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An efficient measurement-driven sequential Monte Carlo multi-Bernoulli filter for multi-target filtering

Key words: Measurement-driven, Gating technique, Sequential Monte Carlo, Multi-Bernoulli filter, Multi-target filtering

Corresponding author: Mei-qin Liu
E-mail: liumeiqin@zju.edu.cn

Main contributions

- We present an analysis of the time consumption of the prediction, update, and resampling steps, respectively, for SMC-MB recursion from the results of Monte Carlo (MC) simulation.
- We use the gating technique for both survival and birth targets to distinguish survival and birth measurements.
- We propose an efficient measurement-driven approach for the SMC-MB filter. The survival measurements are used to update both survival and birth targets, and the birth measurements are used to update only birth targets.

Proposed approach (1/3)

Gating for the survival targets (pseudo-code)

Algorithm 1 Gating technique for survival targets

for $i = 1, \cdots, M_{k-1}$ **do**
  for $j = 1, \cdots, L_{k-1}^{(i)}$ **do**
    sample $\boldsymbol{x}_{k|k-1}^{(i,j)} \sim q_k^{(i)}(\cdot|\boldsymbol{x}_{k-1}^{(i,j)}, Z_k)$;
    $w_{k|k-1}^{(i,j)} = \frac{w_{k-1}^{(i,j)} f_{k|k-1}(\boldsymbol{x}_{k|k-1}^{(i,j)}|\boldsymbol{x}_{k-1}^{(i,j)}) p_{\mathrm{S},k}(\boldsymbol{x}_{k-1}^{(i,j)})}{q_k^{(i)}(\boldsymbol{x}_{k|k-1}^{(i,j)}|\boldsymbol{x}_{k-1}^{(i,j)}, Z_k)}$;
  end for
end for
$r_{k|k-1}^{(i)} = r_{k-1}^{(i)} \sum_{j=1}^{L_{k-1}^{(i)}} w_{k-1}^{(i,j)} p_{\mathrm{S},k}(\boldsymbol{x}_{k-1}^{(i,j)})$;
$w_{k|k-1}^{(i,j)} = \frac{w_{k|k-1}^{(i,j)}}{\sum_{j=1}^{L_{k-1}^{(i)}} w_{k|k-1}^{(i,j)}}$;
$L_{k|k-1}^{(i)} = L_{k-1}^{(i)}$;
$\bar{z}_{k|k-1}^{(i)} = \sum_{j=1}^{L_{k-1}^{(i)}} w_{k|k-1}^{(i,j)} h_k(\boldsymbol{x}_{k|k-1}^{(i,j)})$;
$\boldsymbol{S}_{k|k-1}^{(i)} = \boldsymbol{R}_k + \sum_{j=1}^{L_{k-1}^{(i)}} w_{k|k-1}^{(i,j)} \left(h_k(\boldsymbol{x}_{k|k-1}^{(i,j)}) - \bar{z}_{k|k-1}^{(i)}\right) \cdot \left(h_k(\boldsymbol{x}_{k|k-1}^{(i,j)}) - \bar{z}_{k|k-1}^{(i)}\right)^{\mathrm{T}}$;
set $Z_{\mathrm{P},k} = \varnothing$;
set flag $= 0$;
for $z \in Z_k$ **do**
  for $i = 1, \cdots, M_{k-1}$ **do**
    if $\left(z - \bar{z}_{k|k-1}^{(i)}\right)^{\mathrm{T}} \left(\boldsymbol{S}_{k|k-1}^{(i)}\right)^{-1} \left(z - \bar{z}_{k|k-1}^{(i)}\right) \le U_{\mathrm{P}}$ **then**
      $Z_{\mathrm{P},k} = [Z_{\mathrm{P},k}, z]$;
      flag $= 1$;
      break;
    end if
    if flag $== 1$ **then**
      break;
    end if
  end for
end for
$\tilde{Z}_k = Z_k - Z_{\mathrm{P},k}$;

Proposed approach (2/3)

Gating for the birth targets (pseudo-code)

Algorithm 2 Gating technique for birth targets

for $i = M_{k-1}+1, \cdots, M_{k-1}+M_{\Gamma,k}$ **do**

  for $j = 1, \cdots, L_{\Gamma,k}^{(i-M_{k-1})}$ **do**

    sample $x_{k|k-1}^{(i,j)} \sim b_k^{(i)}(\cdot|Z_k)$;

