Biophysical neurons, energy, and synapse controllability: a review

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1. Simple neural circuit and field energy

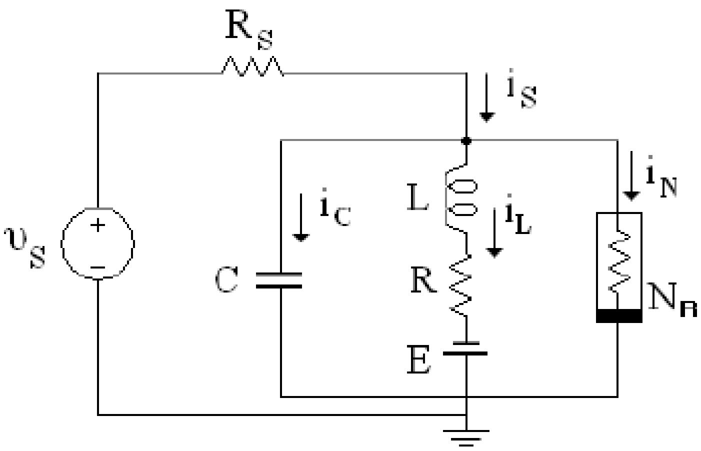

Fig.1 Schematic diagram for a *RLC* neural circuit.

$$\begin{cases} C\dfrac{dV}{dt} = \dfrac{V_S - V}{R_s} - i_L - i_{NR} \\ L\dfrac{di_L}{dt} = V - Ri_L + E \end{cases} ;\ (1)$$

$$i_{NR} = -\frac{1}{\rho}\left(V - \frac{1}{3}\frac{V^3}{V_0^2}\right);\ (2)$$

$$FE = \frac{1}{2}CV^2 + \frac{1}{2}Li_L^2;\ (3)$$

$$\begin{cases} x = \dfrac{V}{V_0}, y = \dfrac{\rho i_L}{V_0}, \tau = \dfrac{t}{\rho C}, a = \dfrac{E}{V_0}, b = \dfrac{R}{\rho} \\ c = \dfrac{\rho^2 C}{L}, \xi = \dfrac{\rho}{R_s}, u_s = \dfrac{V_s \rho}{R_s V_0} = \xi\dfrac{V_s}{V_0} \end{cases} ;\ (4)$$

$$\begin{cases} \dfrac{dx}{d\tau} = x(1-\xi) - \dfrac{1}{3}x^3 - y + u_s \\ \dfrac{dy}{d\tau} = c(x + a - by) \\ H = \dfrac{FE}{CV_0^2} = \dfrac{1}{2}x^2 + \dfrac{1}{2c}y^2 \end{cases} ;\ (5)$$

2. Memristive neuron and energy

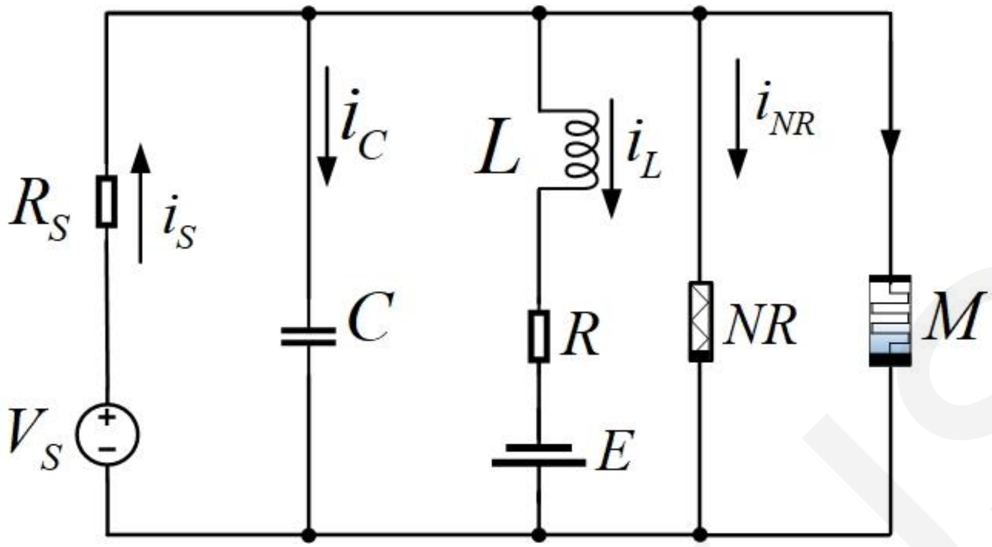

Fig.2 Memistor-coupled neural circuit.

