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Finite volume method-based numerical simulation method for hydraulic fracture initiation in rock around a perforation

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- **Governing equation:**

$$\lambda\nabla^2 T - C_w\rho_w\nabla T\nabla P = C\rho\frac{\partial T}{\partial t},$$

$$\frac{K}{\mu}\nabla^2 P + D_T\nabla^2 T = \left(\beta_0\phi + \alpha_0(1-\phi)\right)\frac{\partial P}{\partial t},$$

- **Time item: Explicit format**

$$\iiint c\rho\frac{\partial T}{\partial t}\mathrm{d}t\mathrm{d}y\mathrm{d}x = (c\rho)_U(T_U^{t+\Delta t} - T_U^t)\Delta x\Delta y,$$

$$\iiint[\beta_0\phi + \alpha_0(1-\phi)]\frac{\partial P}{\partial t}\mathrm{d}x\mathrm{d}y\mathrm{d}t = \left[\beta_0\phi^t + \alpha_0(1-\phi)^t\right]_U\left(P_U^{t+\Delta t} - P_U^t\right)\Delta x\Delta y,$$

- **Space item :**

$$\iiint \lambda\nabla^2 T - C_w\rho_w\nabla T\nabla P\mathrm{d}x\mathrm{d}y\mathrm{d}t = \Sigma\lambda(\nabla T)|S| + \Sigma C_w\rho_w T(\nabla P)|S|,$$

$$\iiint \frac{K}{\mu}\nabla^2 P + D_T\nabla^2 T\mathrm{d}x\mathrm{d}y\mathrm{d}t = \Sigma\frac{K}{\mu}\nabla P|S| + \Sigma D_T\nabla T|S|,$$

- **Thermo-poro-elastic model**

$$\sigma_r = P_w \frac{r_w^2}{r^2} + \delta \frac{\eta}{r^2}\int_{r_w}^{r}(P(r)-P_0)rdr - \phi(P(r)-P_0) - \frac{\eta_T}{r^2}\int_{r_w}^{r} T_{f(r)}rdr + \frac{(\sigma_H-\sigma_h)}{2}(1-\frac{4r_w^2}{r^2}+\frac{3r_w^4}{r^4})\cos 2\theta + \frac{\sigma_H+\sigma_h}{2}(1-\frac{r_w^2}{r^2})$$

$$\sigma_\theta = -P_w \frac{r_w^2}{r^2} - \delta\left[(\eta-\phi)(P(r)-P_0) + \frac{\eta}{r^2}\int_{r_w}^{r}(P(r)-P_0)rdr\right] - \frac{\eta_T}{r^2}(\int_{r_w}^{r} T_{f(r)}rdr - T_{f(r)}) + \frac{\sigma_H+\sigma_h}{2}(1+\frac{r_w^2}{r^2}) - \frac{(\sigma_H-\sigma_h)}{2}(1+\frac{3r_w^4}{r^4})\cos 2\theta$$

$$\sigma_z = \sigma_v - 2v(\sigma_H-\sigma_h)\frac{r_w^2}{r^2}\cos 2\theta + (\eta-\phi)(P(r)-P_0) - \eta_T T_{f(r)}$$

$$\tau_{r\theta} = -\frac{(\sigma_H-\sigma_h)}{2}\left(1+\frac{2r_w^2}{r^2}-\frac{3r_w^4}{r^4}\right)\sin 2\theta$$

