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Numerical simulation of 3D supersonic asymmetric truncated nozzle based on *k-kL* algebraic stress model

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Key words:

Supersonic nozzle; Turbulence model; Numerical simulation; Performance analysis

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k-*kL*-ARSM2018+J Turbulence Model

k-*kL* turbulence model

linear constitutive relationship: $\tau_{ij} = 2\mu_{\mathrm{t}}\left(S_{ij} - \frac{1}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij}\right) - \frac{2}{3}\rho k \delta_{ij}$

$$\frac{\partial \rho k}{\partial t} + \frac{\partial \rho u_j k}{\partial x_j} = P_k + \frac{\partial}{\partial x_j}\left((\mu + \sigma_k \mu_{\mathrm{t}})\frac{\partial \rho k}{\partial x_j}\right) - C_w \mu \frac{k}{d^2} - C_k \rho \frac{k^{2.5}}{(kL)}$$

$$\frac{\partial \rho (kL)}{\partial t} + \frac{\partial \rho u_j (kL)}{\partial x_j} = C_{(kL)}\frac{(kL)}{k}P_{kL} + \frac{\partial}{\partial x_j}\left((\mu + \sigma_{(kL)} \mu_{\mathrm{t}})\frac{\partial \rho (kL)}{\partial x_j}\right) - 6\mu \frac{(kL)}{d^2} f_{(kL)} - C_{(kL)_2}\rho k^{1.5}$$

The Reynolds stress model has high computational cost and is difficult to converge for complex problems.

The linear eddy-viscosity model performs poorly in predicting flow with sudden changes in the mean strain rate, such as flow over curved walls, rotating flows (vortex flows), and three-dimensional flows, etc.

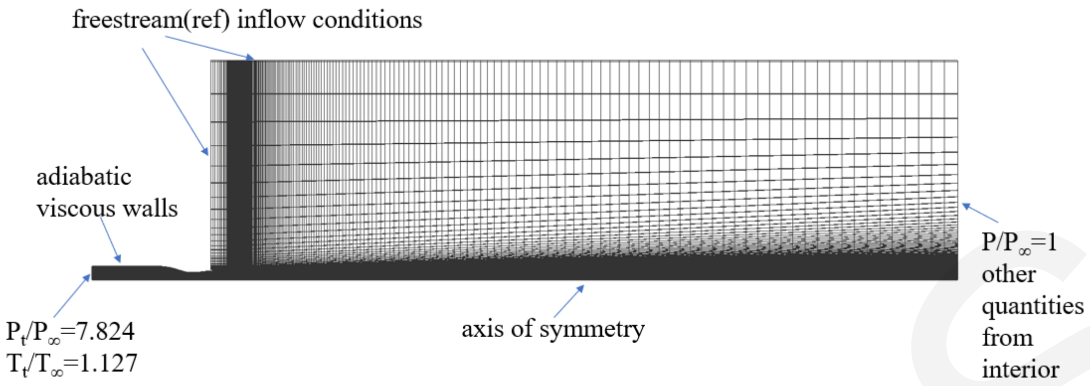
k-*kL*-ARSM2018+J turbulence model

nolinear constitutive relationship: $\tau_{ij} = f_2 \tau_{ij}^{(\mathrm{ARSM})} + (1 - f_2)\tau_{ij}^{(\mathrm{L})}$

$$\tau_{ij}^{(\mathrm{ARSM})} = -\rho k\left(\beta_1 T_{ij}^{(1)} + \beta_2 T_{ij}^{(2)} + \beta_4 T_{ij}^{(4)} + \frac{2}{3}\delta_{ij}\right)$$

$$T_{ij}^{(1)} = \left[S_{ij}^{*} - \frac{1}{3}\mathrm{tr}\left\{S^{*}\right\}\right] \quad T_{ij}^{(2)} = \left[S_{ik}^{*}S_{kj}^{*} - \frac{1}{3}\mathrm{tr}\left\{S^{*2}\right\}\right] \quad T_{ij}^{(4)} = \left[S_{ik}^{*}W_{kj}^{*} - W_{ik}^{*}S_{kj}^{*}\right]$$

