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Analytical solution of ground-borne vibration due to a spatially periodic harmonic moving load in a tunnel embedded in layered soil

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Analytical model description

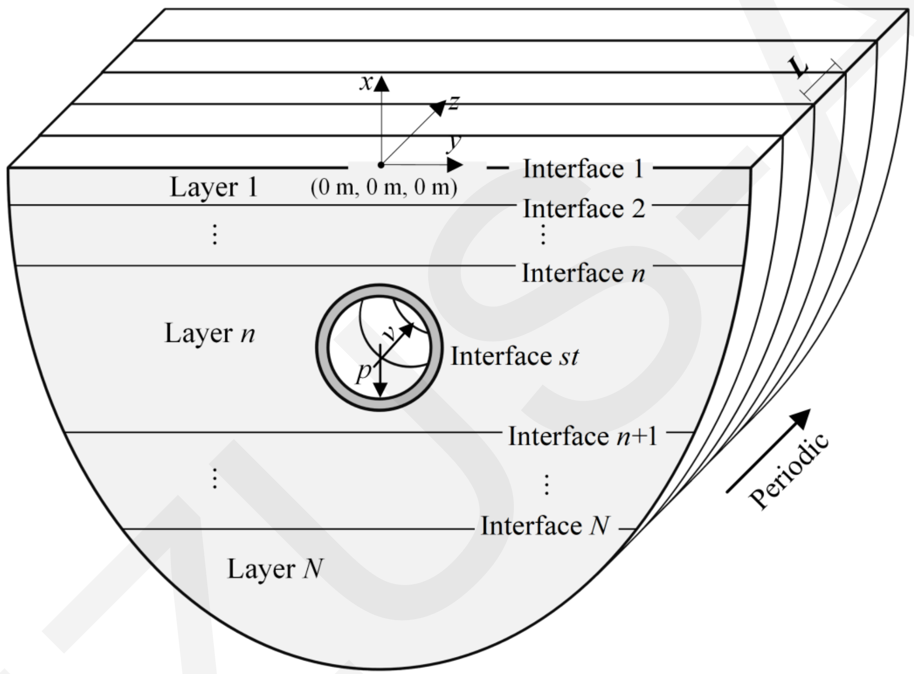

Fig. 1. Tunnel embedded in a multilayered half-space subjected to a spatially periodic harmonic moving load *p* in a global coordinate system.

