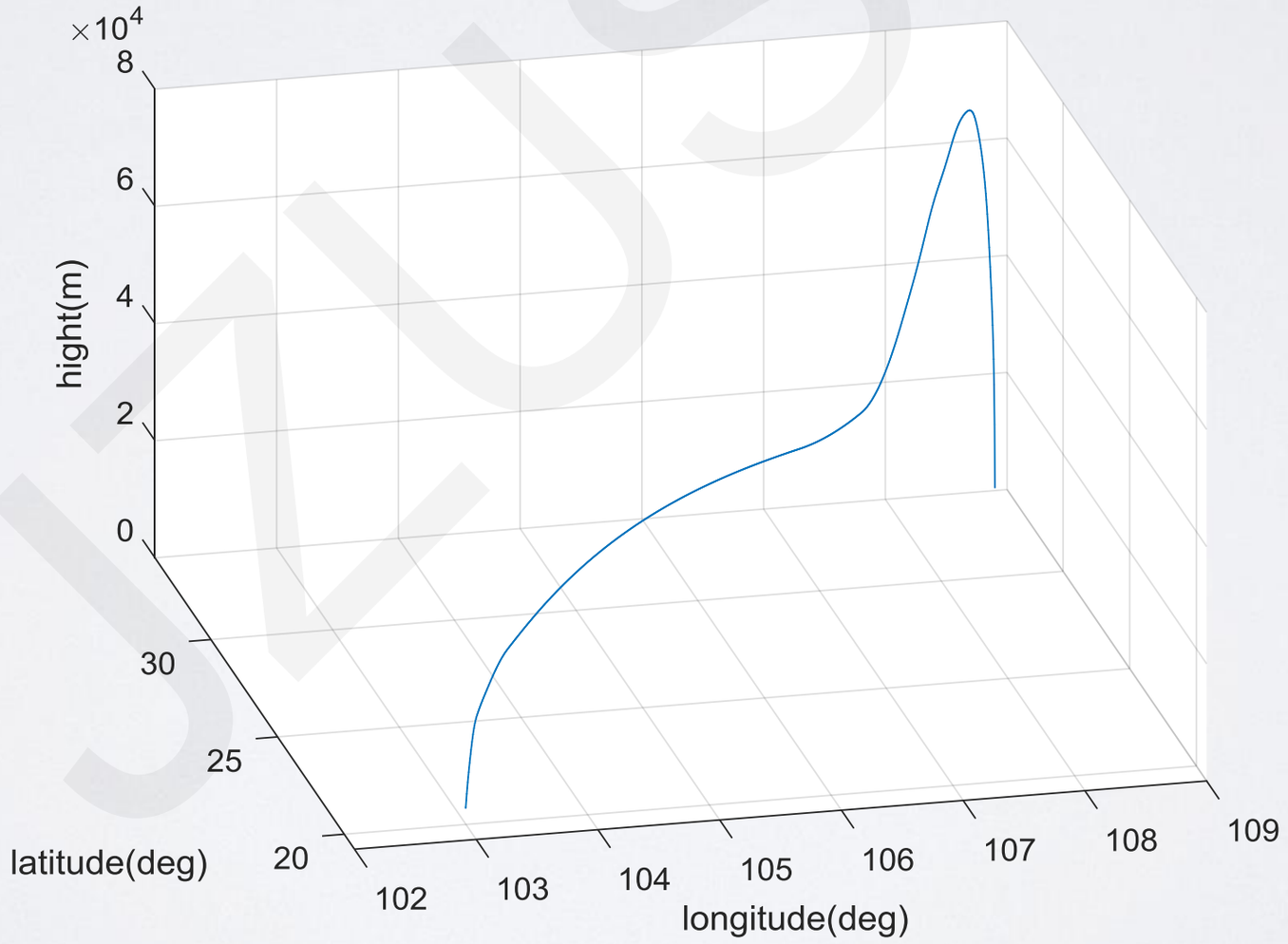

Fig. 4. Simulation trajectory

Simulation

- Only take gravity model error into consider
- Gravity models including J_2, J_4, and J_6, and the EIGEN-6C4 models of maximum order 18 and 200 are used as the navigation gravity models in the navigation solution.

