

# Core-drilling kinematic modeling and analysis of Jiaolong submersible manipulator

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# Research Object

- The Jiaolong submersible is the first Chinese deep-sea manned submersible.
- Jiaolong deep-sea manipulator contains 6 active rotating joints and one end jaw.



**Fig. 1. (a) The Jiaolong manned submersible;  
(b) Manipulator-based core drilling.**

# Manipulator 3-D Model and Link Coordinates

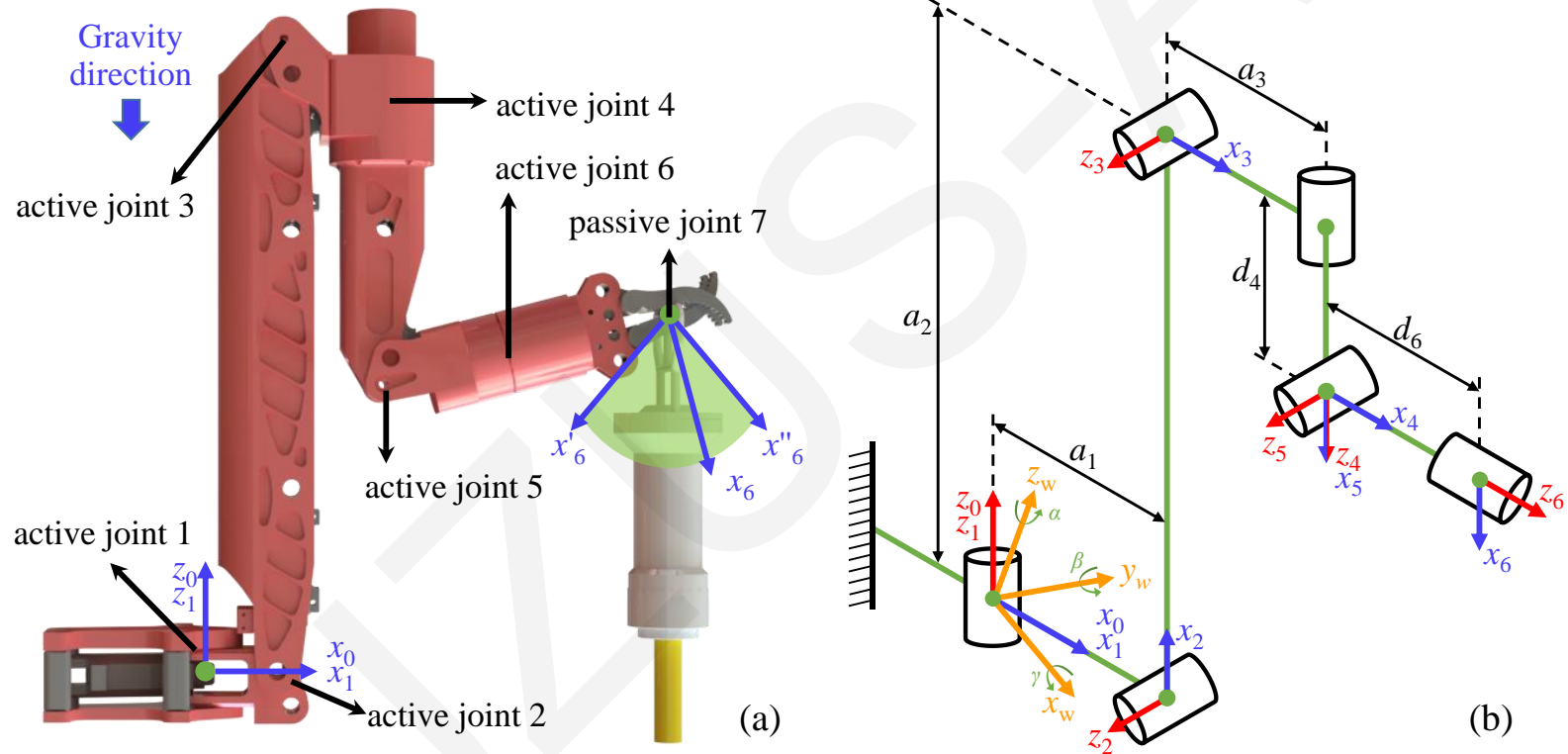


Fig. 2. (a) Jiaolong submersible manipulator-based core drilling; (b) Coordinates of Jiaolong submersible manipulator.