$$\tau_{rz} = \tau_{r\theta} = 0 \qquad \eta_T = \frac{\alpha_m E}{3(1-v)}$$

- **Coupling term:** $K = K_0 e^{-M(\sigma-\alpha P)}$

• Flow chart

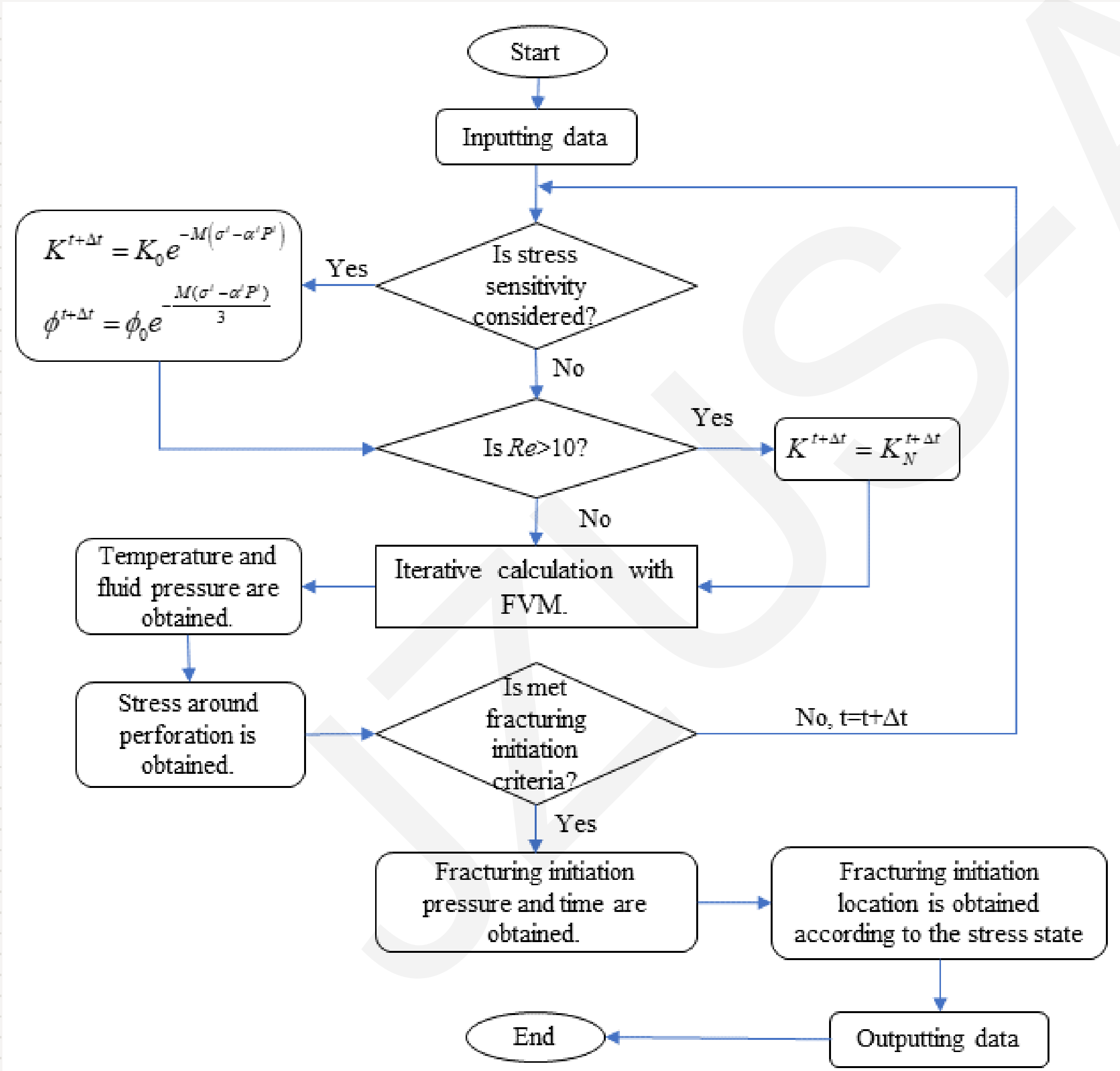
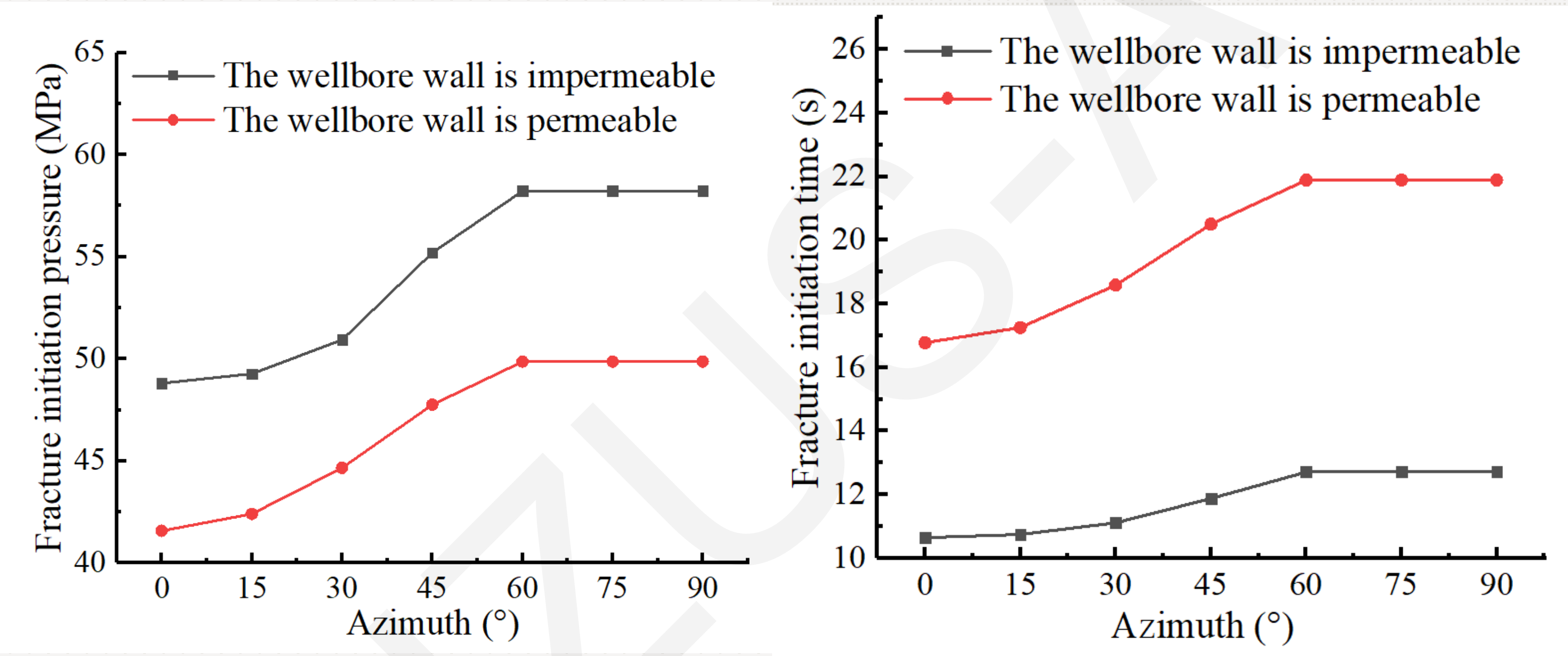


