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Efficient reliability analysis via a nonlinear autoregressive multi-fidelity surrogate model and active learning

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Motivation

1. For complex engineering problems, multi-fidelity modeling has been used to achieve efficient reliability analysis by leveraging multiple information sources. However, most methods require nested training samples to capture the correlation between different fidelity data, which may lead to a significant increase in computational cost.

2. Existing multi-fidelity surrogate-based modeling methods often use a scaling factor and an error term to represent the relationship between different fidelity samples. However, the scaling factor and error term cannot completely consider possible nonlinear relationships, which may lead to an incorrect trend and unreliable predictions.

Main idea

1. To improve the model’ s ability to represent the relationship between information with different fidelities, we introduce the nonlinear autoregressive scheme and construct a multi-fidelity surrogate named the nonlinear autoregressive multi-fidelity Kriging (NAMK) model.

2. In the process of model refinement, the traditional learning function is replaced by a collective multi-fidelity learning function, which selects new sampling points from the multi-fidelity sample space by comprehensively considering the sampling cost and the correlation between multi-fidelity samples.

3. To further reduce the number of samples, instead of directly sampling, nested low-fidelity samples are generated using a constructed residual model when selecting high-fidelity samples.

Proposed method

The main steps of the proposed method are as follows:

1. Initial multi-fidelity samples are selected within the specified parameter range, and the NAMK is used to build the initial surrogate model.

2. The integrated learning function determines the location and fidelity level of new samples.

3. Once a high-fidelity sample is selected, nested low-fidelity samples are generated using a residual model, and the model is updated with the new samples.

4. The active learning process is terminated using a stopping criterion based on relative error estimation, and failure probability results are obtained.

