

Multi-directional wind energy harvesting based on the coupling effect between a piezoelectric beam and an elastic-supported sphere

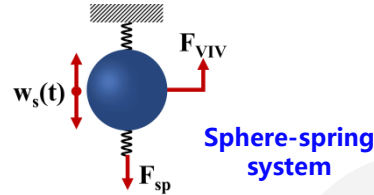
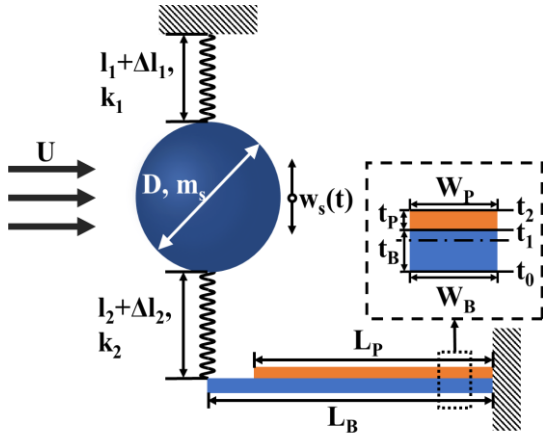
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Modeling and principle



Vortex induced force

$$m_s \ddot{w}_s(t) + c \dot{w}_s(t) + (k_1 + k_2) w_s(t) = F_{VIV}$$

$$F_{VIV} = \frac{\pi C_L \rho_{air} D^2 U^2}{16} q(t) - \frac{\pi C_D \rho_{air} D^2 U}{8} \dot{w}_s(t)$$

$$\ddot{q}(t) + \lambda \omega_s (q^2(t) - 1) \dot{q}(t) + \omega_s^2 q(t) = \frac{A}{D} \ddot{w}_s(t)$$

Euler-Lagrange equation

$$T = \frac{1}{2} \left[\int_{V_b} \rho_b \left[\frac{\partial w_b(x,t)}{\partial t} \right]^2 dV_b + \int_{V_p} \rho_p \left[\frac{\partial w_b(x,t)}{\partial t} \right]^2 dV_p \right]$$

$$U = \frac{1}{2} \left[\int_{V_b} \sigma_x^b \varepsilon_x^b dV_b + \int_{V_p} \sigma_x^p \varepsilon_x^p dV_p - \int_{V_p} E_3 D_3 dV_p \right]$$

$$L = T - U$$

$$w_b(x,t) = \sum_{i=1}^n \Phi_i(x) Y_i(t)$$



Reduced-order model

$$\ddot{Y}_i + 2\zeta_i \omega_i \dot{Y}_i + \omega_i^2 Y_i - \theta_i V = F_{sp}$$

$$C_p \dot{V} + \frac{V}{R} + \sum_{i=1}^n \theta_i \dot{Y}_i = 0$$

Basic parameters:

U represent **inflow velocity**.

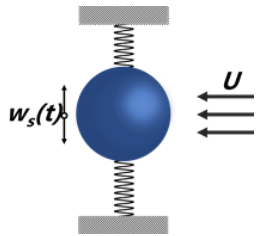
D and **m_s** represent **diameter and mass of the sphere**.

B and **P** represent **metal base and piezoelectric sheet**.

L, **W**, and **t** represent **length, width and thickness of the beam**.

k, **l**, and **\Delta l** represent **stiffness, length and elongation of the spring**.

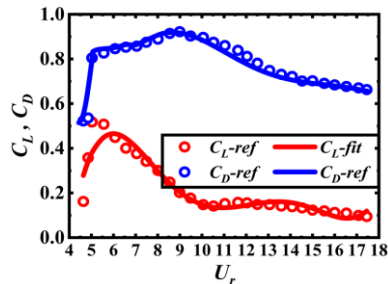
Analytical solution considering the **coupled vibration** effect.



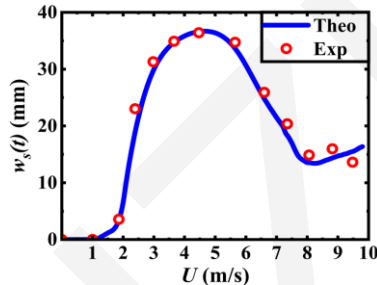
Model of an elastically-mounted sphere



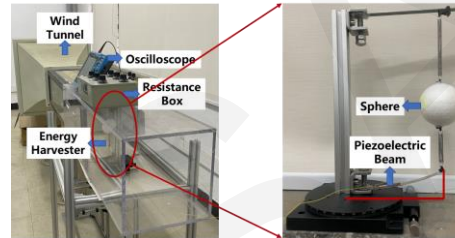
Experimental setup of the elastically-mounted sphere