$$H_M=\frac{E_M}{CV_0^2}=\frac{1}{2}\frac{L_M i_M^2}{CV_0^2}=\frac{1}{2}\frac{\varphi i_M}{CV_0^2}=\frac{1}{2}\frac{\varphi M(\varphi)V_M}{CV_0^2}$$
$$=\frac{1}{2}\frac{k\varphi(a+3b\varphi^2)xV_0}{CV_0^2},\ \varphi'=\frac{\varphi}{\rho CV_0}$$
$$=\frac{1}{2}kx\varphi'\rho(a+3bC^2\rho^2V_0^2\varphi'^2)$$
$$=\frac{1}{2}kx\varphi'(a'+3b'\varphi'^2) \qquad (7a)$$

$$\begin{cases}\dfrac{dx}{d\tau}=x(1-\xi)-\dfrac{1}{3}x^3-y+u_s-kM(\varphi')x;\\ \dfrac{dy}{d\tau}=c(x+a-by);\\ \dfrac{d\varphi'}{d\tau}=kx+\varphi'_{ext};\end{cases} \quad ;\ (6)$$

$$\begin{cases}H=\dfrac{E_{CLM}}{CV_0^2}=\dfrac{1}{2}x^2+\dfrac{1}{2c}y^2+\dfrac{1}{2}k_M\alpha'x\varphi+\dfrac{1}{2}k_M\beta'x\varphi^3;\\ H_M=\dfrac{1}{2}k_M(\alpha'x\varphi+\beta'x\varphi^3);\end{cases} \quad (7b)$$

3. Field coupling and synapse coupling

$$\begin{cases} \dfrac{dx_i}{d\tau} = x_i(1-\xi) - \dfrac{1}{3}x_i^3 - y_i + u'^i_s - kM(\varphi'_i)x_i \\ \qquad + S(x_{i+1} + x_{i-1} - 2x_i); \\ \dfrac{dy_i}{d\tau} = c(x_i + a - by_i); \\ \dfrac{d\varphi'_i}{d\tau} = kx_i + D\sum_{j=1}^{N}(x_j - x_i) + \varphi'_{ext}; \end{cases} \quad ;(8)$$

Neural network composed of memristive neurons under field coupling and synaptic coupling. D represents the coupling intensity for magnetic field, S denotes the intensity for synaptic coupling between neurons subjected to external electromagnetic radiation φ'_{ext}.

$$\begin{cases} \dot{x}_{i,j} = x_{i,j}(1-\xi) - \dfrac{1}{3}x_{i,j}^3 - y_{i,j} + u_s + \\ \qquad D(x_{i+1,j} + x_{i-1,j} + x_{i,j+1} + x_{i,j-1} - 4x_{i,j}); \\ \dot{y}_{i,j} = c[x_{i,j} + a - b(T_{i,j})y_{i,j}] \end{cases} \quad (11)$$

Neural network composed of thermo sensitive neurons under synaptic coupling on square array. D represents the coupling intensity for adjacent neurons in regular network.

$$F = \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}x_{i,j},$$

$$SF = \frac{\langle F^2 \rangle - \langle F \rangle^2}{\frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}(\langle x_{i,j}{}^2 \rangle - \langle x_{i,j} \rangle^2)}; (12)$$

Synchronization factor R is close to 1, perfect synchronization is obtained in the neural network.

4. Josephson junction-based neural circuit

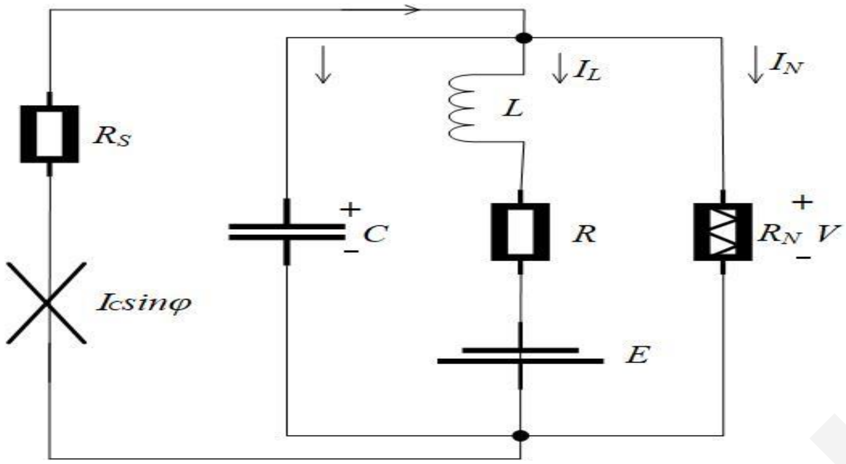

Fig.3 Schematic diagram for neural circuit connected with an ideal Josephson junction(JJ).