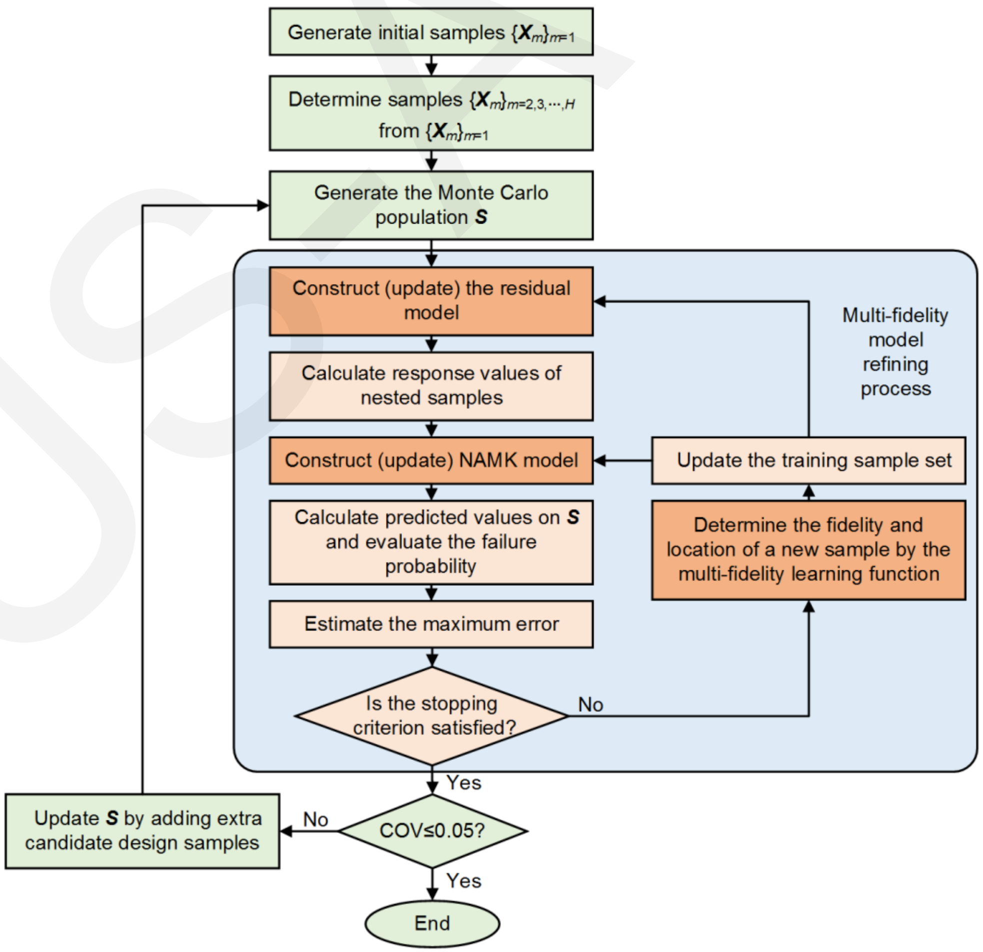

Fig. 1 Flowchart of the proposed method

Case studies

Case 1: multimodal function

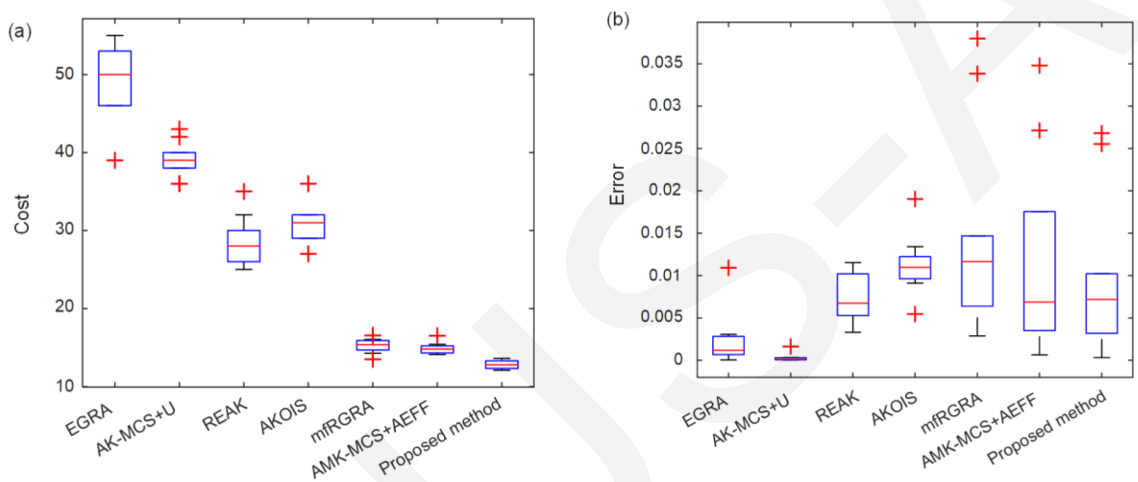

Fig. 2 Boxplots of cost (a) and relative error (b) of different methods

Table 1 Results of different methods for the multimodal function

Methods	Cost	$\hat{p}_f\left(\times10^{-2}\right)$	COV (%)	Error (%)
MCS	10^6	3.13	0.56	-
EGRA	49.3	3.14	3.93	0.24
AK-MCS+U	39.2	3.13	3.93	2.84×10^{-2}
REAK	28.6	3.14	3.92	0.74
AKOIS	30.8	3.16	3.90	1.13
mfEGRA	$15.24(12.1+26.6\times0.1+47.9\times0.01)$	3.09	3.96	1.45
AMK-MCS+AEFF	$14.9(10.3+46\times0.1)$	3.12	3.93	1.15
Proposed method	$12.90(10.9+16.1\times0.1+39.2\times0.01)$	3.12	3.89	0.97

Case studies

Case 2: 4-D PARK function

Table 2 Results of different methods for the 4-D PARK function

Methods	Cost	$\hat{p}_f\left(\times 10^{-2}\right)$	COV (%)	Error (%)
MCS	1×10^6	3.86	0.50	-
EGRA	72.2	3.92	3.50	2.01
AK-MCS+U	59.8	3.86	3.52	0.13
REAK	42.8	3.87	3.52	0.36
AKOIS	47.3	3.84	3.54	0.48
mfEGRA	$40.15(30.6+95.5\times0.1)$	3.73	3.60	5.80
AMK-MCS+AEFF	$38.71(29.8+89.1\times0.1)$	3.81	3.55	2.13
Proposed method	$28.70(22.5+62.0\times0.1)$	3.85	3.54	0.51