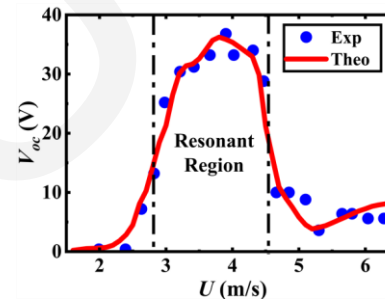
Lift/drag force coefficient



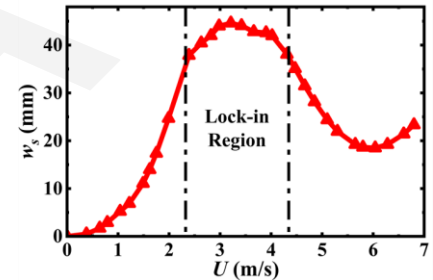
Sphere displacement under varying wind speeds: theory vs. experiment



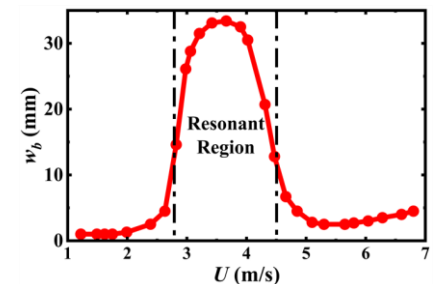
Experimental setup of the proposed device and the sample



The peak open-circuit voltage of the designed device varies



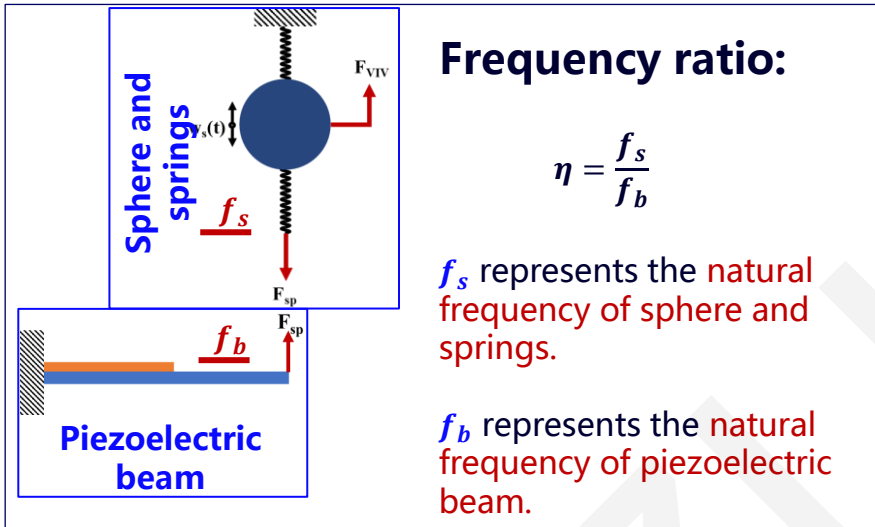
The vibration amplitude of the sphere in the designed device