    $w_{k|k-1}^{(i,j)} = \dfrac{p_{\Gamma,k}(x_{k|k-1}^{(i,j)})}{b_k^{(i)}(x_{k|k-1}^{(i,j)}|Z_k)}$;

  end for

end for

$r_{k|k-1}^{(i)} = r_{\Gamma,k}^{(i-M_{k-1})}$;

$w_{k|k-1}^{(i,j)} = \dfrac{w_{k|k-1}^{(i,j)}}{\sum_{j=1}^{L_{\Gamma,k}^{(i-M_{k-1})}} w_{k|k-1}^{(i,j)}}$;

$L_{k|k-1}^{(i)} = L_{\Gamma,k}^{(i-M_{k-1})}$;

$M_{k|k-1} = M_{k-1} + M_{\Gamma,k}$;

$\bar{z}_{k|k-1}^{(i)} = \sum_{j=1}^{L_{k|k-1}^{(i)}} w_{k|k-1}^{(i,j)} h_k(x_{k|k-1}^{(i,j)})$;

$S_{k|k-1}^{(i)} = R_k + \sum_{j=1}^{L_{k|k-1}^{(i)}} w_{k|k-1}^{(i,j)} \cdot \left(h_k(x_{k|k-1}^{(i,j)}) - \bar{z}_{k|k-1}^{(i)}\right)\left(h_k(x_{k|k-1}^{(i,j)}) - \bar{z}_{k|k-1}^{(i)}\right)^{\mathrm{T}}$;

set $Z_{\Gamma,k} = \varnothing$;

set flag $= 0$;

for $z \in \tilde{Z}_k$ **do**

  for $i = M_{k-1}+1, \cdots, M_{k|k-1}$ **do**

    if $\left(z - \bar{z}_{k|k-1}^{(i)}\right)^{\mathrm{T}}\left(S_{k|k-1}^{(i)}\right)^{-1}\left(z - \bar{z}_{k|k-1}^{(i)}\right) \le U_\Gamma$ **then**

      $Z_{\Gamma,k} = [Z_{\Gamma,k}, z]$;

      flag $= 1$;

      break;

    end if

    if flag$==1$ **then**

      break;

    end if

  end for

end for

Proposed approach (3/3)

The measurement driven approach (pseudo-code)

Algorithm 3 Measurement-driven algorithm

Step 1: Update for survival targets
for each $z \in Z_{\mathrm{P},k}$ **do**
  set $l = 0$;
  for $i = 1, \cdots, M_{k|k-1}$ **do**
    for $j = 1, \cdots, L^{(i)}_{k|k-1}$ **do**
      $l = l + 1$;
      $\psi^{(i,j)}_{k,z} = g_k(z|x^{(i,j)}_{k|k-1}) P_{\mathrm{D},k}(x^{(i,j)}_{k|k-1})$;
      $w^{(i,j)}_{\mathrm{U},k}(z) = w^{(i,j)}_{k|k-1} \dfrac{r^{(i,j)}_{k|k-1}}{1 - r^{(i,j)}_{k|k-1}} \psi^{(i,j)}_{k,z}$;
    end for
    $\varrho^{(i)}_{\mathrm{U},k}(z) = \sum_{j=1}^{L^{(i)}_{k|k-1}} w^{(i,j)}_{k|k-1} \psi^{(i,j)}_{k,z}$;
    $\varrho^{(i)}_{\mathrm{L},k} = \sum_{j=1}^{L^{(i)}_{k|k-1}} w^{(i,j)}_{k|k-1} p_{\mathrm{D},k}(x^{(i,j)}_{k|k-1})$;
  end for
  $L_k(z) = l$;
  $w^{(i,j)}_{\mathrm{U},k}(z) = \dfrac{w^{(i,j)}_{\mathrm{U},k}(z)}{\sum\limits_{i=1}^{M_{k|k-1}} \sum\limits_{j=1}^{L^{(i)}_{k|k-1}} w^{(i,j)}_{\mathrm{U},k}(z)}$;
  $r_k(z) = \dfrac{\sum\limits_{i=1}^{M_{k|k-1}} \dfrac{r^{(i)}_{k|k-1}(1 - r^{(i)}_{k|k-1}) \varrho^{(i)}_{\mathrm{U},k}(z)}{(1 - r^{(i)}_{k|k-1} \varrho^{(i)}_{\mathrm{L},k})^2}}{\kappa_k(z) + \sum\limits_{i=1}^{M_{k|k-1}} \dfrac{r^{(i)}_{k|k-1} \varrho^{(i)}_{\mathrm{U},k}(z)}{1 - r^{(i)}_{k|k-1} \varrho^{(i)}_{\mathrm{L},k}}}$;
end for