# Core-drilling forward kinematic model

## ■ Transformation matrix (w-coordinate to 6-coordinate)

$$\begin{aligned}
 {}^w_6T &= {}^w_0T {}^0_6T \\
 &= \begin{bmatrix} r_{11} & r_{12} & r_{13} & 0 \\ r_{21} & r_{22} & r_{23} & 0 \\ r_{31} & r_{32} & r_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} r_{11}n_x + r_{12}n_y + r_{13}n_z & r_{11}o_x + r_{12}o_y + r_{13}o_z & r_{11}a_x + r_{12}a_y + r_{13}a_z & r_{11}p_x + r_{12}p_y + r_{13}p_z \\ r_{21}n_x + r_{22}n_y + r_{23}n_z & r_{21}o_x + r_{22}o_y + r_{23}o_z & r_{21}a_x + r_{22}a_y + r_{23}a_z & r_{21}p_x + r_{22}p_y + r_{23}p_z \\ r_{31}n_x + r_{32}n_y + r_{33}n_z & r_{31}o_x + r_{32}o_y + r_{33}o_z & r_{31}a_x + r_{32}a_y + r_{33}a_z & r_{31}p_x + r_{32}p_y + r_{33}p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}
 \end{aligned}$$

## ■ Constraint equation of sedimentary core drilling

$${}^w_6T(3,2) = r_{31}o_x + r_{32}o_y + r_{33}o_z = 0$$

## ■ Analytical solution of joint variable $\theta_6$

$$\theta_6 = \psi(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5) = \begin{cases} \forall & \text{if: } g_x = 0 \wedge g_y = 0 \\ \text{Atan2}(g_y, g_x) - \cos^{-1} 0 & \text{if: } g_x \neq 0 \vee g_y \neq 0 \end{cases}$$

$$\begin{aligned}
 g_x &= r_{31}(c_1c_{23}s_4 - s_1c_4) \\
 &\quad + r_{32}(s_1c_{23}s_4 + c_1c_4) \\
 &\quad + r_{33}s_4s_{23}
 \end{aligned}$$

$$\begin{aligned}
 g_y &= r_{31}(c_1s_{23}s_5 - c_1c_{23}c_4c_5 - s_1s_4c_5) \\
 &\quad + r_{32}(s_1s_{23}s_5 - s_1c_{23}c_4c_5 + c_1s_4c_5) \\
 &\quad - r_{33}(s_5c_{23} + c_4c_5s_{23})
 \end{aligned}$$

# Core-drilling inverse kinematic model

■ Based on the forward kinematics equation, the following three analytical solutions are obtained

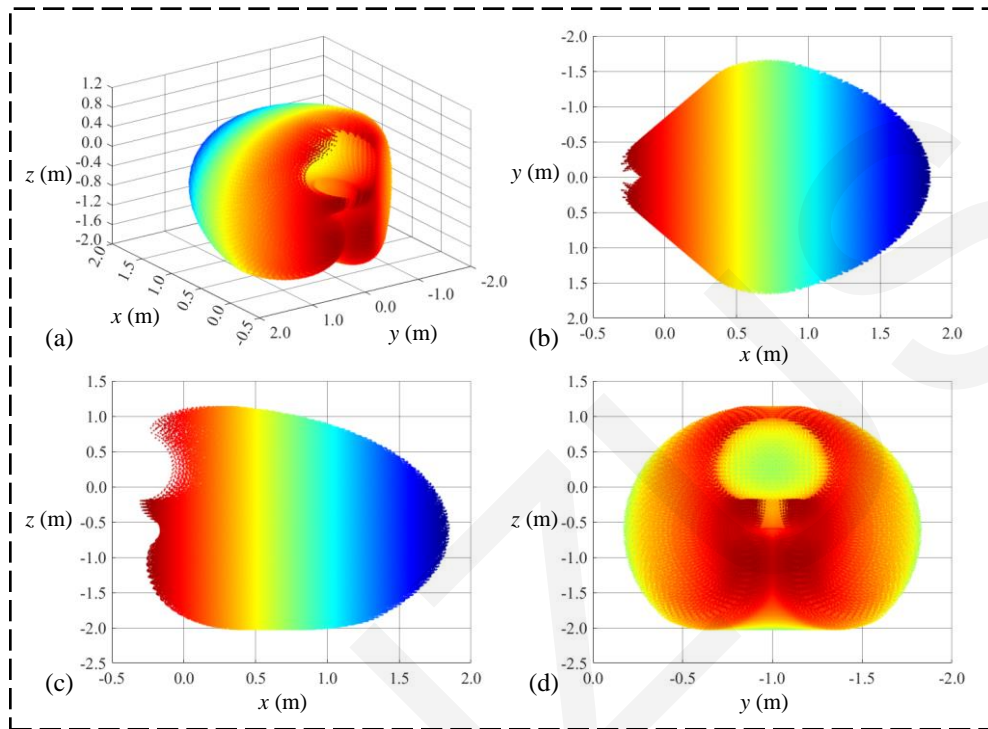
$$\theta_1 = \gamma(\theta_2, \theta_3, \zeta, \eta) = \begin{cases} \forall & \text{if: } w_x = 0 \wedge w_y = 0 \\ \text{Atan2}(w_y, w_x) - \text{Atan2}(k_1, k_2) & \text{if: } w_x \neq 0 \vee w_y \neq 0 \end{cases}$$