The end displacement of the piezoelectric beam

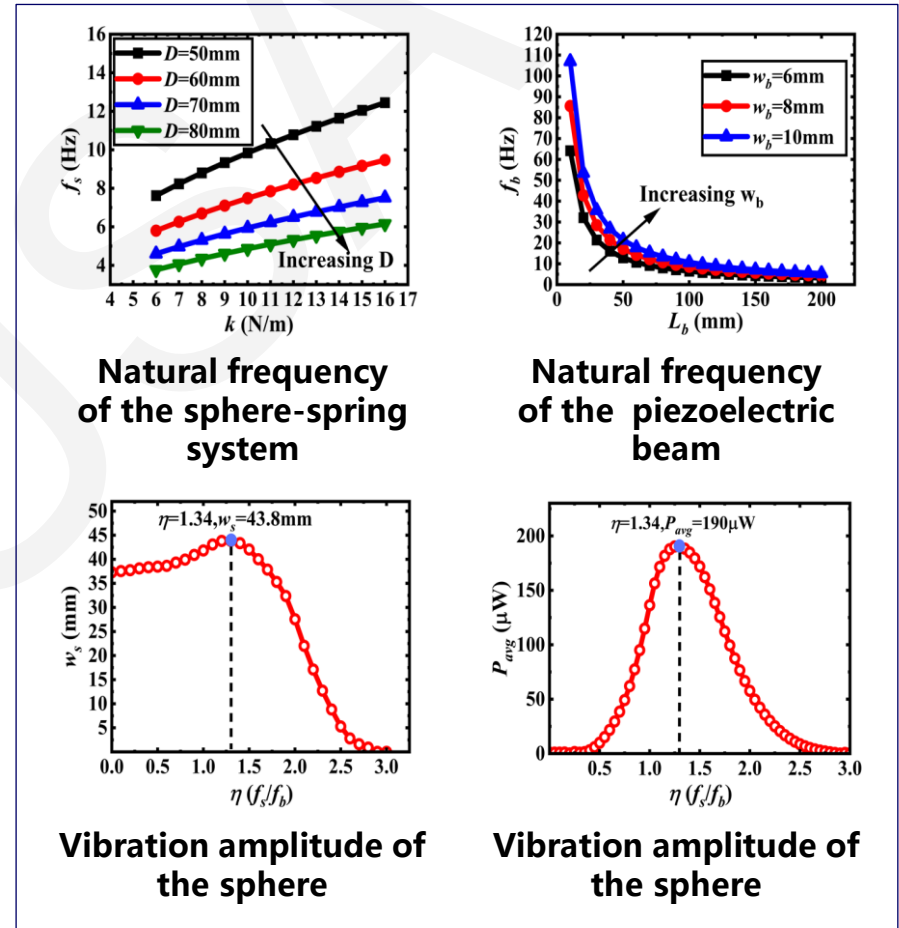
The wind tunnel experiment validated the theoretical model.

➤ Influence of frequencies ration on output performance



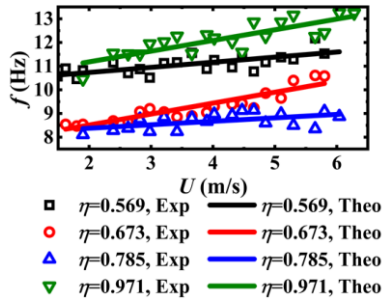
The **sphere diameter (D)**, the **spring stiffness (K)** and the **length (L_b)** and **width (w_b)** of the beam are four **key factors**.

Tuning the **natural frequencies** optimizes **mechanical and electrical performance**.

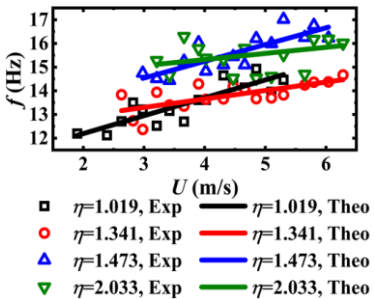


Adjusting the **frequency ratio (η)** enables **coupled vibration**.

➤ Influence of frequencies ratio on output performance

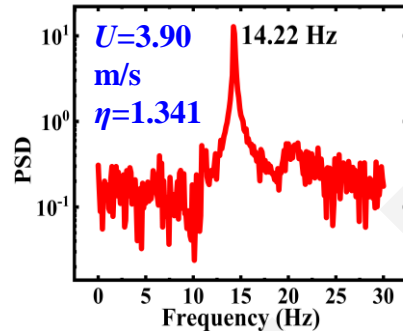


$\eta < 1$



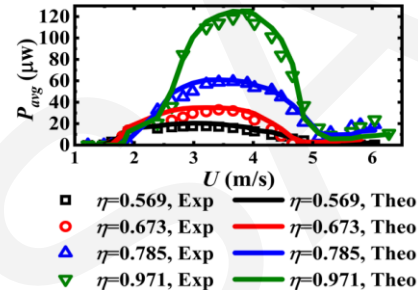
$\eta > 1$

Influence of wind speed (U) on vibrational frequency (f)

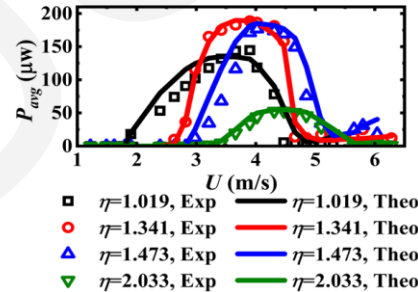


FFT of the output power signal

Frequency ratio (η) affects the resonant frequency (f).



$\eta < 1$



$\eta > 1$

Influence of wind speed (U) on output power (P_{avg})

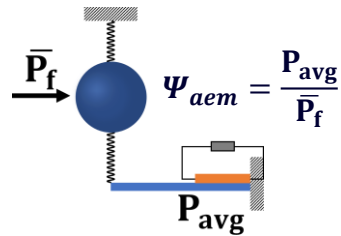
Lower frequency ratio lead to lower critical wind speed for VIV.