k-kL-ARSM2018+J Turbulence Model Validation


freestream(ref) inflow conditions

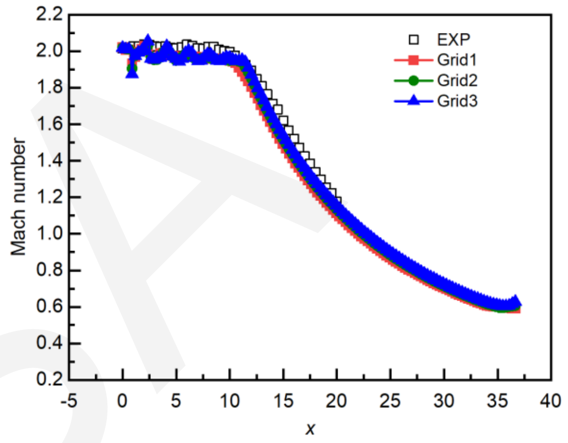
adiabatic
viscous walls

P_t/P_∞=7.824
T_t/T_∞=1.127

axis of symmetry

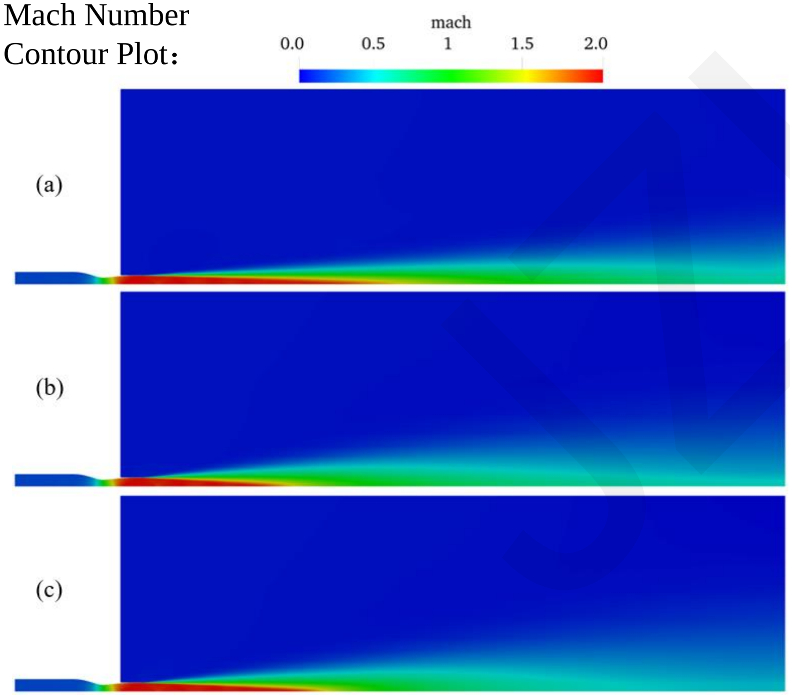
P/P_∞=1
other
quantities
from
interior

Mesh and Boundary Condition Description

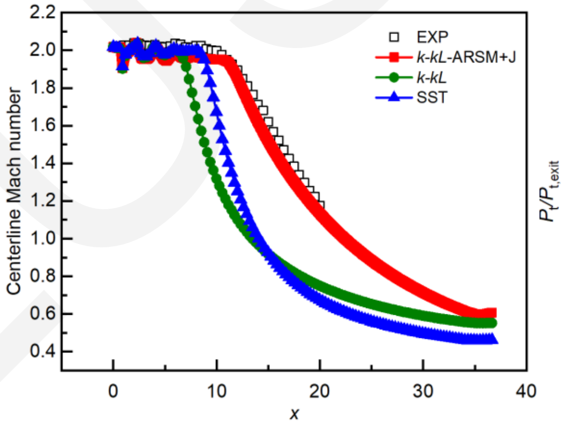


Mesh Independence Verification

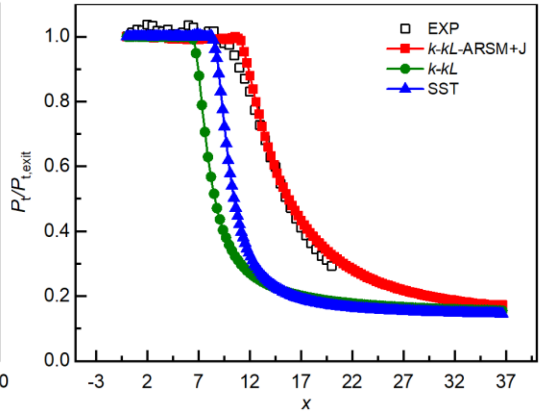
Mach Number
Contour Plot:



(a)*k-kL*-ARSM2018+J (b)*k-kL* (c)SST



Velocity Distribution under
Different Turbulence Models

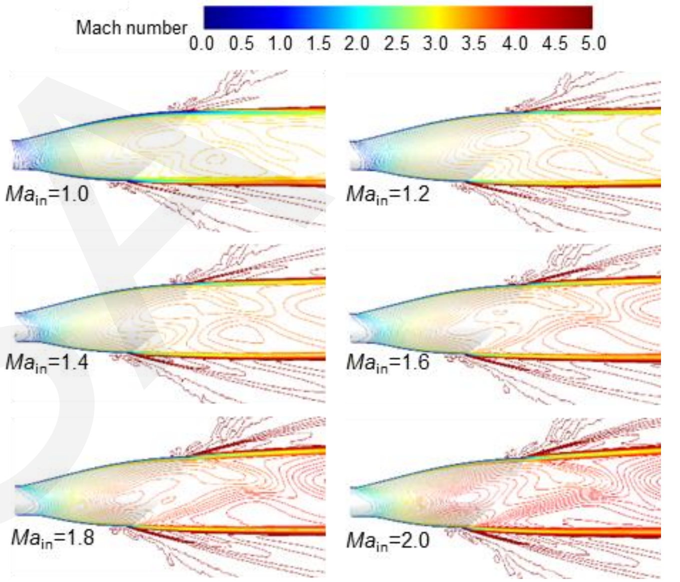


Pressure Distribution under
Different Turbulence Models

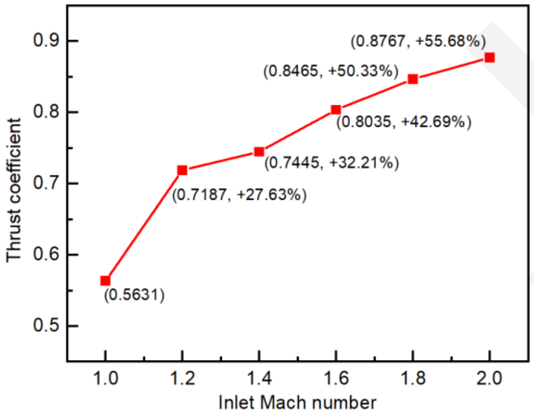
- The *k-kL*-ARSM2018+J model can accurately predict the jet exit velocity and pressure.