$$\theta_4 = \eta(\theta_2, \theta_3, \zeta) = \begin{cases} \forall & \text{if: } s_\zeta = 0 \wedge k_3 = a_2 s_2 + a_3 s_{23} - d_4 c_{23} + c_\zeta c_{23} d_6 \\ & \wedge k_1^2 + k_2^2 = (a_1 + a_2 c_2 + a_3 c_{23} + d_4 s_{23} - d_6 s_{23} c_\zeta)^2 \\ \forall & \text{if: } s_\zeta \neq 0 \wedge s_{23} = 0 \wedge a_1 + a_2 c_2 + a_3 c_{23} = 0 \\ & \wedge k_1^2 + k_2^2 = (d_6 s_\zeta)^2 \\ \cos^{-1} \left[ -\frac{k_1^2 + k_2^2 - (a_1 + a_2 c_2 + a_3 c_{23})^2 - (d_6 s_\zeta)^2}{2d_6 c_{23} s_\zeta (a_1 + a_2 c_2 + a_3 c_{23})} \right] & \text{if: } s_\zeta \neq 0 \wedge s_{23} = 0 \wedge a_1 + a_2 c_2 + a_3 c_{23} \neq 0 \\ \cos^{-1} \left[ \frac{a_2 s_2 + a_3 s_{23} - d_4 c_{23} + c_\zeta c_{23} d_6 - k_3}{s_\zeta s_{23} d_6} \right] & \text{if: } s_\zeta \neq 0 \wedge s_{23} \neq 0 \end{cases}$$

$$\theta_5 = \zeta(\theta_2, \theta_3) = \begin{cases} \forall & \text{if: } a_1 + a_2 c_2 + a_3 c_{23} + d_4 s_{23} = 0 \wedge s_{23} \neq 0 \\ & \wedge k_1^2 + k_2^2 + (a_2 s_2 + a_3 s_{23} - d_4 c_{23} - k_3)^2 - d_6^2 = 0 \\ \cos^{-1} \left[ \frac{k_3 - a_2 s_2 + d_4 c_{23}}{c_{23} d_6} \right] & \text{if: } s_{23} = 0 \\ \cos^{-1} \left[ -\frac{s_{23} (k_1^2 + k_2^2) - s_{23} i^2 + 2c_{23} ij + s_{23} j^2 - s_{23} d_6^2}{2d_6 i} \right] & \text{if: } a_1 + a_2 c_2 + a_3 c_{23} + d_4 s_{23} \neq 0 \wedge s_{23} \neq 0 \end{cases}$$

# Core-drilling workspace analysis

## ■ Horizontal Jiaolong submersible posture.



The core-drilling workspace surrounds the manipulator, the envelope surface of which is like an ellipsoid.

