



# Estimating the time-dependent reliability of aging structures in the presence of incomplete deterioration information

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# Background

- ❖ The safety evaluation and service-life prediction of deteriorating structures should be based on reliability concepts and methods, aiming at preventing the use of these structures beyond an acceptable safety level.
  - ❖ The probabilistic distribution of degraded resistance often remains unaddressed due to the limits of available data.
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- The error of reliability induced if one chooses empirically the deterioration probabilistic model ?
  - How to estimate the structural reliability under incomplete deterioration information?



# Structural reliability analysis

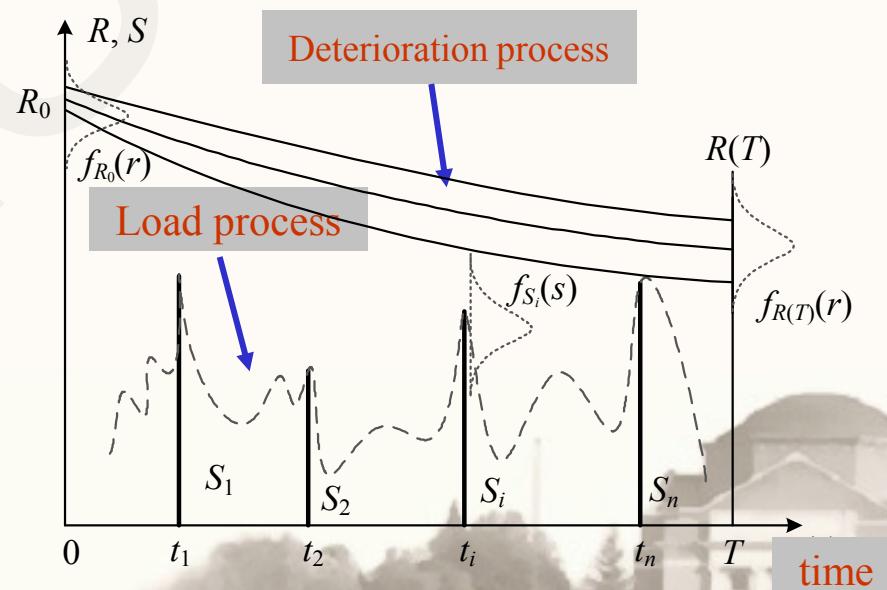
Consider the time-dependent reliability of a structure over time interval  $(0, T]$ ,  $L(T)$ .

$$L(T) = \Pr[R(t_1) > S(t_1) \cap \dots \cap R(t_n) > S(t_n)]$$

where  $\Pr[ ]$  denotes the probability of the event in the bracket,  $R(t_1), R(t_2), \dots, R(t_n)$  are the resistances corresponding to times  $t_1, t_2, \dots, t_n$ ,  $S(t_1), S(t_2), \dots, S(t_n)$  are the load effects occurring at times  $t_1, t_2, \dots, t_n$ , respectively.

For a non-stationary load process with a time-variant occurrence rate of  $\lambda(t)$  and a time-variant cumulative density function of  $F_S(s, t)$  for each load effect (Li et al, 2015),

$$L(T) = \exp \left\{ - \int_0^T \lambda(t) \{1 - F_S[r(t), t]\} dt \right\}$$





# Probabilistic model of resistance deterioration

The resistance deterioration is assumed to be a **fully correlated process**, as it provides a reasonable estimate for structural time-dependent reliability (Li et al, 2015).

$$R(t) = R_0 \cdot [1 - G(t)]$$

where  $R(t)$  is the resistance at time  $t$ ,  $R_0$  is the initial resistance, and  $G(t)$  is the deterioration function.

Taking into account the uncertainty associated with  $G(T)$ ,

$$L(T) = \int_{-\infty}^{+\infty} \exp \left[ - \int_0^T \lambda(t)(1 - F_S[r(t|g), t]) dt \right] \cdot f_G(g) dg,$$

While the mean value and standard deviation of  $G(T)$  can be estimated according to limited observed data, the probability distribution of  $G(T)$  is often unknown.

Candidate distributions for  $G(T)$ :

Uniform, Normal, Beta, Gamma and inversed-lognormal.





# Reliability under incomplete deterioration information

c.f. Eq.(18) and appendix B

$$L(T) = \frac{a_0}{2} + \sum_{m=1}^{\infty} \left[ a_m \sum_{j=0}^{\infty} c_{2j} \cdot (m\pi)^{2j} (-1)^j \right] = \frac{a_0}{2} + \sum_{m=1}^{\infty} [a_m f(m\pi)]$$

where the coefficients  $a_i$  can be obtained according to Eq. (14a), and the function  $f$  can be solved numerically by

$$f''(x) = -f(x) + Af(\gamma_4 x) + Bf(\gamma_4^2 x)$$

The proposed reliability analysis method:

- (1) is independent of the probability distribution of  $G(T)$ ;
- (2) provides an “averaged” reliability considering all possible distributions of  $G(T)$ .