The output power peaks at $\eta=1.341$ and $U=3.90$ m/s.

At $\eta=1.341$, the vibration frequency of the piezoelectric beam matches the vortex shedding frequency.

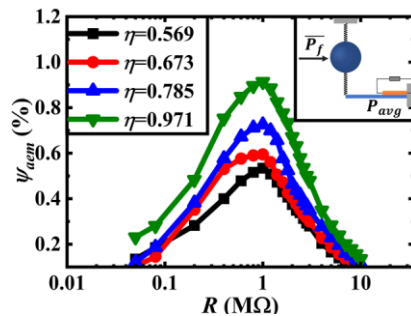
Coupled vibration optimizes vibration modes through frequency adjustment.

➤ Energy conversion efficiency

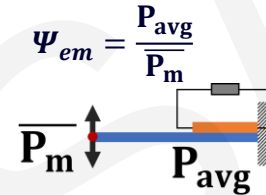
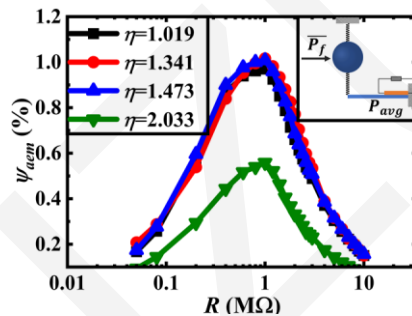