Nozzle Simulation - Impact of Inlet Velocity

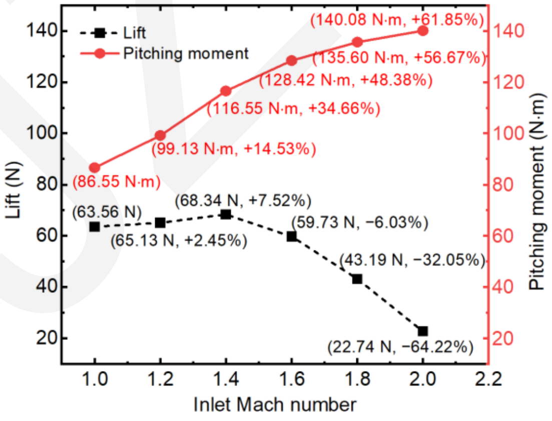
- The thrust increases with the increase in inlet Mach number, with a maximum increase of 55.68% in thrust at Mach 1.
- As the inlet Mach number increases, the core region of the nozzle expands, and the uniform region moves backward. This leads to an increase in the overall high-pressure region (>5000 Pa) inside the nozzle, resulting in an increase in nozzle thrust.
- The pressure increase on the lower wall of the nozzle is faster than that on the upper wall. This difference in pressure growth leads to a lift that first increases and then decreases, with the maximum lift occurring at a Mach number of 1.4.



