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# Stochastic averaging of quasi partially integrable Hamiltonian systems under fractional Gaussian noise

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Key words: Fractional Brownian motion (fBm); Fractional Gaussian noise (fGn); Quasi partially integrable Hamiltonian system; Stochastic averaging method; Stationary response

# Fractional Guassian noise (fGn)

- Many real excitations in nature have property of long-range spatial and/or temporal correlations (long memory). These excitations can be modeled as fGn.
- FGn has already been applied as excitation model in physics, finance, and biology, etc.

characteristics

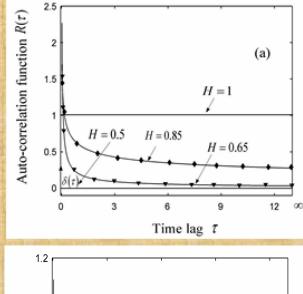
\$\left\$ self-similarity
\$\left\$ long range dependence (see Fig.1: Auto-correlation function *R*(τ) of fGn)

1/2 < H < 1

The response of dynamical system to fGn is not Markov process.

Hurst index of fGn

 The power spectral density(PSD) of fGn is meaningful physically only when 1/2<H<1. Thus, only 1/2<H<1 is considered in this paper. (see Fig.1: PSD S(ω) of fGn)



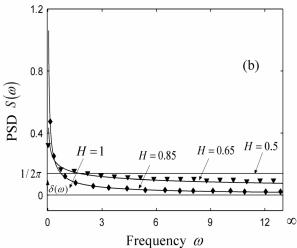


Fig. 1 Auto-correlation function  $R(\tau)$  (a) and PSD  $S(\omega)$  (b) of fGn  $W^{H}(t)$  with the simulated results ( $\checkmark$  indicate simulated results)

## Stochastic averaging method

Original system(MODF strongly nonlinear system excited by fGn)

Based on fractional stochastic integral for fBm Modeled as

FGn excited Quasi-partially integrable Hamiltonian system

Numerical simulation

Stationary response

Agree well

Consider the non-resonant case

Associated Hamiltonian  $\mathcal{H}$ 

Apply fractional differential rule for fBm

Stochastic averaging

Fractional averaged SDEs(the dimension is much less than original system )

Numerical simulation

Approximated Stationary response

Original system is approximated by fractional averaged SDEs

### Results and conclusion

• Two examples are worked out to illustrate the proposed stochastic averaging method.

Advantages

- Effectiveness: The probability density and statistics of first integrals calculated from averaged SDEs and those from the original system agree well while the error is acceptable.
- Efficiency: The dimension of the original system is greatly reduced. Thus, the computation time for simulating averaged fractional SDEs is much less than that for original system.

• Thus, in the future, it is promising to apply the proposed averaging method to do more study work.