max Ψ_{aem}
 $\eta = 1.341$
 $R = 1.0 \text{ M}\Omega$
 $\Psi_{aem} = 1.05\%$

Schematic diagram of Ψ_{aem}

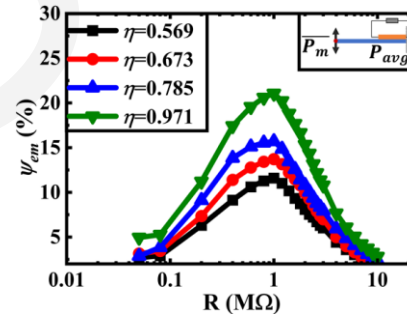


The aero-electro-mechanical efficiency

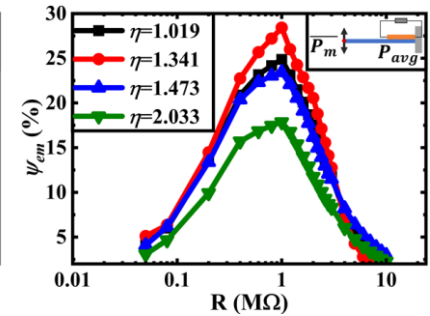


max Ψ_{em}
 $\eta = 1.341$
 $R = 1.0 \text{ M}\Omega$
 $\Psi_{em} = 28.4\%$

Schematic diagram of Ψ_{em}



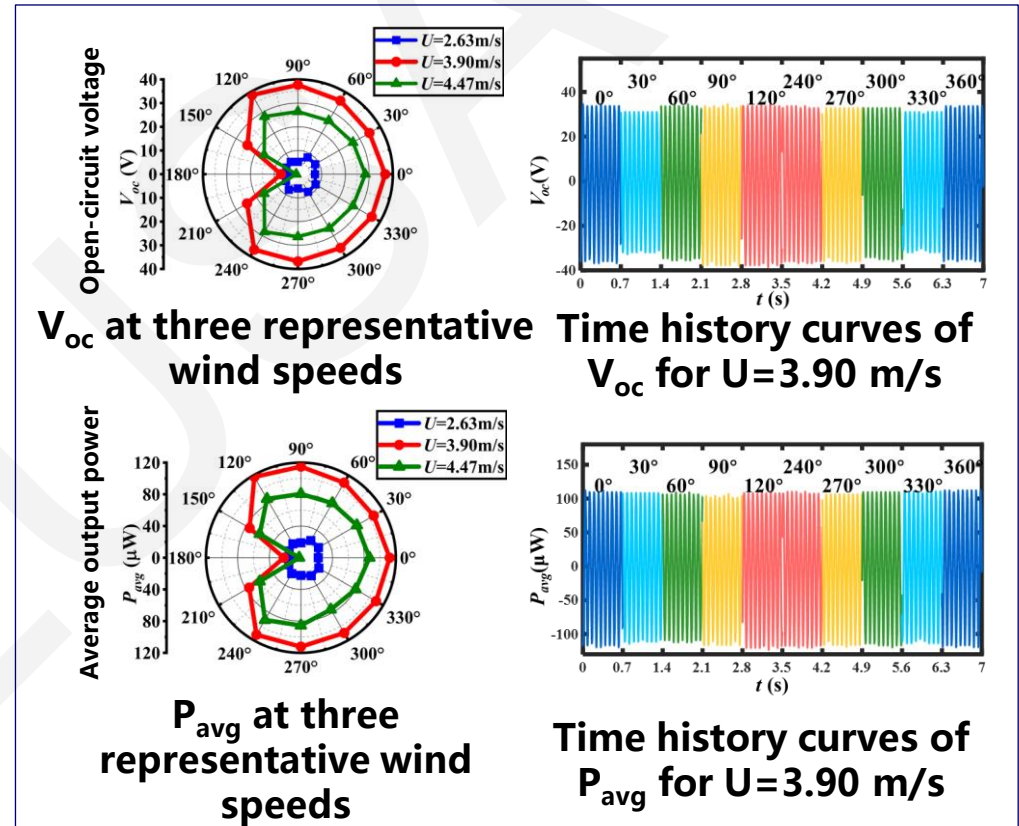
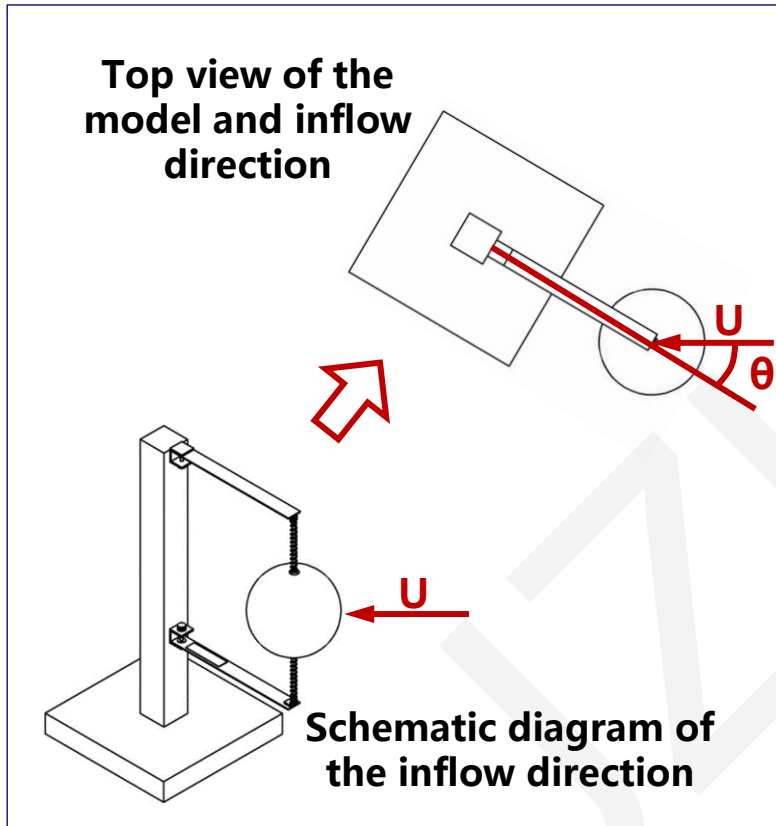
The electro-mechanical efficiency



Energy conversion efficiency peaks at $\eta = 1.341$ and $R = 1.0 \text{ M}\Omega$.

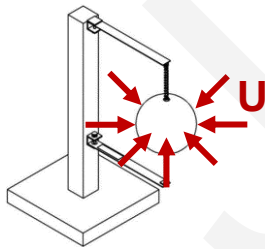
Output Performance Testing and Analysis

➤ Influence of wind direction on output performance

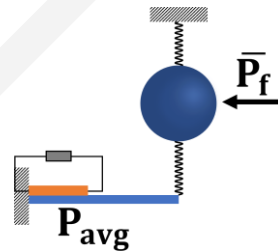


The voltage and power are **consistent** across **varying flow directions**.

- Developed a novel piezoelectric energy harvester, capable of maintaining **stable performance under complex wind direction**.
- Proposed a **nonlinear coupled dynamic theoretical model based on the Euler-Lagrange equations and the Van der Pol oscillator**.
- Explored **the optimal frequency ratio** and tested **the multi-directional performances**.



Omnidirectionality



Higher efficiency



Engineering application