Step 2: Update for birth targets
for each $z \in Z_{\Gamma,k}$ **do**
  set $l = 0$;
  for $i = M_{k-1} + 1, \cdots, M_{k|k-1}$ **do**
    for $j = 1, \cdots, L^{(i)}_{k|k-1}$ **do**
      $l = l + 1$;
      $\psi^{(i,j)}_{k,z} = g_k(z|x^{(i,j)}_{k|k-1}) P_{\mathrm{D},k}(x^{(i,j)}_{k|k-1})$;
      $w^{(i,j)}_{\mathrm{U},k}(z) = w^{(i,j)}_{k|k-1} \dfrac{r^{(i,j)}_{k|k-1}}{1 - r^{(i,j)}_{k|k-1}} \psi^{(i,j)}_{k,z}$;
    end for
    $\varrho^{(i)}_{\mathrm{U},k}(z) = \sum_{j=1}^{L^{(i)}_{k|k-1}} w^{(i,j)}_{k|k-1} \psi^{(i,j)}_{k,z}$;
    $\varrho^{(i)}_{\mathrm{L},k} = \sum_{j=1}^{L^{(i)}_{k|k-1}} w^{(i,j)}_{k|k-1} p_{\mathrm{D},k}(x^{(i,j)}_{k|k-1})$;
  end for
  $L_k(z) = l$;
  $w^{(i,j)}_{\mathrm{U},k}(z) = \dfrac{w^{(i,j)}_{\mathrm{U},k}(z)}{\sum_{i=M_{k-1}}^{M_{k|k-1}} \sum_{j=1}^{L^{(i)}_{k|k-1}} w^{(i,j)}_{\mathrm{U},k}(z)}$;
  $r_k(z) = \dfrac{\sum\limits_{i=M_{k-1}}^{M_{k|k-1}} \dfrac{r^{(i)}_{k|k-1}(1 - r^{(i)}_{k|k-1}) \varrho^{(i)}_{\mathrm{U},k}(z)}{(1 - r^{(i)}_{k|k-1} \varrho^{(i)}_{\mathrm{L},k})^2}}{\kappa_k(z) + \sum\limits_{i=M_{k-1}}^{M_{k|k-1}} \dfrac{r^{(i)}_{k|k-1} \varrho^{(i)}_{\mathrm{U},k}(z)}{1 - r^{(i)}_{k|k-1} \varrho^{(i)}_{\mathrm{L},k}}}$;
end for
$M_k = M_{k|k-1} + |Z_{\mathrm{P},k}| + |Z_{\Gamma,k}|$;

Simulation results: linear example

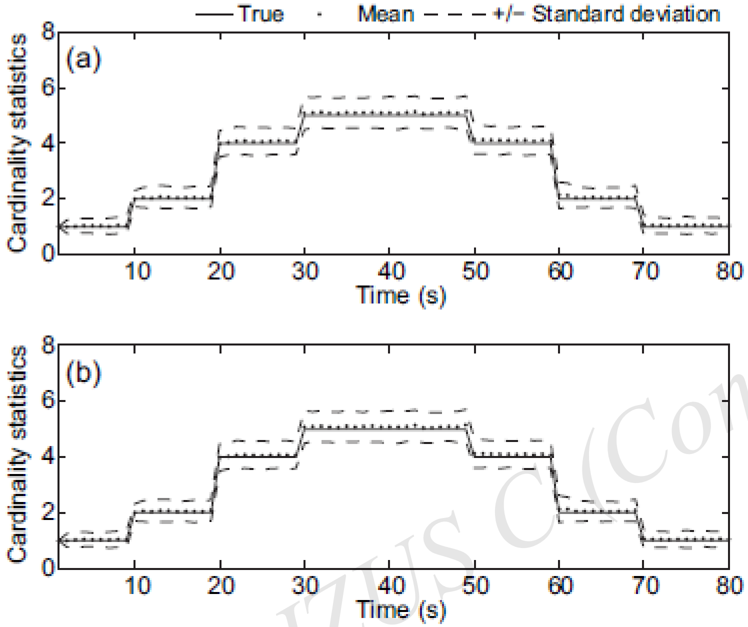

Fig. 5 The average of cardinality statistics of 500 Monte Carlo runs versus time for the SMC-MB filter (a) and the proposed approach (b)

