Stationary response of stochastically excited nonlinear systems with continuous-time Markov jump

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Research Background

- The operation of **complex dynamical systems** is often accompanied by abrupt changes in their configurations caused by component or interconnection failure, or by the onset of environmental disturbance.
- When these sudden changes in the operating rules occur in accordance with a Markov process, the associated stochastic system is referred to as a continuous-time **Markov jump system** (MJS).
- MJSs have many applications in a variety of fields, including **air vehicles**, **economics**, **power systems**, **satellite dynamics**, **communication**, etc.
- Since Krasovskii et al first introduced MJSs in 1960s, considerable attention has been devoted to the analysis and synthesis of MJSs.
- Previous study on MJSs mainly focused on the stability and optimal control. Little effort has been given to studying the response of MJSs, especially for stochastically excited nonlinear MJSs. Development of the methodology for analyzing nonlinear MJS is thus much deserving.

Formulation of Problem

- Consider a single-degree-of-freedom (SDOF) stochastically excited nonlinear system with continuous-time Markov jump

- $$\ddot{x} + g(x) = \varepsilon f(x, \dot{x}, s(t)) + \varepsilon^{1/2} h_k(x, s(t)) W_k(t), \qquad (1)$$
  $$x(t_0) = x_0, s(t_0) = s_0$$

- $s(t)$ is a continuous-time Markov jump process which takes discrete values in a given finite set $S = \{1,2,\cdots l\}$. Each $s \in S$ denotes the mode in which the system operates.
- Eq.(1) can be used, for instance, to model a class of linear or nonlinear systems whose random changes in their structures may be a consequence of abrupt phenomena such as component and/or interconnection failure. Our primary concern here is the stationary response of this system.

Stationary Response

- By using the stochastic averaging method, the original jump system can be approximately substituted by an averaged Itô equation of the total energy H with the Markov jump process as parameter
- $$dH = \varepsilon m(H,s)dt + \varepsilon^{1/2}\bar{\sigma}(H,s)dB(t) \qquad (2)$$
- Where $m(H,s)$ and $\bar{\sigma}(H,s)$ denote the drift and diffusion coefficients, respectively.
- Based on the averaged equation, the following FPK equation can be deduced
- $$\frac{\partial}{\partial t}p(H,s,t) = -\frac{\partial}{\partial H}[m(H,s)p(H,s,t)] + \frac{1}{2}\frac{\partial^2}{\partial H^2}[\{\bar{\sigma}(H,s)\}^2 p(H,s,t)] - \sum_{\substack{r=1 \\ r\neq s}}^{l}[\lambda_{sr}p(H,s,t) - \lambda_{rs}\int_0^\infty p(H',r,t)q(H,s,t\,|\,H',r,t)dH'] \qquad (3)$$

Numerical Example

- Note that the FPK equation does not admit an easy solution, analytically or numerically. In this case, FPK equation (3) is simplified by letting $\partial p / \partial t = 0$. Then, the stationary probability density p(H,s) is obtained readily from solving Eq.(3) using the finite difference method.
- Consider a stochastically excited Duffing oscillator with independent Markov jump process as parameter and governed by the equation
- $$\ddot{x} + \omega^2 x + \alpha x^3 = \beta(s(t))\dot{x} + h(s(t))\xi(t) \qquad (4)$$
- Where $\beta(s(t))$ is the Markov jump coefficient of linear damping; and $h(s(t))$ is the Markov jump amplitude of external random excitation; $\xi(t)$ is Gaussian white noise with zero mean and intensity 2D. $s(t)$ is a continuous-time Markov jump process. takes discrete values in a given finite set $S = \{1, 2, \cdots l\}$.