$$\begin{cases} C\dfrac{dV}{dt} = I_C sin\phi - i_L - i_{NR}; \\ L\dfrac{di_L}{dt} = V + E - Ri_L; \\ V - R_S I_C sin\varphi = \dfrac{\hbar}{2e}\dfrac{d\phi}{dt}; \end{cases} \quad (18)$$

$$\begin{cases} x = \dfrac{V}{V_0}, \quad y = \dfrac{\rho i_L}{V_0}, z = \phi, \quad \tau = \dfrac{t}{\rho C}, \\ a = \dfrac{E}{V_0}, \quad b = \dfrac{R}{\rho}, c = \dfrac{\rho^2 C}{L}, \\ d = \dfrac{\rho I_C}{V_0}, g = \dfrac{2e\rho C V_0}{\hbar}, m = \dfrac{2e\rho C R_S I_C}{\hbar}; \end{cases} \quad (19)$$

$$\begin{cases} \dfrac{dx}{d\tau} = x - \dfrac{1}{3}x^3 - y + dsinz; \\ \dfrac{dy}{d\tau} = c(x + a - by); \\ \dfrac{dz}{d\tau} = gx - msinz; \end{cases} \quad (20)$$

5. Hamilton energy and growth of synapse

$$\begin{cases} \dfrac{dX}{dt} = F_c(X) + F_d(X) = [J(X) + R(X)]\nabla H,\ X \subset R^N; \\ \dfrac{dH}{dt} = \nabla H^T [J(X) + R(X)]\nabla H; \\ \nabla H^T J(X)\nabla H = \nabla H^T F_c(X) = 0; \end{cases} \quad (28)$$

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} x(1-\xi) - \frac{1}{3}x^3 - y + u_s \\ c(x + a - by) \end{pmatrix} = F_c + F_d$$

$$= \begin{pmatrix} -y \\ cx \end{pmatrix} + \begin{pmatrix} x(1-\xi) - \frac{1}{3}x^3 + u_s \\ c(a - by) \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -c \\ c & 0 \end{pmatrix} \begin{pmatrix} x \\ \frac{y}{c} \end{pmatrix} + \begin{pmatrix} (1-\xi) - \frac{1}{3}x^2 + \frac{u_s}{x} & 0 \\ 0 & c^2(\frac{a}{y} - b) \end{pmatrix} \begin{pmatrix} x \\ \frac{y}{c} \end{pmatrix}; \quad (29)$$

$$\begin{cases} \dfrac{dx_1}{d\tau} = x_1(1-\xi) - \frac{1}{3}x_1^3 - y_1 + u_s^1 + k(x_2 - x_1); \\ \dfrac{dy_1}{d\tau} = c_1(x_1 + a - by_1); \\ \dfrac{dx_2}{d\tau} = x_2(1-\xi) - \frac{1}{3}x_2^3 - y_2 + u_s^1 + k(x_1 - x_2); \\ \dfrac{dy_2}{d\tau} = c_2(x_2 + a - by_2); \\ \dfrac{dk}{d\tau} = \sigma.k\vartheta(\Delta H - \varepsilon),\ \vartheta(p) = 1,\ p \geq 1,\ \vartheta(p) = 0,\ p < 0; \end{cases} \quad (32)$$

$$(-y)\frac{\partial H}{\partial x} + cx\frac{\partial H}{\partial y} = 0\,; \quad (30)$$

$$\Delta H = |H_1 - H_2| = \frac{1}{2}\left|(x_1^2 + \frac{1}{c_1}y_1^2) - (x_2^2 + \frac{1}{c_2}y_2^2)\right|; \quad (31)$$

Growth of synapse is controlled by energy diversity between neurons, complete synchronization is accompanied with energy balance.

6. Formation of Heterogeneity and defects

$$\begin{cases} \frac{dx_i}{dt} = f(x_i, y_i, \mu_i) + D \sum_{j=1, i \neq j}^{N} \varepsilon_{ij}(x_j - x_i); \\ \frac{dy_i}{dt} = g(x_i, y_i, \mu_i); \\ \frac{d\mu_i}{dt} = \pm\delta\mu_i \vartheta(\varepsilon - \Delta H_i), \\ \vartheta(p) = 1, p \geq 0, \vartheta(p) = 0, p < 0; \end{cases} \quad ; (33)$$

External energy injection and asymmetrical energy propagation can induce shape deformation of the media, and parameters shifts are induced under energy diversity. Nonuniform distribution of energy can develop local defects and heterogeneity in the neural network and media.

7. Scientific contribution and conclusion

1. Presented a list of functional neuron models by connecting specific electric components into FHN neural circuit.
2. Field energy is described by equivalent Hamilton energy, which is confirmed from Helmholtz theorem.
3. Self-adaptability mechanism for biophysical neurons are explained from physical viewpoint, and it claimed energy flow controls the growth of synapse and connection.
4. Nonuniform distribution of energy creates heterogeneity and local defects, and parameters shifts are induced to keep energy balance between neurons.