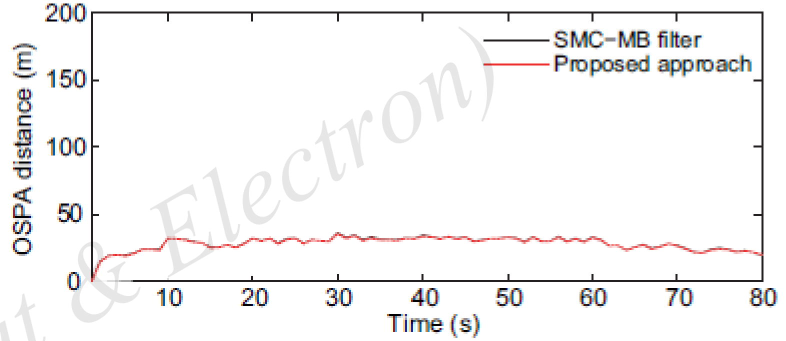

Fig. 6 The average OSPA distance for both approaches (p=2, c=200)

Table 1 Comparison between the SMC-MB filter and the proposed approach for the linear example

Filtering algorithm	Time averaged OSPA distance (m)	Average computing time (s)
SMC-MB filter	27.95	213.59
Proposed approach	27.90	76.09

Simulation results: nonlinear example

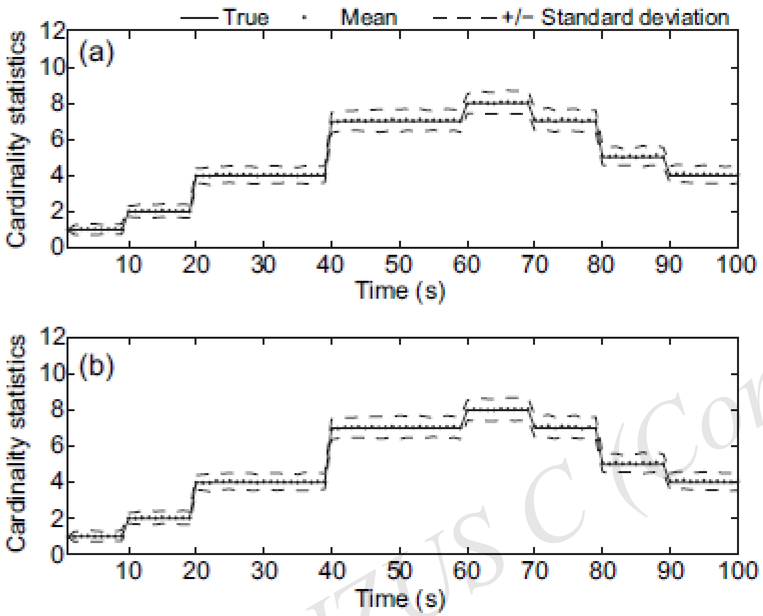

Fig. 11 The average cardinality statistics of 500 Monte Carlo runs versus time for the SMC-MB filter (a) and the proposed approach (b)

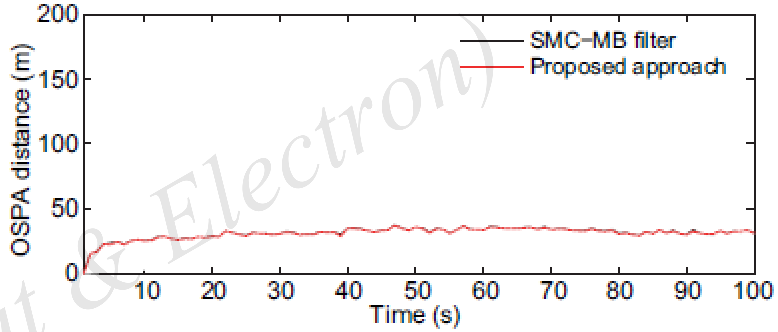

Fig. 12 The average OSPA distance for both approaches (p=2, c=200)

Table 2 Comparison between the SMC-MB filter and the proposed approach for the nonlinear example

Filtering algorithm	Time averaged OSPA distance (m)	Average computing time (s)
SMC-MB filter	31.15	901.13
Proposed approach	31.17	404.15