Each part of the model

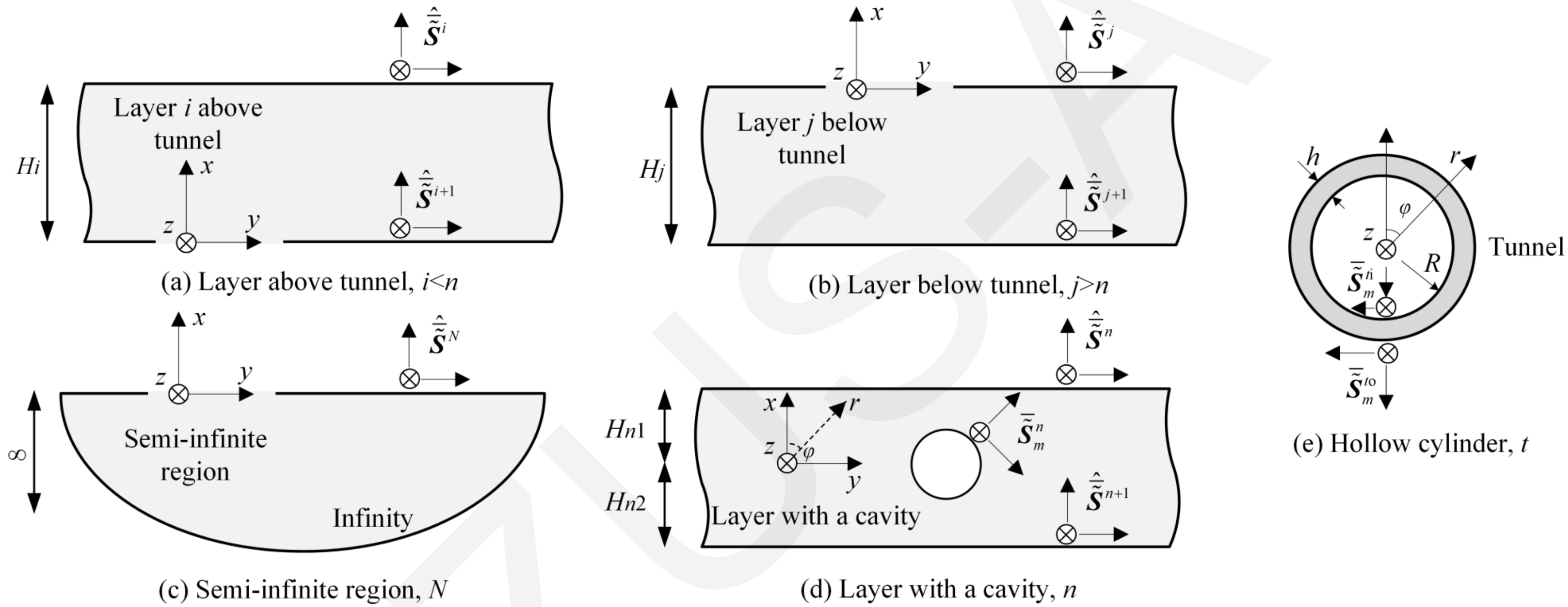

Fig. 2. The geometry, local coordinate system, and state variable at the corresponding interface of the soil layer (a) above and (b) below the tunnel, (c) in the semi-infinite region, (d) in a soil layer with a cavity, and (e) in a hollow cylinder for tunnel lining.

Main formulas

The outgoing cylindrical wave should be converted into ascending or descending plane waves, obeying the following relationships

$$\bar{\tilde{\chi}}_{\mathrm{o}j}^{m} = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{\hat{\bar{\tilde{\phi}}}_{\mathrm{a}j}\mathrm{e}^{\mathrm{i}k_y y}}{k_{xj}} T_{mj}^{-}\mathrm{d}k_y$$

$$\bar{\tilde{\chi}}_{\mathrm{o}j}^{m} = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{\hat{\bar{\tilde{\phi}}}_{\mathrm{d}j}\mathrm{e}^{\mathrm{i}k_y y}}{k_{xj}} T_{mj}^{+}\mathrm{d}k_y$$

The ascending and descending plane wave potentials should be expanded in terms of regular cylindrical wave potentials, expressed as

$$\hat{\bar{\tilde{\phi}}}_{\mathrm{a}j}\mathrm{e}^{\mathrm{i}k_y y} = \sum_{m=0}^{M} \varepsilon_m \bar{\tilde{\chi}}_{\mathrm{r}j}^{m} T_{mj}^{+}$$

$$\hat{\bar{\tilde{\phi}}}_{\mathrm{d}j}\mathrm{e}^{\mathrm{i}k_y y} = \sum_{m=0}^{M} \varepsilon_m \bar{\tilde{\chi}}_{\mathrm{r}j}^{m} T_{mj}^{-}$$

The force applied at the inverted arch of the tunnel structure can be mathematically expressed as

$$p\left(r,\varphi,z,t\right) = \frac{1}{R}\delta(r-R)\delta(\varphi-\pi)\delta\left(z-vt\right)\mathrm{e}^{\mathrm{i}\xi_n z}\mathrm{e}^{\mathrm{i}\omega_l t}$$

$$\xi_n = \frac{2\pi n}{L}$$

The governing equation of motion can be derived as

$$\begin{bmatrix} \bar{\tilde{\eta}}_{\mathrm{o}}^{m}(r=R) & \bar{\tilde{\eta}}_{\mathrm{r}}^{m}(r=R) \end{bmatrix} \bullet \begin{bmatrix} \bar{\tilde{\chi}}_{\mathrm{o}}^{m}(r=R+h) & \bar{\tilde{\chi}}_{\mathrm{r}}^{m}(r=R+h) \\ \bar{\tilde{\eta}}_{\mathrm{o}}^{m}(r=R+h) & \bar{\tilde{\eta}}_{\mathrm{r}}^{m}(r=R+h) \end{bmatrix}^{-1} \bullet \begin{bmatrix} \boldsymbol{C}_{m}^{n}(r=R+h) \\ \boldsymbol{D}_{m}^{n}(r=R+h) \end{bmatrix} \boldsymbol{A}_{\mathrm{o}} = \bar{\tilde{\boldsymbol{t}}}_{m}(r=R)$$

