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Analysis of large deformation geotechnical problems using implicit generalized interpolation material point method

Key words:

Material point method, Large deformation, Implicit GIMP, Volumetric locking, B-bar method

Wei-hai YUAN

BACKGROUND

- ▶The traditional MPM suffers from so-called cell crossing instabilities: the gradients of the shape functions result in discontinuities between elements.
- ▶Volumetric locking for incompressible material introduces a spurious increase in stiffness and numerical oscillations, which eventually lead to numerical instability.
- ▶MPM has so far been formulated mostly in an explicit manner: the time step size is limited by many factors. the explicit formulation may suffer the problem of inaccuracy for some particular problems.

GIMP-METHOD

A quasi-static iGIMP framework is proposed by combining the GIMP interpolation function, the B-bar approach and the implicit integration scheme.

GIMP

$$S_{ip}(x_p) = \frac{1}{V_p} \int_{\Omega_p} \chi_p(x - x_p) N_i(x) dx.$$

S_{ip} -shape function

$$N_i(x) = \begin{cases} 1 + (x - x_i)/h & -h < x - x_i \leq 0, \\ 1 - (x - x_i)/h & 0 < x - x_i \leq h, \\ 0 & \text{otherwise,} \end{cases}$$

N_i -the grid shape function

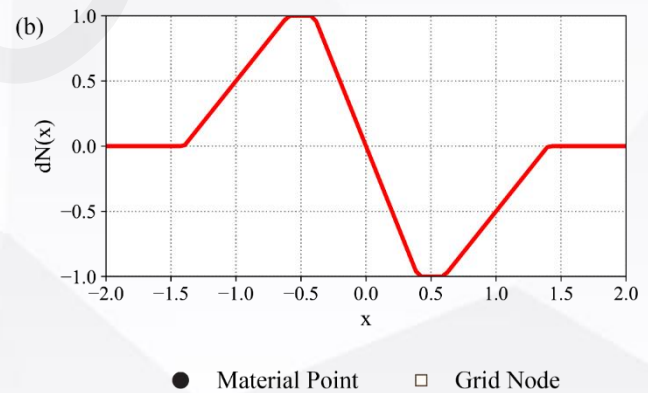
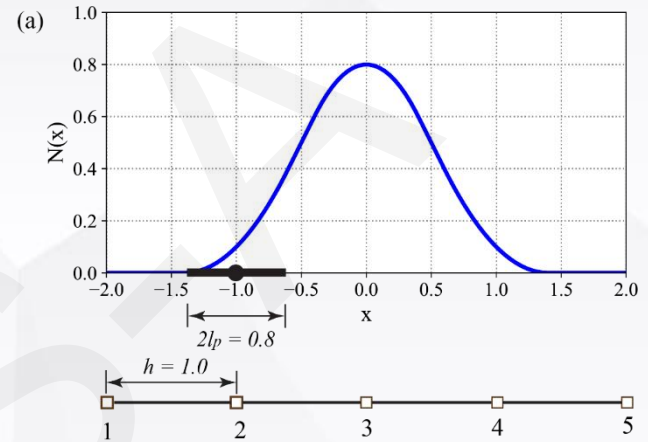
$$\chi_p(x) = \begin{cases} 1 & |x - x_p| \leq l_p, \\ 0 & \text{otherwise,} \end{cases}$$

χ_p -general characteristic function

GIMP-METHOD

$$S_i(x_p) = \begin{cases} 1 - \frac{(x_p - x_i)^2 + l_p^2}{2hl_p} & |x_p - x_i| < l_p, \\ 1 - \frac{|x_p - x_i|}{h} & l_p \leq |x_p - x_i| < h - l_p, \\ \frac{(h + l_p - |x_p - x_i|)^2}{4hl_p} & h - l_p \leq |x_p - x_i| < h + l_p, \\ 0 & \text{otherwise.} \end{cases}$$

$$\nabla S_i(x_p) = \begin{cases} \frac{(x_p - x_i)}{hl_p} & |x_p - x_i| < l_p, \\ -\text{sign}(x) \frac{1}{h} & l_p \leq |x_p - x_i| < h - l_p, \\ -\text{sign}(x) \frac{h + l_p - |x_p - x_i|}{2hl_p} & h - l_p \leq |x_p - x_i| < h + l_p, \\ 0 & \text{otherwise.} \end{cases}$$



Typical GIMP shape function (a) and space derivative of the shape function (b)

B-BAR-METHOD

To overcome volumetric locking, the B-bar concept splits the \mathbf{B} matrix into the dilatational and deviatoric parts.