Gravity models	Velocity error sum of squares /m·s^{-1}	Position error sum of squares /m	CPU time of each gravity calculation/s
J_2	0.742	387.18	0.000003
J_4	0.686	351.28	0.000003
J_6	0.686	351.19	0.000004
EIGEN-6C4 (N=18)	0.181	117.52	0.000214
EIGEN-6C4 (N=200)	0.060	35.88	0.004449

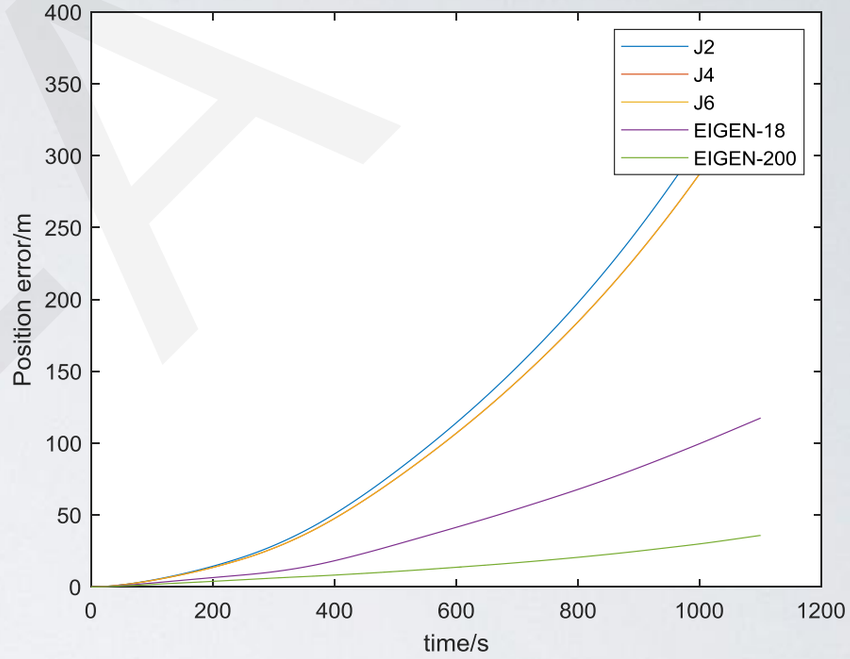

Fig. 5. Position error

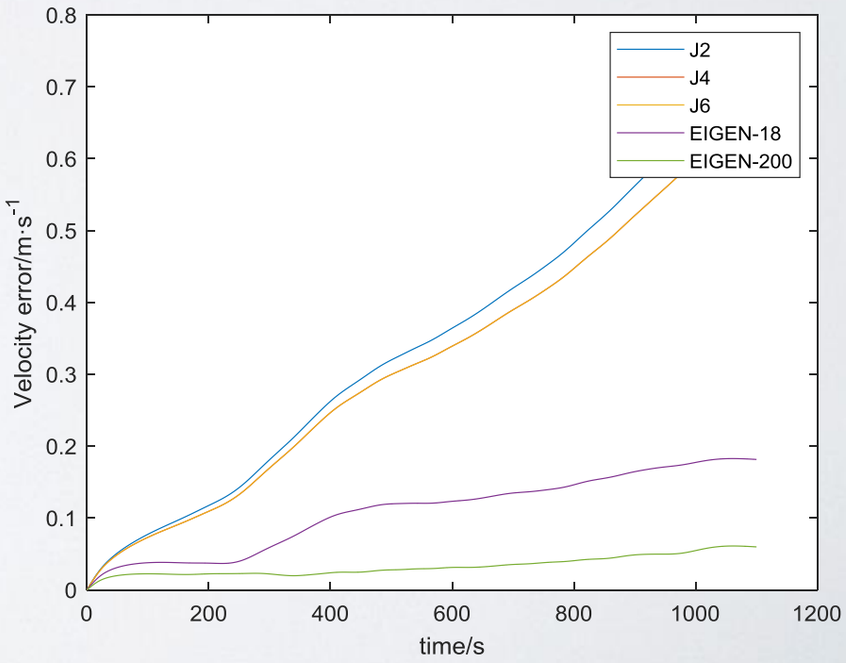

Fig. 6. Velocity error

Simulation

- Consider both gravity model error and IMU error

Simulation parameters	Index	Simulation parameters	Index
Gyroscope constant bias	0.003°/h	Accelerometer constant bias	10μg/h
Gyroscope white noise	0.0003°/h	Accelerometer white noise	1μg/h
Gyroscope installation error	0.5”	Accelerometer installation error	0.5”
Gyroscope scale factor accuracy	10ppm	Accelerometer scale factor accuracy	10ppm