Validation of the proposed model

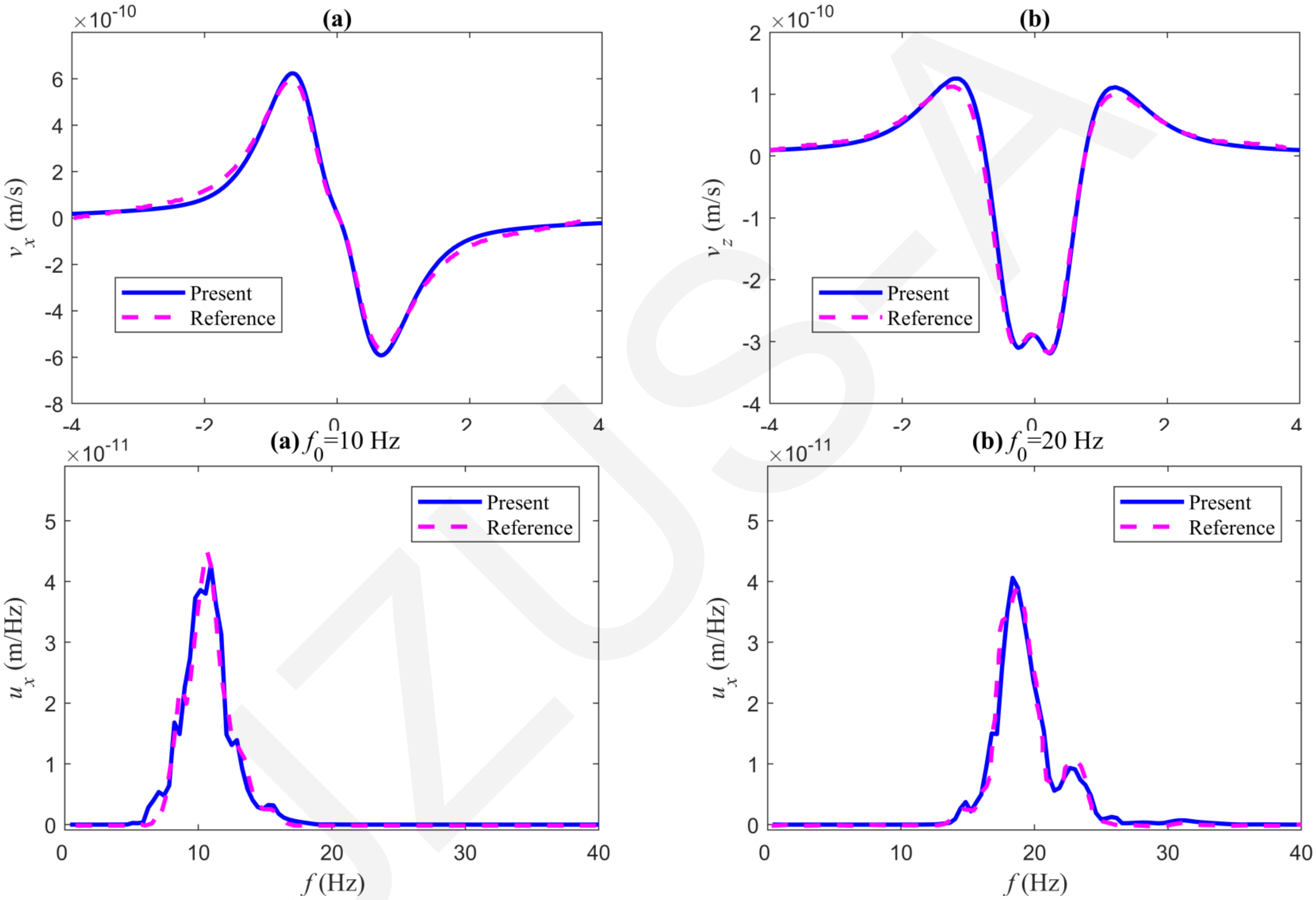

Fig. 3. Comparison of the proposed model with other published models to ensure its correctness and accuracy.