# Illustrative examples

Configuration

Load combination (ACI Committee 318-14, 2014)

$$0.9R_n = 1.2D_n + 1.6L_n$$

where  $R_n$ ,  $D_n$  and  $L_n$  are the nominal resistance, dead load and live load, respectively.

Time-variant mean value of live load

$$\mu_L(t) = (0.4 + 0.005t) \cdot L_n$$

Initial resistance

$R_0$  is assumed to be deterministic, and equal to  $1.05R_n$ ;

Dead load

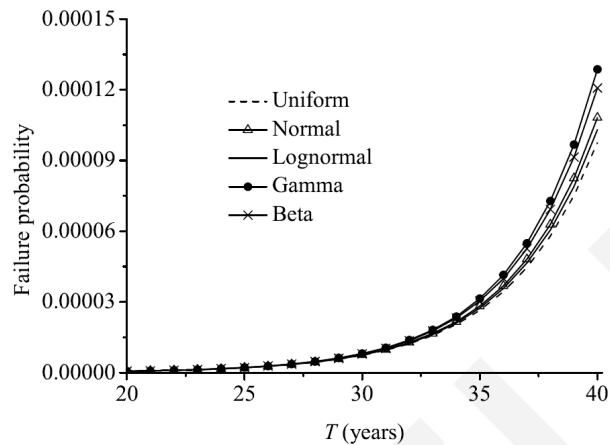
The dead load,  $D$ , equals its nominal value ( $D_n$ ), and is assumed to be deterministic.

The mean load occurrence rate of the live load is 1/year, and a reference period of 40 years is considered.

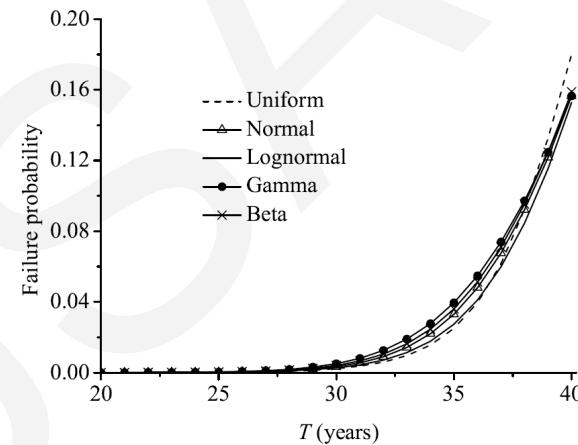




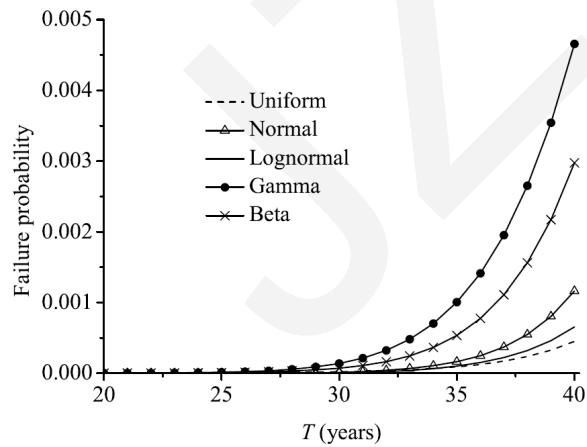
# Dependence of failure probability on resistance deterioration



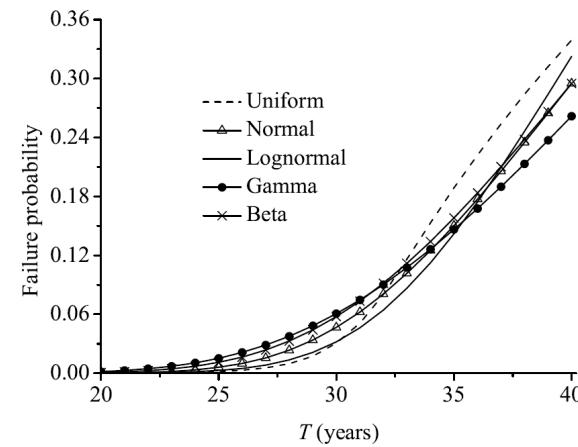
(a)  $\mu_{G(40)} = 0.2, c_{G(40)} = 0.2$