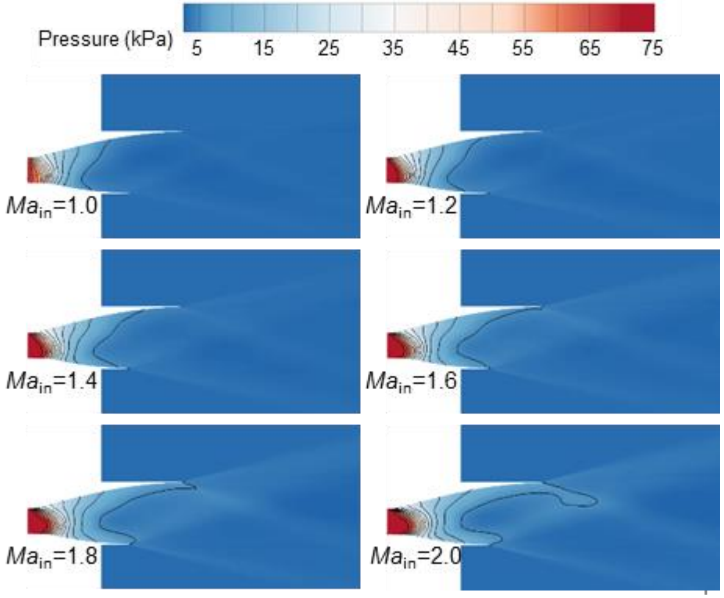
Mach Number Contour Plot at Different Inlet Velocities



Variation of Thrust Coefficient with Inlet Velocity



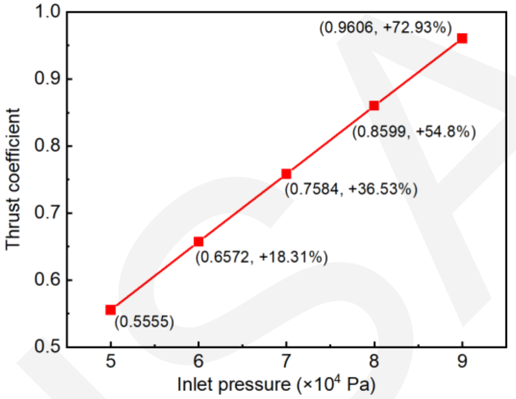
Variation of Lift and Pitching Momemt with Inlet Velocity



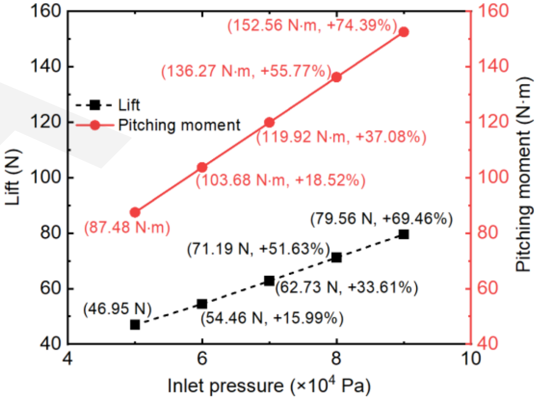
Pressure Contour Plot at Different Inlet Velocities