Numerical Example

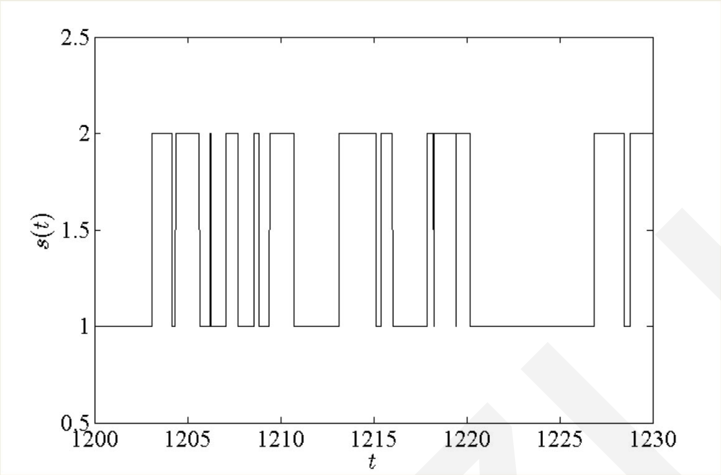

Fig. 1

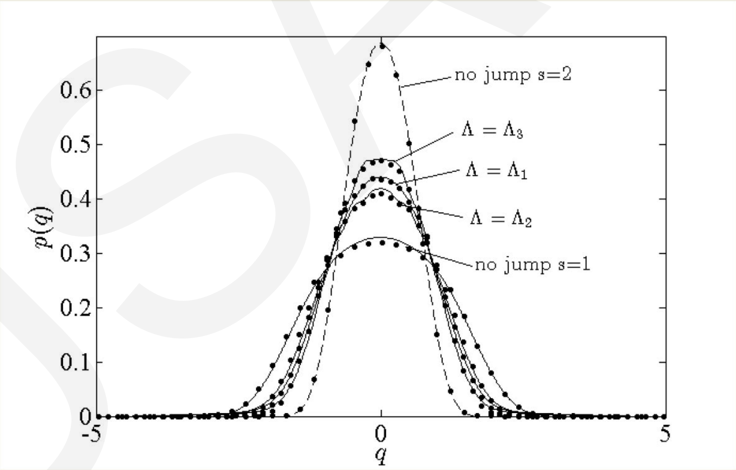

Fig. 2

- Fig. 1 The sample of jump process of **2-mode** jump system.
- Fig. 2 Stationary probability density p(q) of displacement of 2-mode jump system, with s(t)=1 and s(t)=2. The lines are obtained from numerical solution while the dots are obtained from direct simulation of original system.

Numerical Example

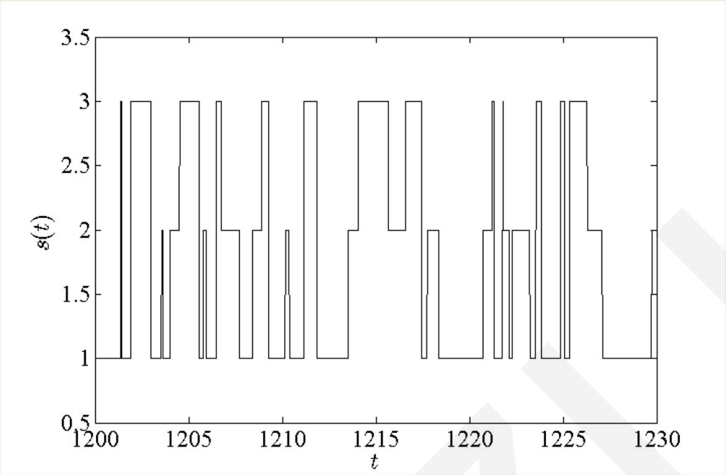

Fig. 3

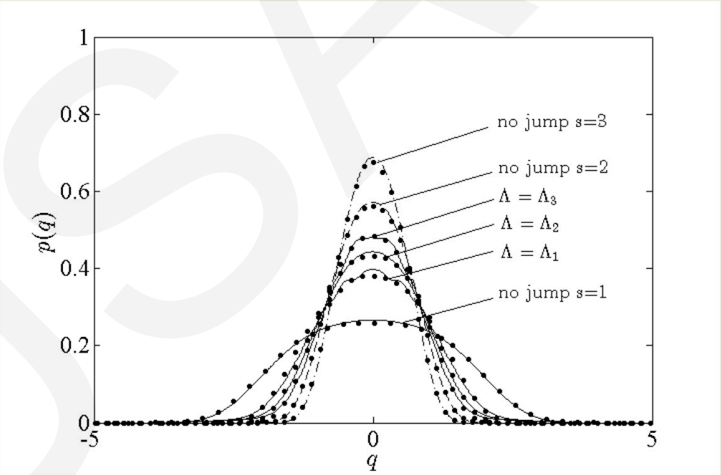

Fig. 4

- Fig. 3 The sample of jump process of **3-mode** jump system.
- Fig. 4 Stationary probability density p(q) of displacement of 3-mode jump system, with s(t)=1, s(t)=2 and s(t)=3. The lines are obtained from numerical solution while the dots are obtained from direct simulation of original system.

Conclusions

- In this paper, an approximate method for predicting the stationary response of stochastically excited nonlinear systems with continuous-time Markov jump has been proposed.
- In the case of a small transition rate, the original system was reduced to one governed by a one-dimensional averaged Itô equation with the Markov jump process as parameter using the stochastic averaging method. The FPK equation governing the probability density of the total energy has been derived.
- The comparison of the analytical results obtained by using the proposed method with those from digital simulation of the original system indicates that the proposed method is feasible and effective for solving the random vibration problem of a nonlinear Markov jump system.