The reachable core-drilling motion range was:

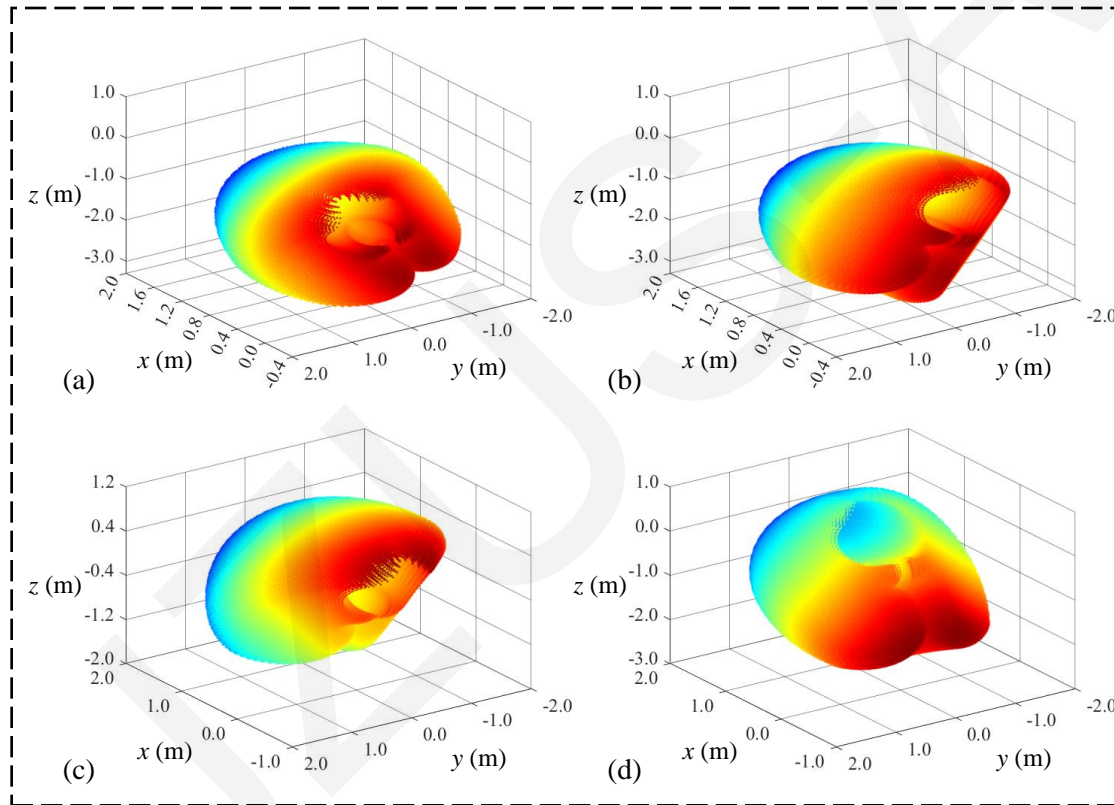
- 300.5~1847.0 mm (X direction)
- 1656.6~1656.6 mm (Y direction)
- 2023.7~1133.9 mm (Z direction)

**Fig. 3. Core-drilling workspace of the Jiaolong submersible manipulator without incline and rotation (yaw angle  $\alpha=0^\circ$  , pitch angle  $\beta=0^\circ$  , roll angle  $\gamma=0^\circ$  ) (a) 3D view; (b) x-y view; (c) x-z view; (d) y-z view.**

# Core-drilling workspace analysis

## ■ Different Jiaolong submersible posture.

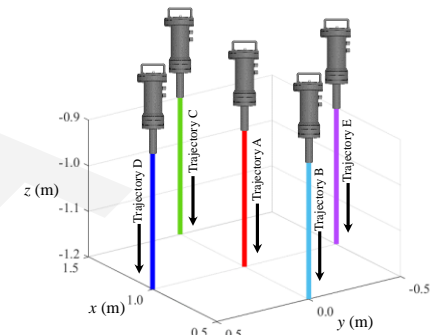
The core-drilling workspace is influenced by the posture of the Jiaolong submersible.