Nozzle Simulation - Impact of Inlet Pressure

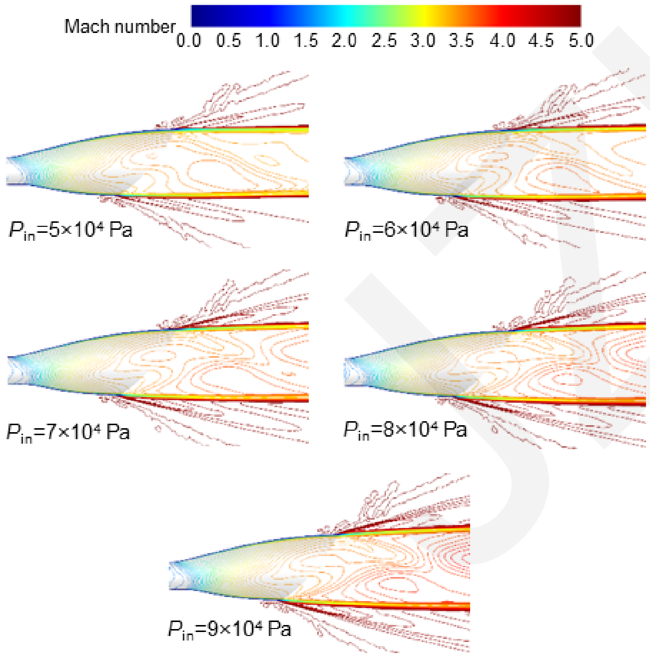
- With the change in inlet pressure, thrust, lift, and pitching moment all increase approximately in proportion
- With every 10 kPa increase in inlet pressure, the to-tal thrust coefficient increases by about 18.3%, lift increases by about 17.3%, and the pitching moment increases by about 18.5%.。



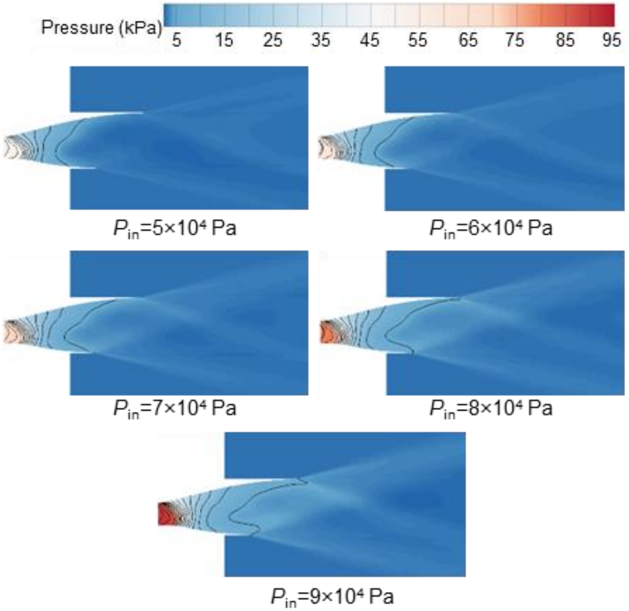
Variation of Thrust Coefficient with Inlet Pressure



Variation of Lift and Pitching Momemt with Inlet Pressure



Mach Number Contour Plot at Different Inlet Pressures on the Symmetry Plane



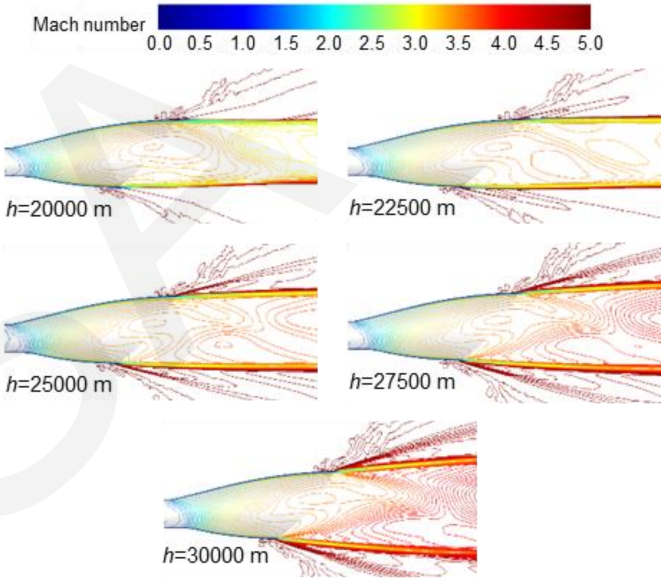
Pressure Contour Plot at Different Inlet Pressures on the Symmetry Plane