The strain at material point locations can be calculated by replacing the \mathbf{B} matrix with the $\bar{\mathbf{B}}$ matrix

$$\bar{\mathbf{B}} = \mathbf{B}_{\text{dev}} + \bar{\mathbf{B}}_{\text{dil}}$$

$$\mathbf{B}_{\text{dev}} = \mathbf{B} - \mathbf{B}_{\text{dil}}$$

$$\bar{\mathbf{B}} = \begin{bmatrix} \frac{\partial N_i}{\partial x} + B_4 & B_6 \\ B_4 & \frac{\partial N_i}{\partial y} + B_6 \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} \\ B_4 & B_6 \end{bmatrix} \text{ with}$$

$$B_4 = \frac{\frac{\partial \bar{N}_{ic}}{\partial x} - \frac{\partial N_i}{\partial x}}{3},$$

$$B_6 = \frac{\frac{\partial \bar{N}_{ic}}{\partial y} - \frac{\partial N_i}{\partial y}}{3},$$

EXTEND TO THE iGIMP

$$\bar{S}_i(x_c) = \frac{1}{V_p} \int_{\Omega_p} \frac{1}{2} dx,$$

$$\bar{S}_i(x_c) = \begin{cases} \frac{1}{2} & |x_p - x_i| < h - l_p, \\ \frac{(h + l_p - |x_p - x_i|)^2}{4l_p} & h - l_p \leq |x_p - x_i| < h + l_p, \\ 0 & \text{otherwise.} \end{cases}$$

The shape function at the centre of the element

$$\bar{\mathbf{B}} = \begin{bmatrix} \frac{\partial S_i(x, y)}{\partial x} + B_4 & B_6 \\ B_4 & \frac{\partial S_i(x, y)}{\partial y} + B_6 \\ \frac{\partial S_i(x, y)}{\partial y} & \frac{\partial S_i(x, y)}{\partial x} \\ B_4 & B_6 \\ \frac{\partial \bar{S}_i(x_c, y_c)}{\partial x} - \frac{\partial S_i(x, y)}{\partial x} & \\ B_4 = \frac{\frac{\partial \bar{S}_i(x_c, y_c)}{\partial x} - \frac{\partial S_i(x, y)}{\partial x}}{3}, & \end{bmatrix},$$

with

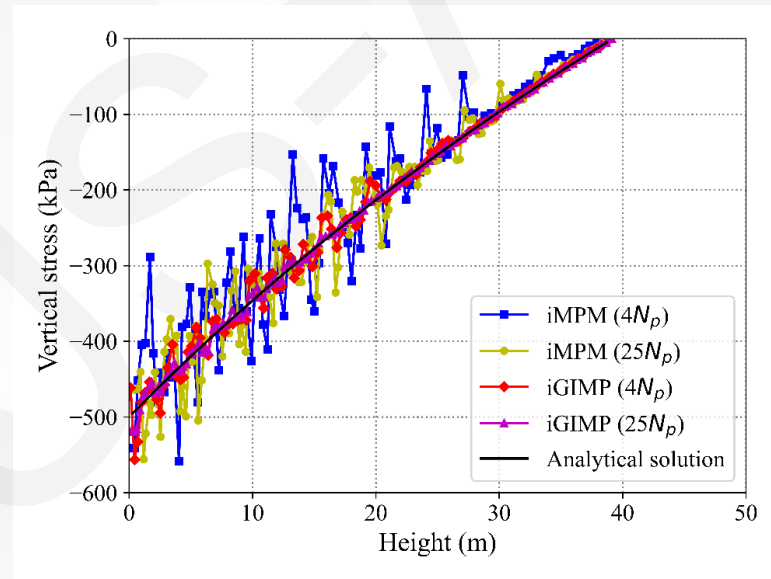
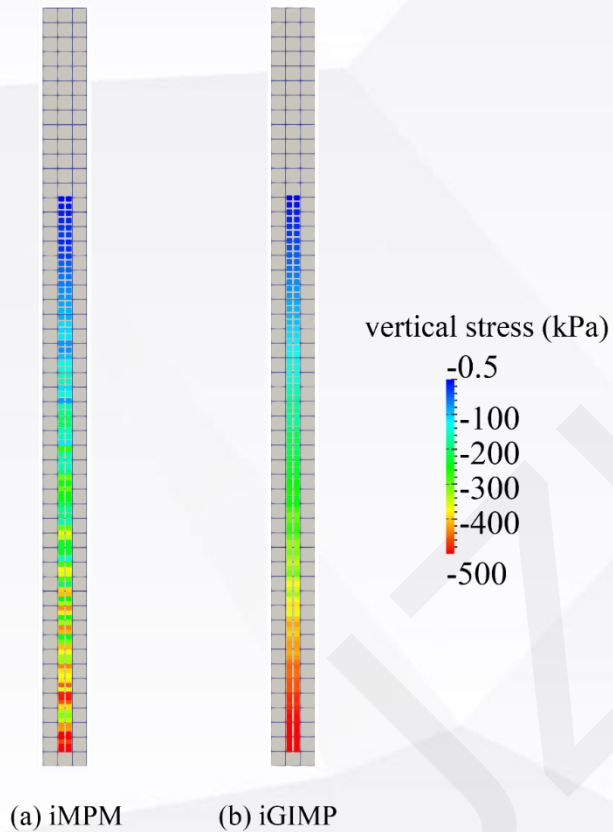
$$B_6 = \frac{\frac{\partial \bar{S}_i(x_c, y_c)}{\partial y} - \frac{\partial S_i(x, y)}{\partial y}}{3}.$$