Ground displacement distribution

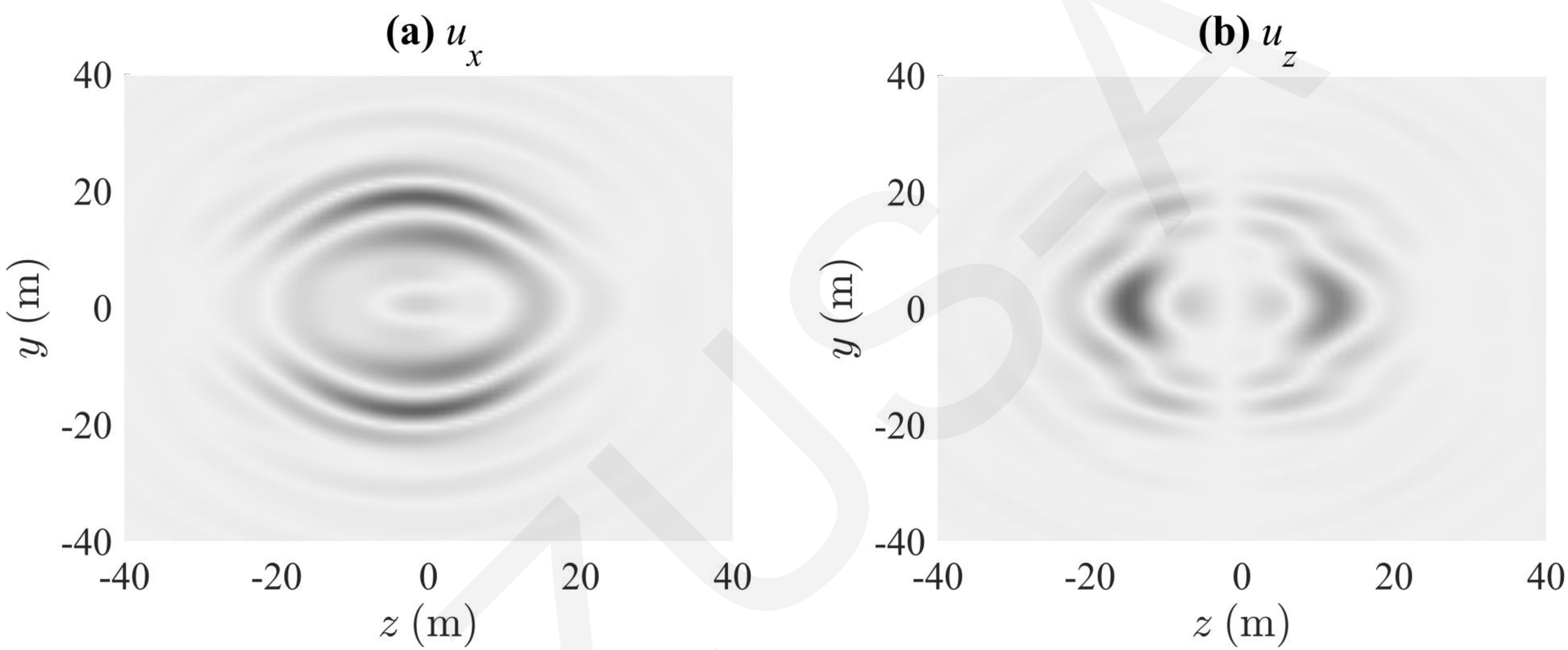

Fig. 4. Absolute instantaneous (a) vertical displacement u_x and (b) longitudinal displacement u_z on the ground surface at the time instant t=0s.