Nozzle Simulation - Impact of Flight Altitude

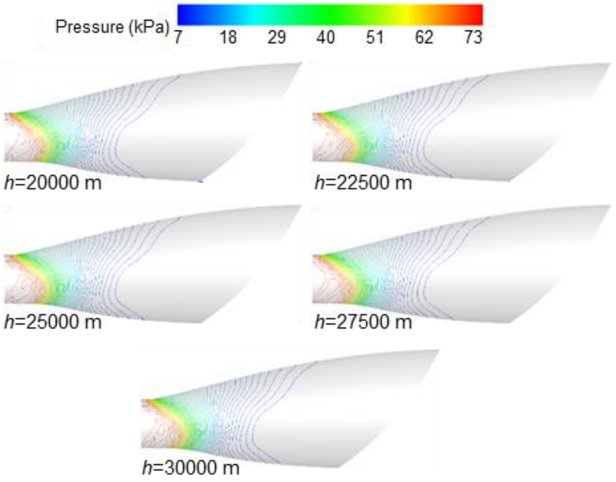
Atmospheric Parameters at Different Flight Altitudes

Flight Altitudes （m）	Pressure （Pa）	Temperature （K）
20000	5474.89	216.65
22500	3699.54	219.15
25000	2511.02	221.65
27500	1711.75	224.15
30000	1171.87	226.65

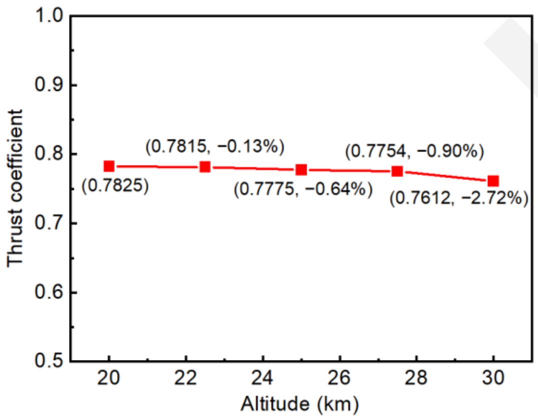
- With the increase in altitude, the variations in nozzle thrust, lift, and pitching moment are relatively small.
- The pressure distribution on the nozzle symmetry plane is largely consistent, indicating that the impact of altitude variation on nozzle performance is limited.



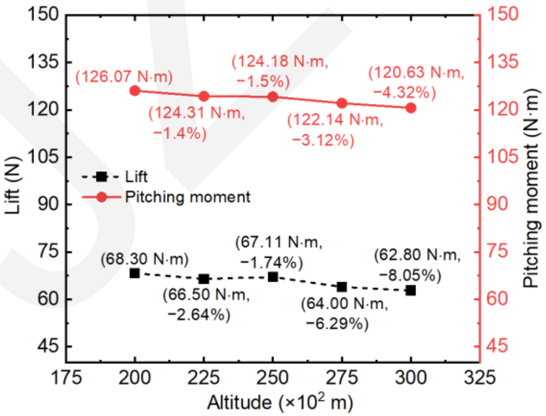
Mach Number Contour Plot at Different Flight Altitudes on the Symmetry Plane



Pressure Contour Lines of the Flow Field on the Symmetry Plane



Variation of Thrust Coefficient with Flight Altitude



Variation of Lift and Pitching Momemt with Flight Altitude

Conclusions

- The NASA standard test cases showed that the k-kL-ARSM+J algebraic stress model provides accurate simulations of supersonic nozzles, closely matching experimental data. This model enables precise calculations of nozzle performance.

- With increasing inflow Mach number, the thrust and pitching moment increase while the rate of increase decreases. The lift reaches its peak near the design Mach number and then decreases rapidly. As inlet pressure increases, the nozzle thrust, lift, and pitching moment all show linear growth.

- With increasing flight altitude, the internal flow field of the nozzle remains essentially the same due to the consistent supersonic nozzle inlet conditions. However, external to the nozzle, changes in external flow pressure cause the nozzle exit to transition from over-expansion to under-expansion. As a result, the shear layer behind the nozzle first converges towards the center of the nozzle and then diverges.