the $\bar{\mathbf{B}}$ matrix for GIMP

The quasi-static iGIMP with B-bar scheme is computationally efficient and can greatly reduce cell crossing errors and volumetric locking, thereby improving the accuracy of the algorithm.

NUMERICAL EXAMPLES

One-dimensional column in compression

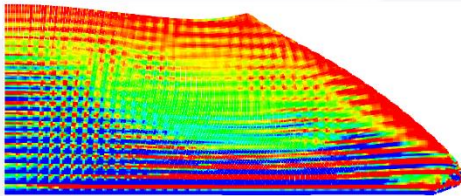


Vertical stress plot along the column

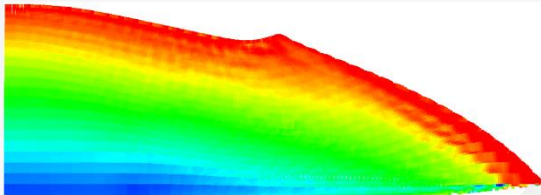
NUMERICAL EXAMPLES

Slope stability in von Mises soil

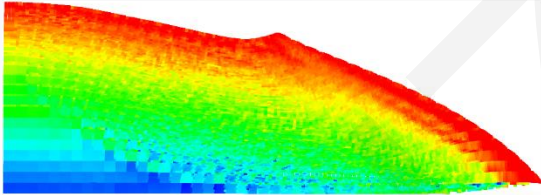
(a) iGIMP



(b) iGIMP with B-bar



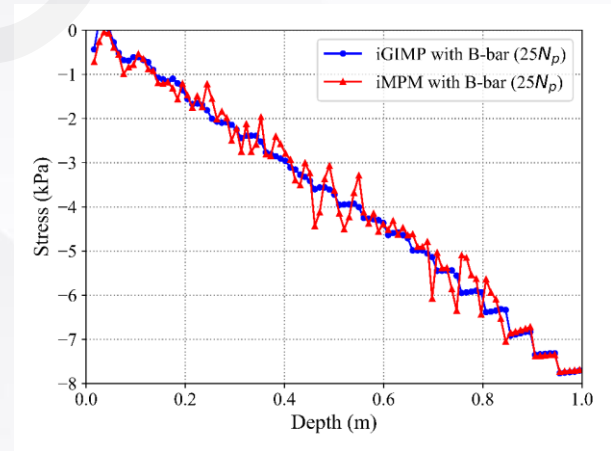
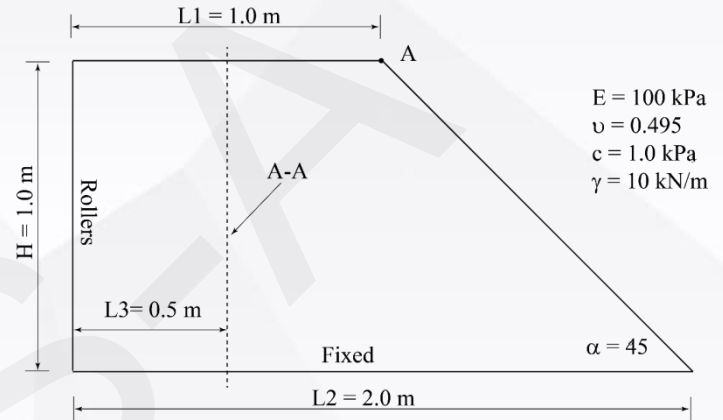
(c) iMPM with B-bar



