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A heterogeneous cyclic Hopfield neural network without self-connections

Key words:

Activation function; Analog circuit; Chaos; Cyclic Hopfield neural network; Multi-scroll chaotic

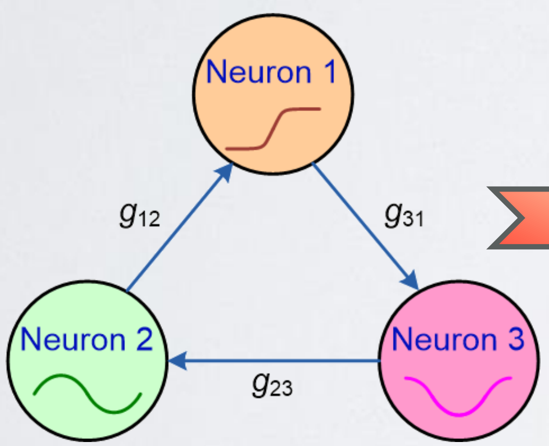
Objective

The unidirectional cyclic Hopfield neural network with three neurons fails to present chaotic dynamics.

This paper aims to propose a three-neuron heterogeneous cyclic Hopfield neural network without self-connections, which can display chaotic dynamics and generate multi-scroll chaotic attractors.

Heterogeneous CHNN with three neurons

- Model descriptions
- Proof of boundedness
- Equilibrium points with stability types



$$\begin{cases} \dot{x}_1 = -x_1 + g_{12}\sin x_2, \\ \dot{x}_2 = -x_2 + g_{23}\cos x_3, \\ \dot{x}_3 = -x_3 + g_{31}\tanh x_1. \end{cases}$$

Fig. 1 Framework of the three-neuron het-CHNN without self-connections

We set the equilibrium point of the 3D het-CHNN as P=(X_1, X_2, X_3). Thus, we obtain:

$$\begin{cases} -X_1 + g_{12}\sin X_2 = 0, \\ -X_2 + g_{23}\cos X_3 = 0, \\ -X_3 + g_{31}\tanh X_1 = 0. \end{cases}$$

$$P = (X_1, X_2, g_{31}\tanh X_1),$$

$$h_1(X_1, X_2) = -X_1 + g_{12}\sin X_2,$$

$$h_2(X_1, X_2) = -X_2 + g_{23}\cos(g_{31}\tanh X_1).$$

$$\boldsymbol{J} = -\boldsymbol{I} + \boldsymbol{G}\boldsymbol{f}'(\boldsymbol{X}),$$

$$P(\lambda) = \det(\lambda\boldsymbol{I} - \boldsymbol{J}) = 0,$$

Bifurcation dynamics and chaotic attractors

- Bifurcation dynamics
- Multi-scroll chaotic attractors
- Evolution of bi-stability

