

# Continuous energy exchange between magnetic fields supporting memristive neuron firing

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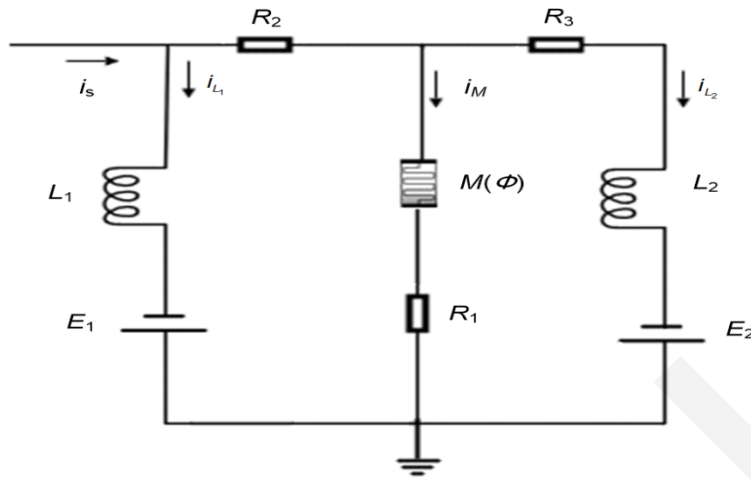
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# Main contents

1. Two inductors are connected with a magnetic flux-controlled memristor for building a neural circuit.
2. Combination of resistance and inductance is used as new reference unit as capacitance.
3. Continuous energy exchange in magnetic field is effective to maintain firing patterns.
4. Adaptive law is proposed to control electrical activities in the new neuron.

# 1. Memristor-coupled neural circuit



**Fig. 1** Proposed MFCM coupled neural circuit driven by forcing current  $i_s$ .  $R_1, R_2, R_3, L_1, L_2, E_1$ , and  $E_2$  denote three resistors, two inductors, and two constant voltage sources, respectively.

Capacitors are not used in the neural circuit, and energy in magnetic field is exchanged in the electric components.

Circuit equations

$$\begin{cases} L_1 \frac{di_{L_1}}{dt} = (i_s - i_{L_1})R_2 + V_M + i_M R_1 + E_1, \\ L_2 \frac{di_{L_2}}{dt} = V_M + i_M R_1 + E_2 - i_{L_2} R_3, \\ \frac{d\phi}{dt} = -K\phi + \delta V_M. \end{cases}$$

Field energy function

$$\begin{cases} W_{L_1} = \frac{1}{2} L_1 i_{L_1}^2, & W_{L_2} = \frac{1}{2} L_2 i_{L_2}^2, \\ W_M = \frac{1}{2} \phi i_M, \\ W = \frac{1}{2} L_1 i_{L_1}^2 + \frac{1}{2} L_2 i_{L_2}^2 + \frac{1}{2} \phi i_M. \end{cases}$$

# 2 Scale transformation

Reference time and capacitance

$$[T] = \left[ \frac{Q}{I} \right] = \left[ \frac{CV}{I} \right] = [RC] = \left[ \frac{\Phi}{V} \right] = \left[ \frac{LI}{IR} \right] = \left[ \frac{L}{R} \right], \quad [C] = \left[ \frac{L}{R^2} \right].$$

$L/(R \cdot R)$  has the same physical unit as capacitance.  $[T]$  obtains the reference time by removing the physical unit for time.  $[*]$  means unit calculation for a variable.

**Scale transformation:** Physical variables are converted into dimensionless variables

$$\begin{cases} x = \frac{R_1 i_{L_1}}{E_1}, & y = \frac{R_1 i_{L_2}}{E_1}, & z = \frac{R_1 \phi}{L_1 E_1}, & \tau = \frac{t R_1}{L_1}, \\ i'_s = \frac{i_s R_1}{E_1}, & a = \frac{E_2}{E_1}, & b = \frac{R_3}{R_1}, & c = \frac{L_1}{L_2}, \\ r = \frac{R_2}{R_1}, & k = K \frac{L_1}{R_1}, & \sigma = \frac{L_1 E_1}{R_1}, & \beta' = \frac{1}{\beta R_1}. \end{cases}$$

# 3. Theoretical model and control law

**Memristive neuron model without capacitive variables**

$$\begin{cases} \frac{dx}{d\tau} = ri'_s - rx + (i'_s - x - y)(\beta' \sin(\sigma z) + 1) + 1, \\ \frac{dy}{d\tau} = c \left[ (i'_s - x - y)(\beta' \sin(\sigma z) + 1) - by + a \right], \\ \frac{dz}{d\tau} = -kz + \delta(i'_s - x - y)\beta' \sin(\sigma z). \end{cases}$$

**Tracked signals**

$$\begin{cases} V_M = \frac{i'_s - i_{L_1} - i_{L_2}}{\beta \csc(\phi)}, \\ u = \frac{V_M}{E_1} = (i'_s - x - y)\beta' \sin(\sigma z), \end{cases}$$

**Hamilton energy function**

$$\begin{cases} H_{L_1} = \frac{1}{2}x^2, \quad H_{L_2} = \frac{1}{2c}y^2, \\ H_M = \frac{1}{2}z(i'_s - x - y), \\ H = \frac{R_1^2 W}{L_1 E_1^2} = \frac{1}{2}x^2 + \frac{1}{2c}y^2 + \frac{1}{2}z(i'_s - x - y). \end{cases}$$

**Statistical analysis**

$$\begin{cases} C_v = \frac{\sqrt{\langle A_{Is}^2 \rangle - \langle A_{Is} \rangle^2}}{\langle A_{Is} \rangle}, \\ \langle H \rangle = \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} H d\tau = \frac{1}{N} \sum_{i=1}^N H_i, \end{cases}$$

**Adaptive parameter growth**

$$\begin{cases} \frac{d\sigma}{d\tau} = -g\sigma \cdot \theta \left( \left| \frac{H_M(\tau)}{H(\tau)} \right| - \varepsilon \right), \\ \theta(v) = 1, v \geq 0, \quad \theta(v) = 0, v < 0, \quad \tau > 650. \end{cases}$$

## 4. Scientific contribution and conclusion

1. Capacitance is replaced by combining resistance and inductance, and new reference value criterion is proposed as  $[T]=[L/R]$ , and  $[C]=[L/(R*R)]$ .
2. When capacitor is not available or breakdown, MFCM and inductors can be connected to build a effective neural circuit.
3. Continuous energy exchange is crucial for supporting different kinds of firing patterns in neural circuits.
4. Energy level controls the firing modes and changes of energy flow can modify the firing patterns in neurons in adaptive way.