vertical stress (kPa)



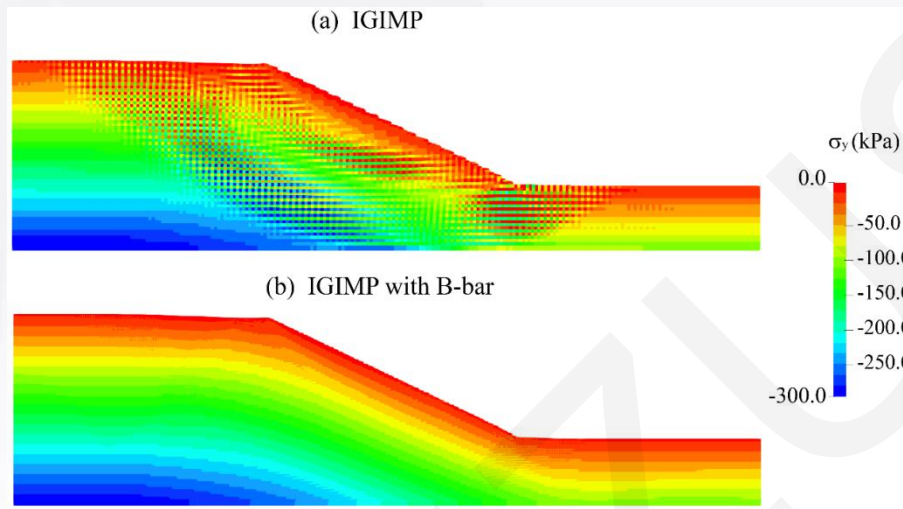
Contour of vertical stress



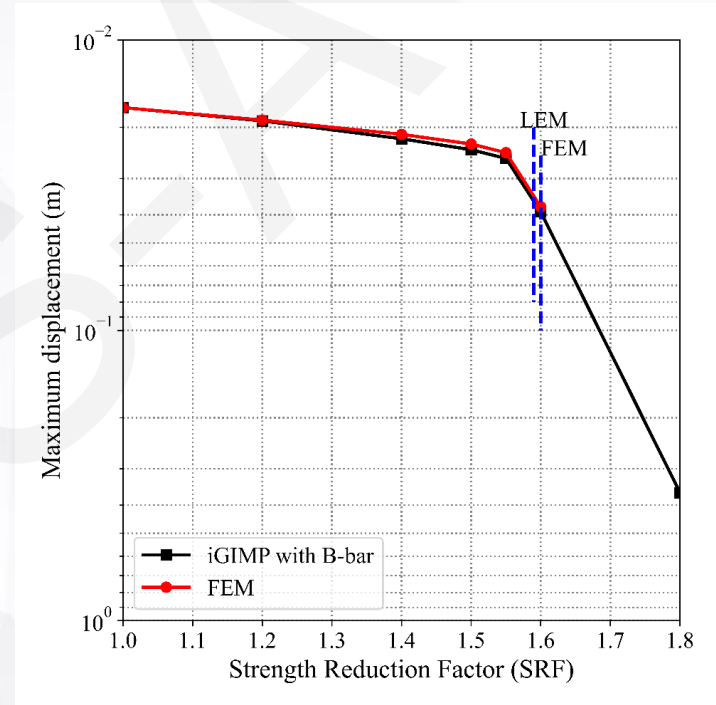
Vertical stress along the cross-section A-A