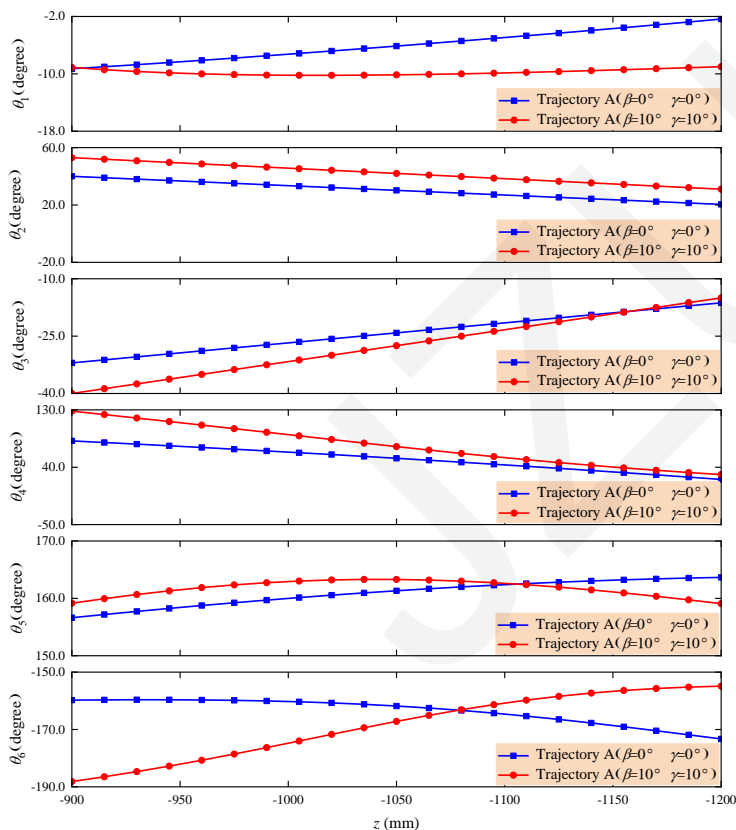
**Fig. 4. Core-drilling workspace of the Jiaolong submersible with different posture (a) yaw angle  $\alpha=0^\circ$  , pitch angle  $\beta=0^\circ$  , roll angle  $\gamma=-30^\circ$  ; (b) yaw angle  $\alpha=0^\circ$  , pitch angle  $\beta=0^\circ$  , roll angle  $\gamma=30^\circ$  ; (c) yaw angle  $\alpha=0^\circ$  , pitch angle  $\beta=-30^\circ$  , roll angle  $\gamma=0^\circ$  ; (d) yaw angle  $\alpha=0^\circ$  , pitch angle  $\beta=30^\circ$  , roll angle  $\gamma=0^\circ$**

# Core-drilling trajectory analysis

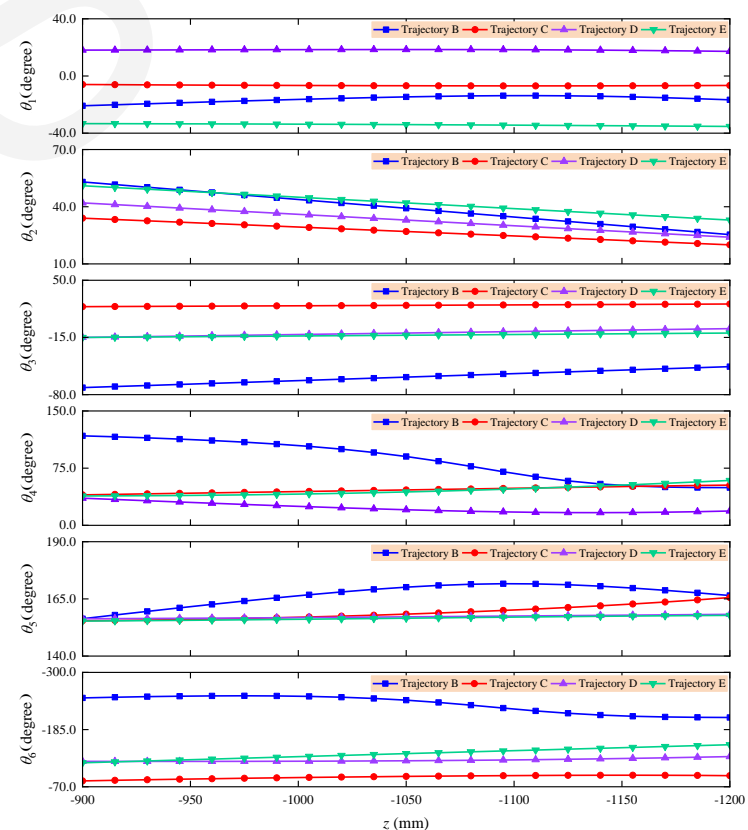
- Select five core drilling trajectory: A, B, C, D and E.
- The double-redundancy inverse kinematic model is computationally effective for deriving joint variable.



different posture



different trajectory



# Conclusions

- This paper proposes a core-drilling kinematic model for the Jiaolong submersible manipulator, comprised of two key parts: forward kinematics and inverse kinematics. The established forward and inverse kinematic models are constructed with clear analytic equations, and thus are directly applicable to the Jiaolong submersible's manipulator-based core-drilling task.