Case studies

Case 3: vehicle side impact problem

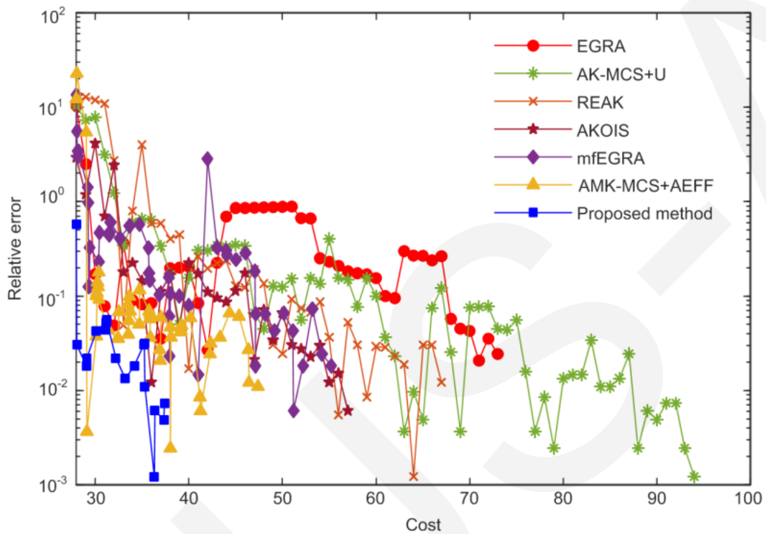

Fig. 3 Relative error of failure probability of different methods (shown in log-scale)

Table 3 Results of different methods for the vehicle side impact problem

Methods	Cost	$\hat{p}_f\left(\times 10^{-4}\right)$	COV (%)	Error (%)
MCS	5×10^6	1.64	3.49	-
EGRA	79.9	1.74	3.39	6.86
AK-MCS+U	91.8	1.65	3.49	0.55
REAK	65.2	1.62	3.51	1.09
AKOIS	58.0	1.65	3.48	0.82
mfEGRA	$53.9(48.4+109.3\times0.05)$	1.60	3.54	2.41
AMK-MCS+AEFF	$49.5(42.6+137.2\times0.05)$	1.61	3.52	1.28
Proposed method	$37.6(33.6+80.4\times0.05)$	1.67	3.48	1.19

Case studies

Engineering application: aircraft tubing

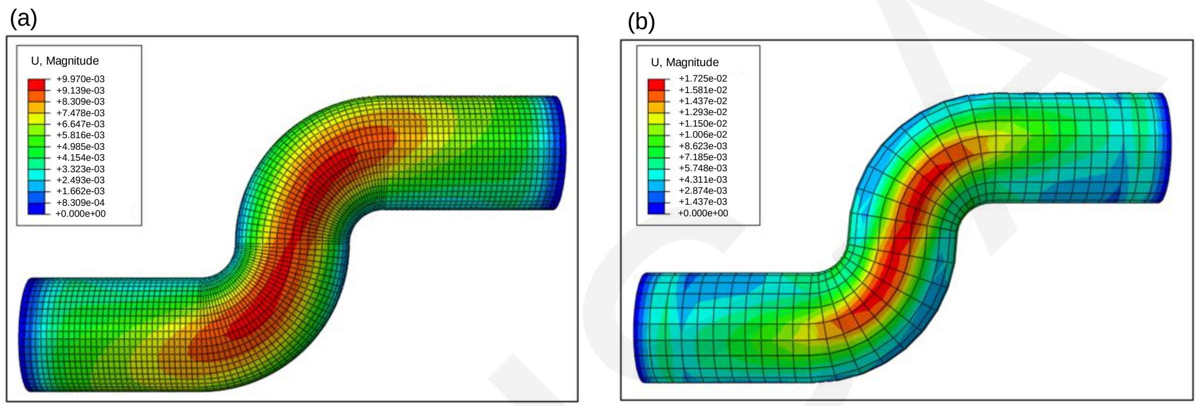

Fig. 4 Simulation results of the high-fidelity model (a) and low-fidelity model (b)

Table 4 Results of different methods for the aircraft tubing problem

Methods	Cost	$\hat{p}_f\left(\times 10^{-2}\right)$	COV (%)	Error (%)
MCS	1×10^4	4.92	4.41	-
EGRA	43.9	4.72	4.51	5.09
AK-MCS+U	38.5	4.93	4.39	0.66
REAK	34.1	4.95	4.36	0.80
AKOIS	37.9	4.96	4.35	1.12
mfEGRA	$26.2(20.6+44.8\times 1/8)$	4.87	4.64	2.10
AMK-MCS+AEFF	$23.6(17.6+48.2\times 1/8)$	4.96	4.36	1.41
Proposed method	$18.8(14.7+32.6\times 1/8)$	4.90	4.43	1.07

Conclusions

1. The proposed multi-fidelity modeling and active learning-based reliability analysis method improves the efficiency and accuracy of failure probability estimation.

2. The NAMK model effectively captures the nonlinear relationships between multi-fidelity samples, enhancing the accuracy of the surrogate model.

3. The multi-fidelity learning function, considering both multi-fidelity sample correlation and sampling cost, adaptively determines the location and fidelity level of new samples.

4. The use of the residual model to generate nested low-fidelity samples reduces the number of low-fidelity model calls when high-fidelity samples are selected by the learning function.