NUMERICAL EXAMPLES

Slope stability in Mohr-Coulomb soil



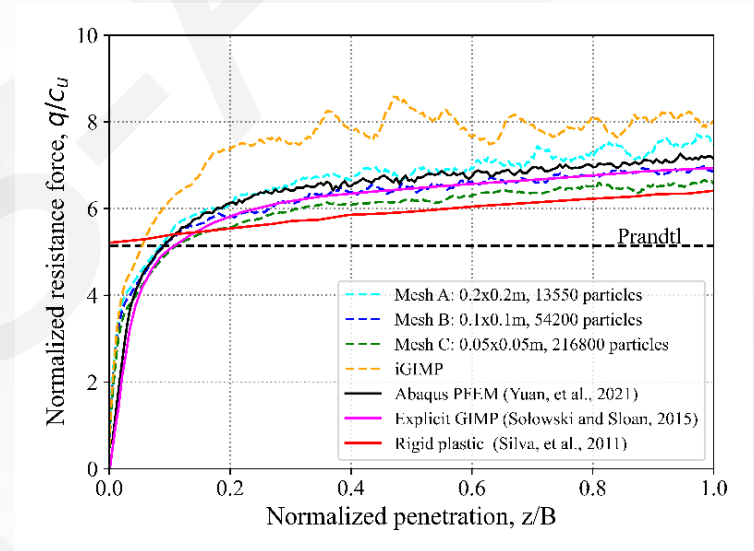
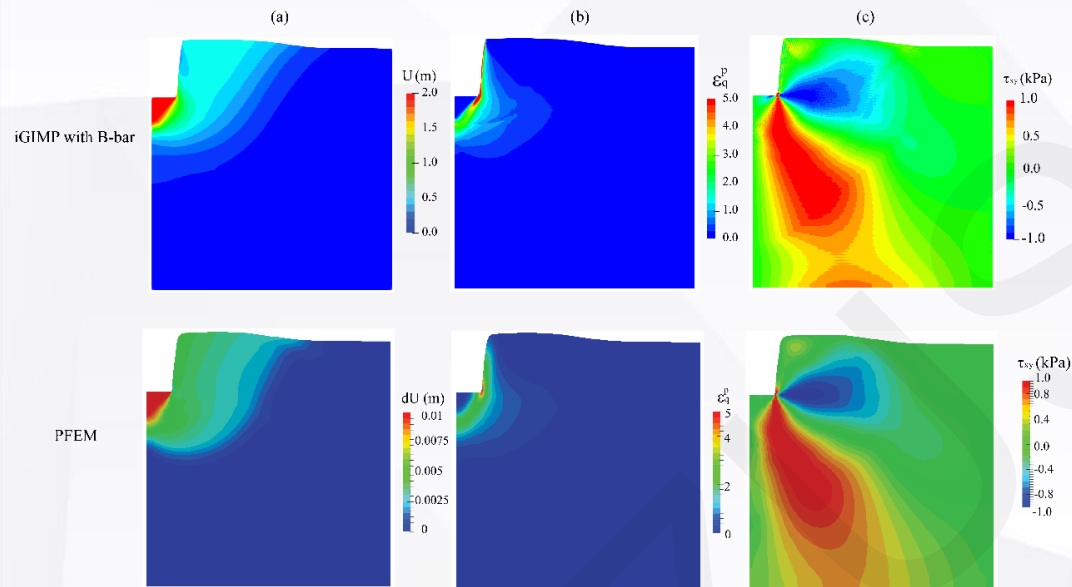
Contour of vertical stress



Maximum displacement versus strength reduction factor

NUMERICAL EXAMPLES

Penetration of rigid footing in Tresca soil



Contour of displacements (a), accumulated plastic strain invariant (b) and shear stress (c) at a penetration depth of $1B$

Normalized resistance force versus penetration depth

CONCLUSION

A new implicit generalized interpolation material point method (iGIMP) framework with B-bar has been developed for large deformation geotechnical problems.

- ▶The B-bar method has been used to overcome volumetric locking in the GIMP method for near-incompressible large deformation geotechnical problems.
- ▶The performance of the iGIMP approach was demonstrated via several numerical examples, including one-dimensional column compression problem, elasto-plastic slope stability problem, slope stability problem with shear strength reduction and rigid footing penetration.
- ▶Compared with the traditional iMPM and iGIMP methods, the proposed iGIMP with B-bar approach can reduce cell crossing and overcome volumetric locking due to incompressibility.
- ▶All the results show that the proposed iGIMP with B-bar approach can handle large deformation problems well in geotechnical engineering.