(b)  $\mu_{G(40)} = 0.4, c_{G(40)} = 0.2$



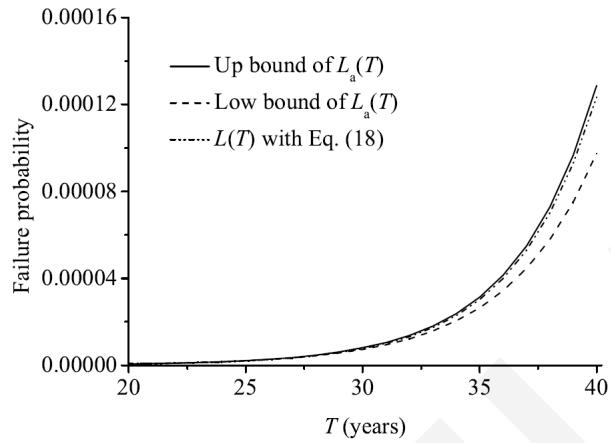
(c)  $\mu_{G(40)} = 0.2, c_{G(40)} = 0.4$



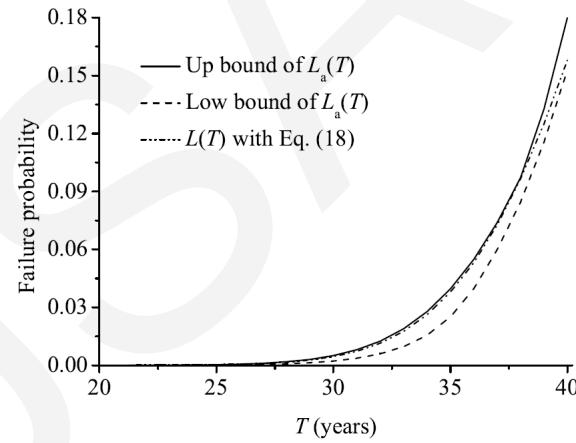
(d)  $\mu_{G(40)} = 0.4, c_{G(40)} = 0.4$



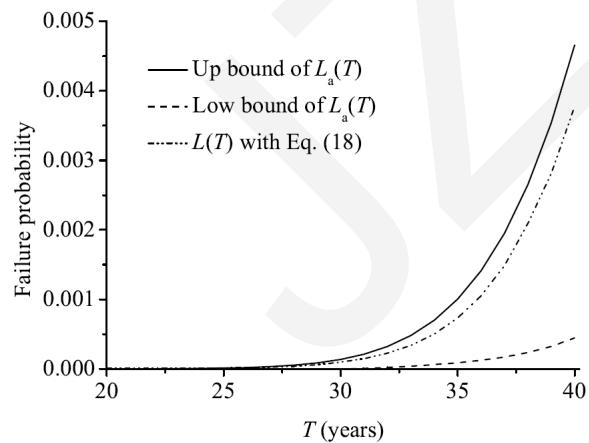
# Impact of resistance deterioration on structural averaged failure probability



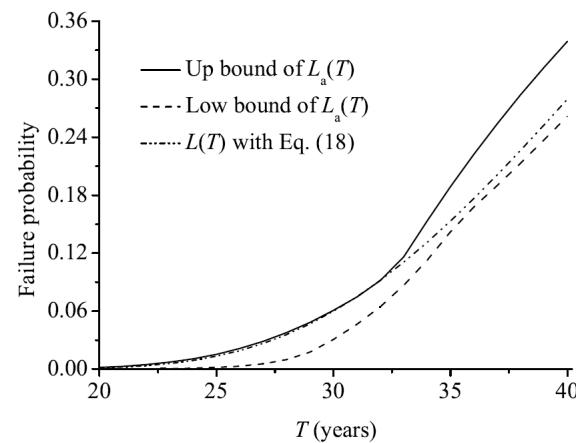
(a)  $\mu_{G(40)} = 0.2, c_{G(40)} = 0.2$



(b)  $\mu_{G(40)} = 0.4, c_{G(40)} = 0.2$



(c)  $\mu_{G(40)} = 0.2, c_{G(40)} = 0.4$



(d)  $\mu_{G(40)} = 0.4, c_{G(40)} = 0.4$



# Conclusions

- (1) With the increase of the variation associated with resistance deterioration, the reliabilities associated with different deterioration models differ significantly.
- (2) No such a deterioration model exists that results in the greatest or smallest failure probability for all deterioration conditions.
- (3) A new method is developed to estimate the averaged time-dependent reliability of aging structures. It is **independent** of the probabilistic mass scenario of the candidate deterioration models.

Thank you.