Fig. 1. Iterative calculation process of the numerical simulation method

• Result one



(a) (b)

Fig. 2 Fracture initiation of rock around perforation under different perforation azimuth. (a) the fracture initiation pressure; (b) the fracture initiation time.

- **Result two**

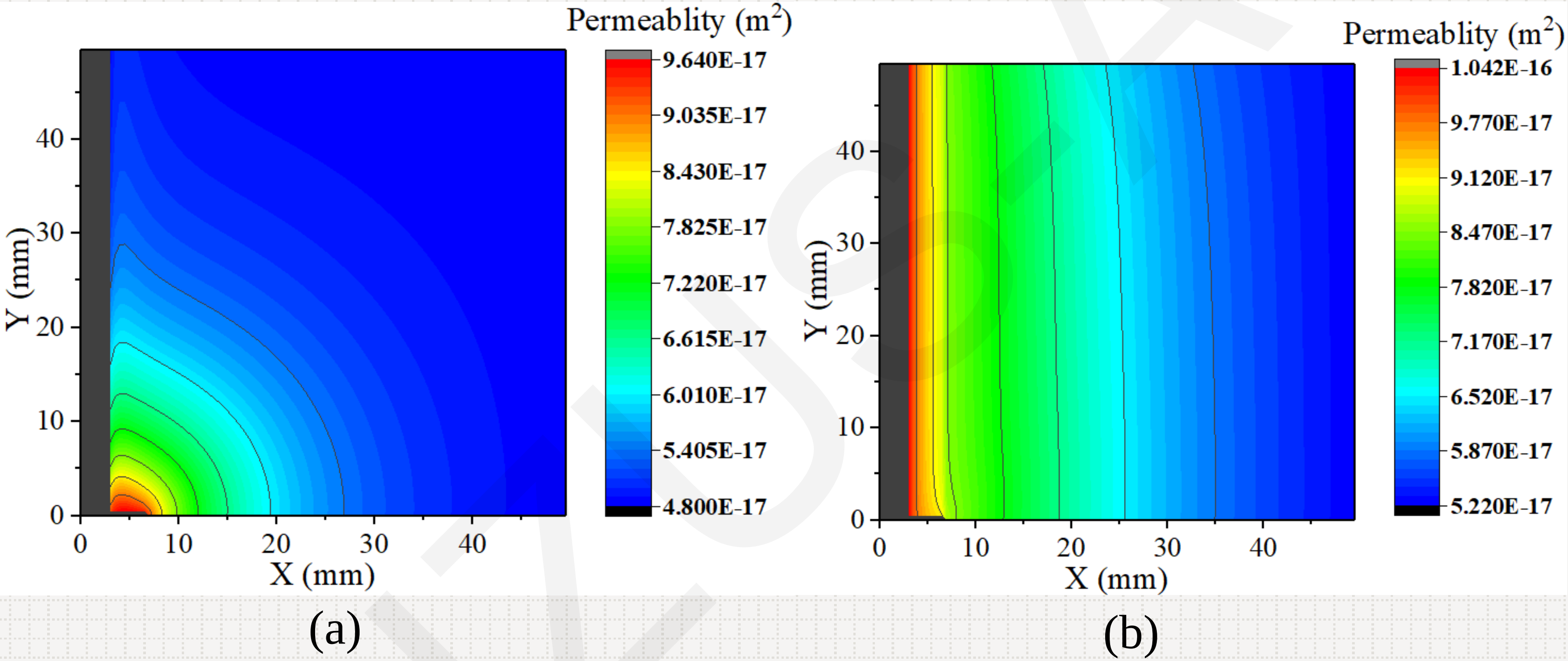


Fig. 3 Distribution of permeability when fracture initiates（θ=0°）. (a) the wellbore wall is impermeable; (b) the wellbore wall is permeable.

• Conclusion

- As the perforation azimuth rise, more injection time and higher fluid pressure are required to reach fracture initiation. The fracture initiation pressure is higher when the wellbore wall is impermeable than permeable.
- Fluid pressure is distributed in an ellipse right before the perforation and reduces gradually from the perforation to the far field when the wellbore wall is impermeable.
- The stress sensitivity of permeability and porosity increased fluid pressure and permeability in the around–well area, which caused a wider range of fluid flow and a reduction in both fracture initiation pressure and time.