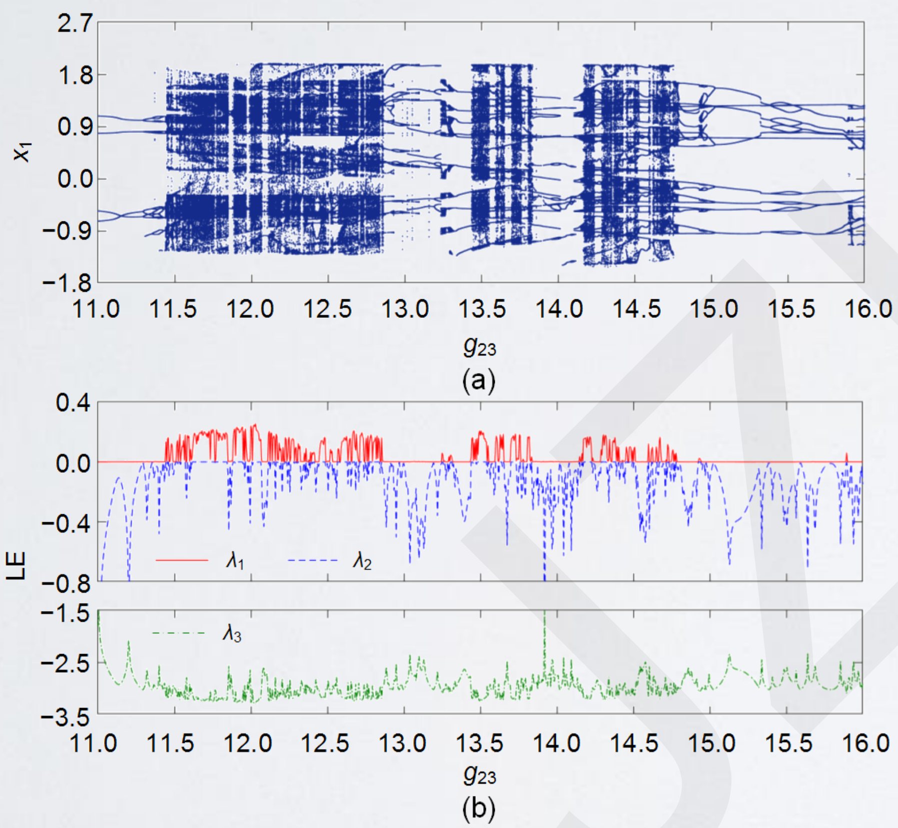


Fig. 2 Bifurcation behaviors when g_{23} is varied in the region [11, 16] for fixed g_{12}=4.2, g_{31}=−8.6, and IS=(0.001, 0, 0): (a) bifurcation plot of x_1; (b) three LE spectra

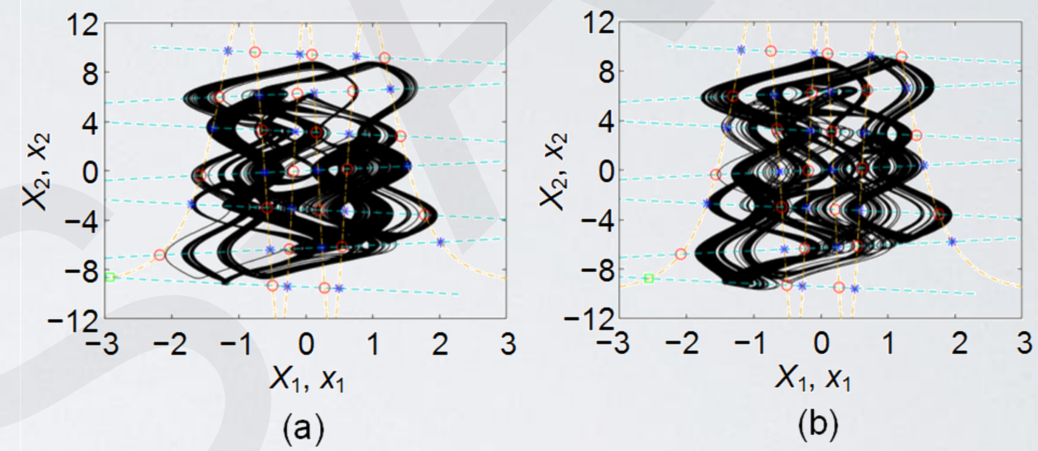


Fig. 3 Equilibrium points with stability types as well as the phase orbits of the multi-scroll chaotic attractor for fixed g_{12}=4.2, g_{31}=−8.6, and IS=(0.001, 0, 0) with two different g_{23} values: (a) g_{23}=13.5; (b) g_{23}=14.6

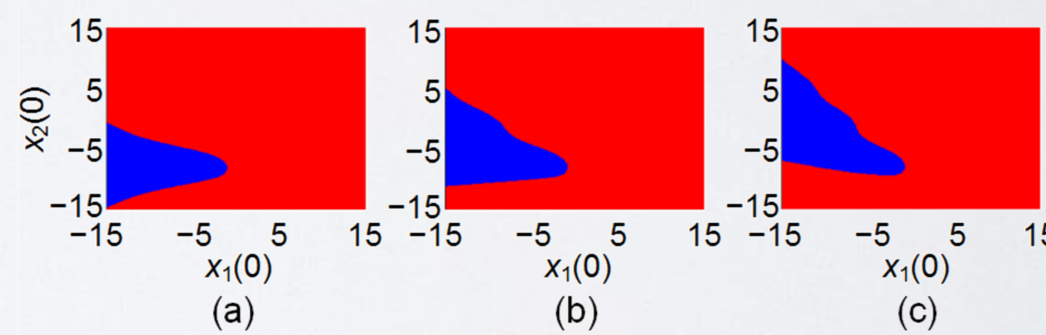


Fig. 4 Basins of attraction in the $x_1(0)$-$x_2(0)$ initial plane for IS=[$x_1(0)$, $x_2(0)$, 8.6] with different g_{23} values: (a) g_{23}=12.0; (b) g_{23}=13.5; (c) g_{23}=14.6. The multi-scroll chaotic at-tractor and the point attractor are marked with red and blue regions, respectively. References to color refer to the online version of this figure