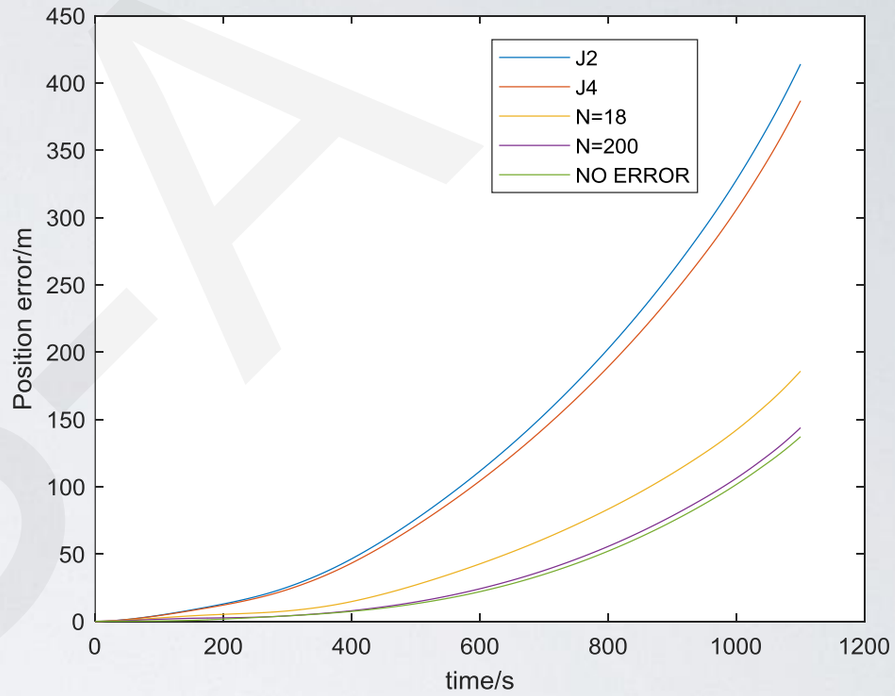

Fig. 7. Position error

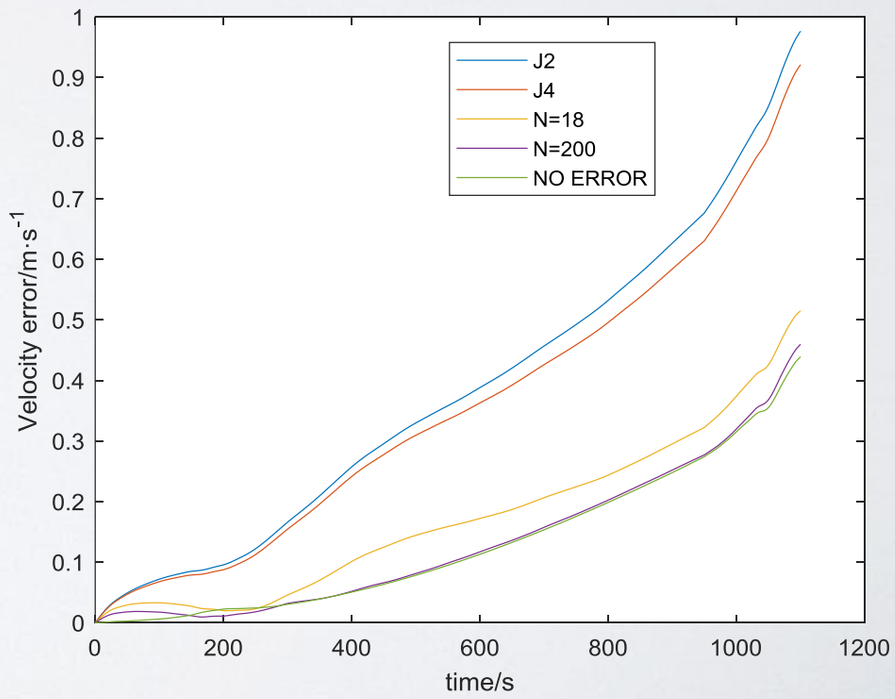

Fig. 8. Velocity error

Conclusion

- Gravitational disturbances ranging from 0.01 mg to 0.1 m*g* exist in most areas of near space.
- The correction of the J4 term itself in near space is of the magnitude of 0.01 m*g*, and this magnitude is the same as that of the gravity disturbance in most regions of the world.
- For INS, the J2 or J4 normal gravity model is applicable if the used accelerometer constant bias is above 0.1 m*g*.
- When the gyroscope has sufficiently high precision and the precision of the accelerometer reaches 10 μg or higher, the model error of the normal gravity model becomes the main cause of error, and the J2 or J4 model is no longer applicable. This conclusion is verified by simulations without gyroscope error and with a gyroscope constant bias of 0.003°/h.