

# **Bifurcation control of solid angle car-following model through a time-delay feedback method**

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# Research Background

## *Car-Following Model*

### Traffic Engineering Perspective

- Stimulus-response Model
- Safe Spacing Model
- Physiological Mental Model
- Artificial Intelligence Model

### Statistical Physics Perspective

- Optimized Speed Model
- Intelligent Driving Model
- Metacellular Automata Model

# Research Background

## *The proposal of the angle car-following model*

**Michaels perspective change model:**

$$\frac{d\theta_n(t)}{dt} = -W \frac{\Delta V_n(t)}{(S_n(t))^2}$$

**Vehicle conservation**

**Bando car-following model:**

$$\dot{v}_n(t) = a[V(\Delta x_n(t)) - v_n(t)]$$

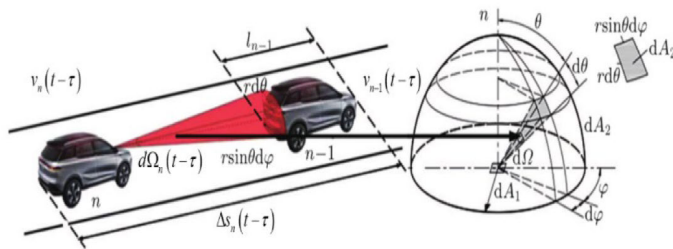
**Speed tends towards optimal speed**

$$a_n(t) = \alpha \{V[\Omega_n(t)] - u_n(t)\} - \lambda \frac{d}{dt} \Omega_n(t) \text{ **solid angle car-following model**}$$

$$V(\Omega_n(t)) = V_1 + V_2 \tanh\left(C_1 \sqrt{S_{n-1}(t)/\Omega_n(t)} - C_2\right)$$

# Research Content

## Actual scenario



## Controlled model

$$\frac{dv_n(t)}{dt} = \alpha [V(\Omega_n(t)) - v_n(t)] + \varepsilon M (v_n(t) - v_n(t - \tau))$$

$$V(\Omega_n(t)) = V_1 + V_2 \tanh\left(C_1 \sqrt{S_{n-1}(t)/\Omega_n(t)} - C_2\right)$$

$$M = 2A_{n-1}/(h - l_{n-1})^3$$

## Linear analysis

$$V'(\Omega_0) > \frac{\alpha}{2(Mp\varepsilon_1\tau_1 + M(1-p)\varepsilon_2\tau_2 - 1)M}$$

Stability conditions of the centerline

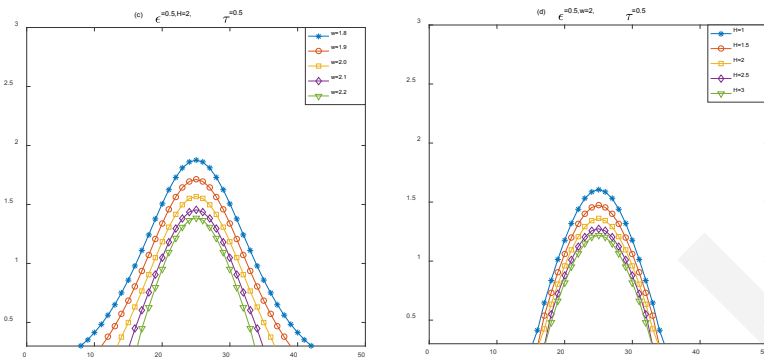
## Bifurcation analysis

$$\begin{cases} -\omega^2 + \varphi_1\omega s_{\tau_1} + \beta - \varphi_2c_{\tau_2} = \beta c_k - \varphi_2c_{\tau_2}c_k - \varphi_2s_{\tau_2}s_k \\ -\delta\omega + \varphi_1\omega c_{\tau_1} + \varphi_2s_{\tau_2} = \beta s_k - \varphi_2c_{\tau_2}s_k + \varphi_2s_{\tau_2}c_k \end{cases}$$

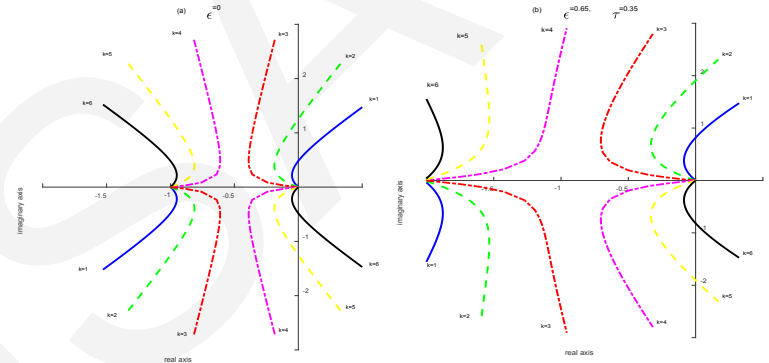
Hopf bifurcation critical condition

# Research Results

## Numerical simulation



Neutral stability curve

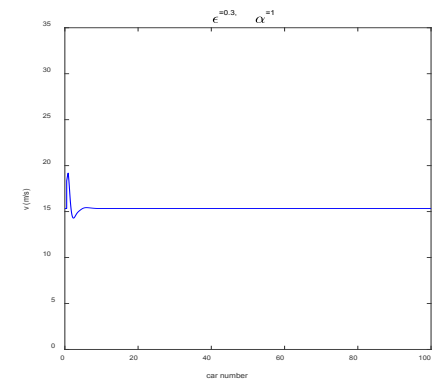
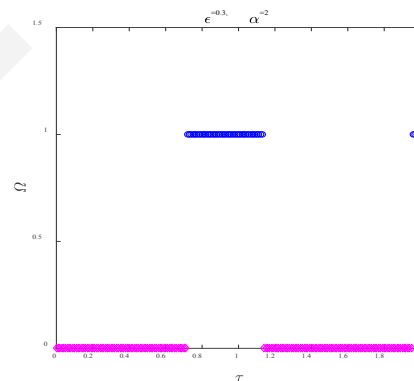


Absolute value distribution curve

## Case studies

Table 2 Calibration parameters<sup>⊖</sup>

parameters <sup>⊖</sup>	范围 <sup>⊖</sup>	SAM <sup>⊖</sup>	受控 SAM <sup>⊖</sup>
$\alpha(s^{-1})$ <sup>⊖</sup>	[0, 2] <sup>⊖</sup>	0.5164 <sup>⊖</sup>	0.4823 <sup>⊖</sup>
$h(m)$ <sup>⊖</sup>	[8, 40] <sup>⊖</sup>	31.5834 <sup>⊖</sup>	27.0565 <sup>⊖</sup>
$v_{max}(m \cdot s^{-1})$ <sup>⊖</sup>	[20, 40] <sup>⊖</sup>	29.4532 <sup>⊖</sup>	34.4623 <sup>⊖</sup>
$\varepsilon(s^{-1})$ <sup>⊖</sup>	[-1, 1] <sup>⊖</sup>	0 <sup>⊖</sup>	0.7746 <sup>⊖</sup>
$\tau(s)$ <sup>⊖</sup>	[0, 2] <sup>⊖</sup>	0 <sup>⊖</sup>	0.7179 <sup>⊖</sup>
$PJ$ <sup>⊖</sup>	<sup>⊖</sup>	0.6624 <sup>⊖</sup>	0.6237 <sup>⊖</sup>



# Conclusions

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- A feedback control with response delay is designed for SAM to suppress bifurcation caused by unstable factors, and suppress traffic flow oscillations;
- Reasonable feedback gain and delay settings can effectively improve the stability of traffic flow.