Analog circuit design and experiments

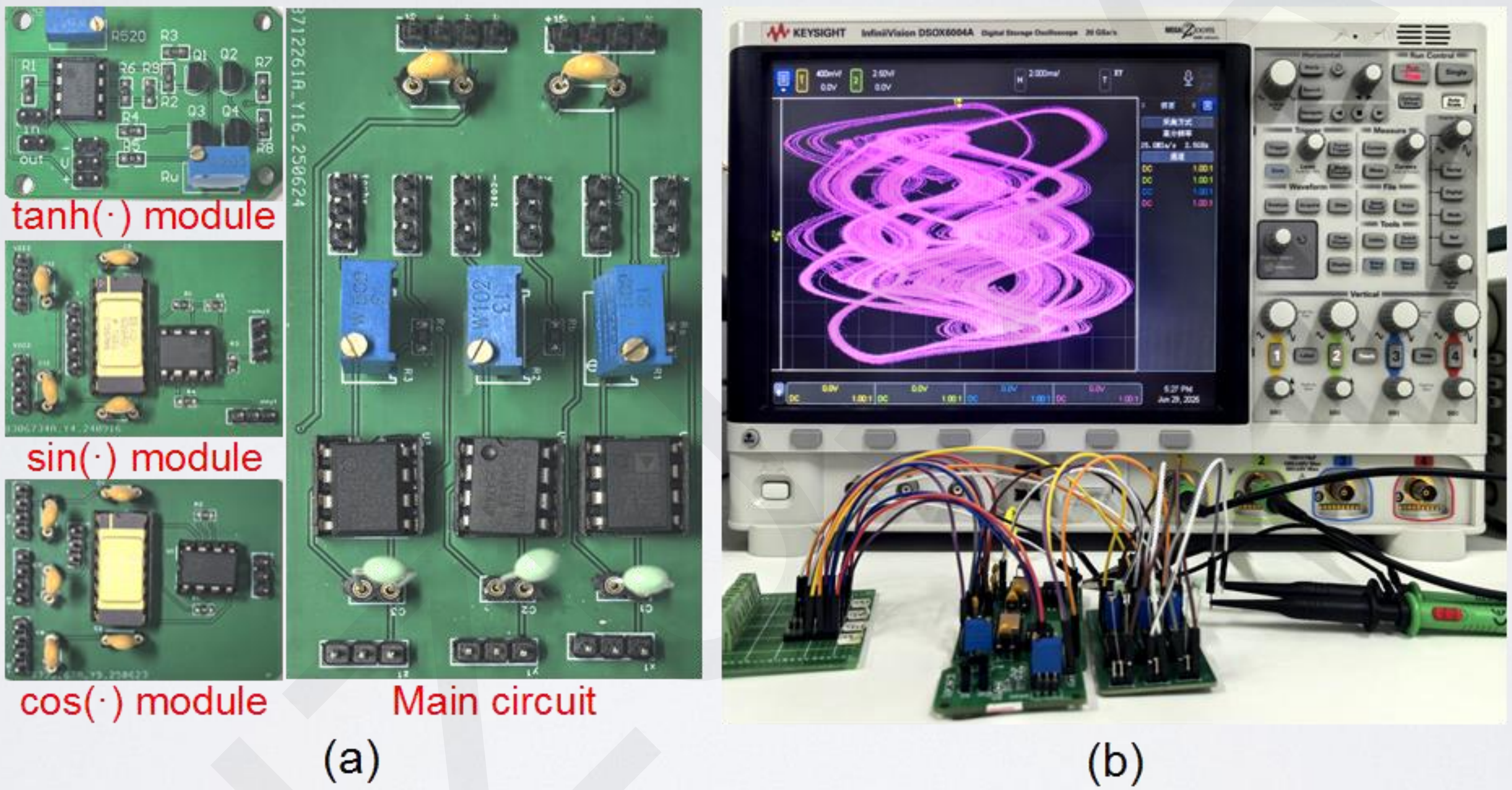


Fig. 5. Hardware setup: (a) hardware circuit board; (b) ex-perimental platform.

Research Priorities

Research Priorities:

- Propose a three-neuron heterogeneous cyclic Hopfield neural network without self-connections.
- Prove the globally consistent ultimate boundedness and determine the final boundary.
- Reveal chaotic dynamics and multi-scroll chaotic attractor.
- Design an analog circuit and verify the results through hardware experiments.

The hyperbolic tangent, sine, and cosine functions were used as the activation functions of the network’s three neurons. Through theoretical proofs and numerical simulations, the boundedness, dynamical behaviors, and multi-scroll chaotic attractors of the network were presented and an-alyzed.Next, the 3D het-CHNN was implemented in an analog circuit, and hardware experiments were conducted with this circuit to confirm the previous numerical results. Notably, the proposed 3D het-CHNN successfully addresses the issue of the absence of chaos in a three-neuron CHNN. As far as the authors know, the 3D het-CHNN is the simplest three-neuron HNN that can generate multi-scroll chaotic at-tractors.