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Preservation of local linearity by neighborhood subspace scaling for solving the pre-image problem

Key words: Kernel method, Pre-image problem, Nonlinear denoising, Kernel PCA, Local linearity preserving

Corresponding author: Hai-bin Shen
E-mail: shb@vlsi.zju.edu.cn

Method

An indirect method is proposed to solve the pre-image problem. In our algorithm, an inverse mapping process is constructed based on a novel framework which preserves local linearity. To preserve the local linearity between feature space and input space, we perform a local nonlinear transformation in input space, which is implicitly conducted by neighborhood subspace scaling. With a smoothness assumption, this transformation can be extended from training data to test data. Thereby, an inverse mapping process for test data can be constructed, and the pre-image problem is solved.

Denoising results

Table 1 MSE comparison with Gaussian noise

Method	$\overline{\text{MSE}}$			Optimal k		
	$\sigma^2 = 0.3$	$\sigma^2 = 0.4$	$\sigma^2 = 0.5$	$\sigma^2 = 0.3$	$\sigma^2 = 0.4$	$\sigma^2 = 0.5$
Kwok	22.3979	24.6594	26.9366	10	5	5
Zheng	24.3877	43.1526	65.8261	205	215	215
Honeine	36.1106	69.6174	109.3635	500*	500*	500*
Huang	19.5596	23.9229	28.2292	15	10	10
This paper	**18.7613**	**21.8739**	**24.6841**	20	10	10

* Since global mapping was used in Honeine's method, the value of k was equal to the number of training samples

Table 2 MSE comparison with salt-and-pepper noise

Method	$\overline{\text{MSE}}$			Optimal k		
	$p = 0.3$	$p = 0.4$	$p = 0.5$	$p = 0.3$	$p = 0.4$	$p = 0.5$
Kwok	58.7487	97.1309	135.8279	10	10	5
Zheng	49.2949	70.0785	92.5192	105	140	160
Honeine	86.7215	120.1639	150.6930	500*	500*	500*
Huang	57.4134	92.5925	134.6848	35	35	15
This paper	**35.5646**	**61.3593**	**87.8186**	60	35	20

* Since global mapping was used in Honeine's method, the value of k was equal to the number of training samples

Execution time and time complexity

Table 3 The time complexity of different algorithms

Method	Time complexity
Kwok	$O(k^3 + d^2k + n^2 + dn)$
Zheng	$O(k^3 + n^2 + dn)$
Honeine	$O(n^2 + dn)$
Huang	$O(n^3 + k^3 + k^2n + kn^2 + dk^2 + dn)$
This paper	$O(k^3 + n^2 + dn)$

Table 4 The time complexity of different algorithms with $d \gg N \gg k$

Method	Time complexity
Kwok	$O(d^2k)$
Zheng	$O(k^3 + dn)$
Honeine	$O(dn)$
Huang	$O(n^3 + dk^2 + dn)$
This paper	$O(k^3 + dn)$

Table 5 Execution time of different algorithms in seconds

Method	Gaussian noise			Salt-and-pepper noise			Average
	$\sigma^2 = 0.3$	$\sigma^2 = 0.4$	$\sigma^2 = 0.5$	$p = 0.3$	$p = 0.4$	$p = 0.5$	
Kwok	45.34	25.98	26.58	46.17	45.82	27.61	36.25
Zheng	6.52	6.41	6.48	5.63	5.79	5.98	6.14
Honeine	3.98	3.68	4.22	5.03	4.62	4.60	4.36
Huang	7.40	7.41	7.22	14.30	14.24	9.58	10.03
This paper	5.03	4.54	4.53	7.06	5.61	5.19	5.33

Conclusions

- ✓ In the image denoising scenario, compared with other algorithms, the proposed algorithm achieved the lowest MSE, and also had obvious advantages in reduced computational complexity.

- ✓ The proposed method is non-iterative, and can be used for any kernel function.