Typical results

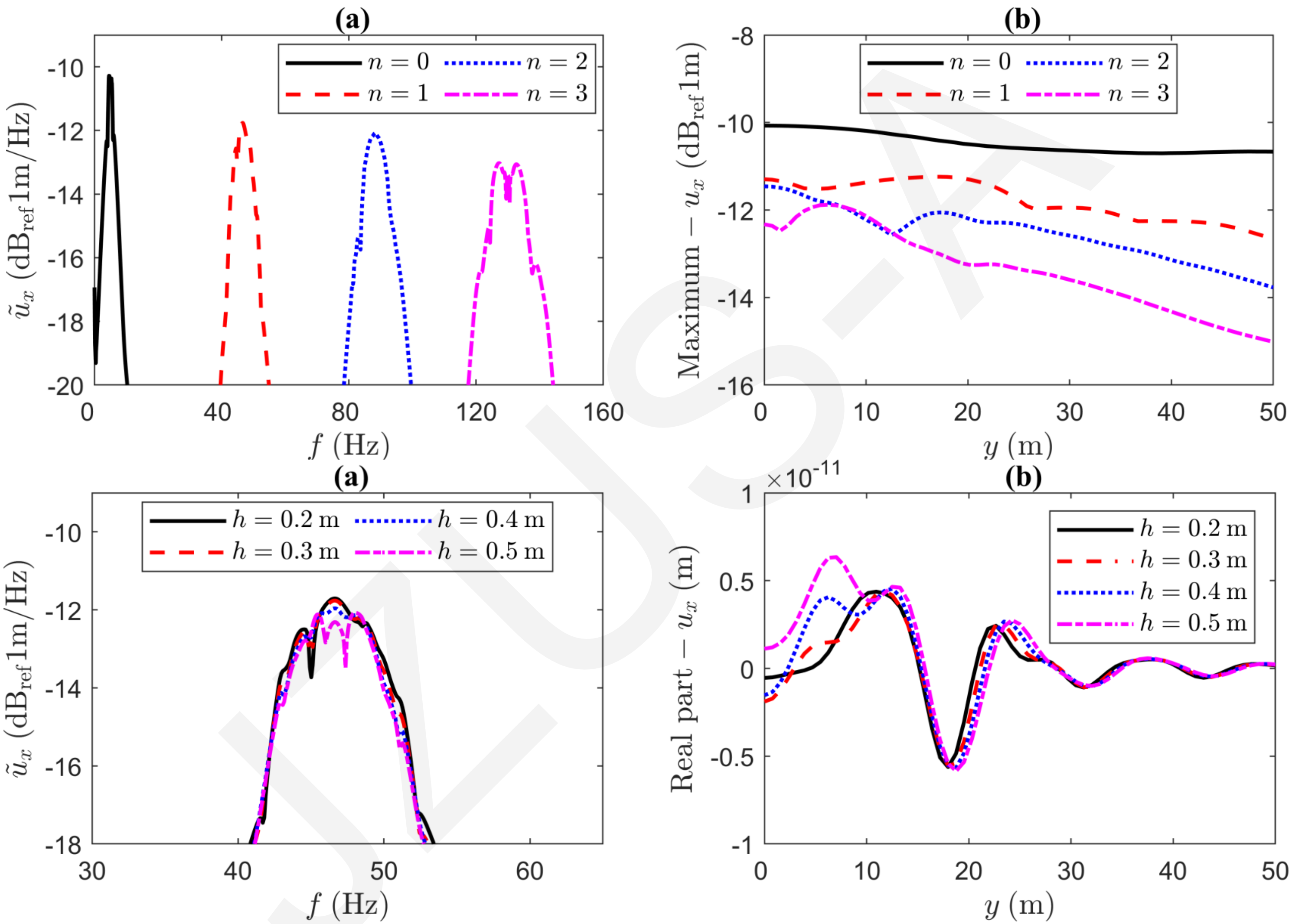

Fig. 5. Effect of wavenumber in one periodicity length and tunnel thickness on the vertical displacement response.

Conclusions

- The proposed coupled periodic tunnel-soil analytical model is highly accurate, computationally efficient, and can be used to predict the ground-borne vibrations induced by train operations within a tunnel.
- Both moving and Doppler effects can be excited by a spatially periodic harmonic moving load.
- Increasing the tunnel depth was found to be an efficient way to reduce the level of ground-borne vibration.
- A vibration amplification area exists at a certain horizontal distance from the tunnel axis. This should be considered to avoid potential excessive train-induced vibration that disturbs residents.