

Research Article

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Adaptive practical fixed-time tracking control with prescribed performance for robotic manipulators

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Abstract: Robotic manipulators often suffer from insufficient positioning accuracy due to dynamic uncertainties, while existing control algorithms struggle to simultaneously achieve fast response and high tracking precision and often rely on complex dynamic models that demand excessive computational power, hindering practical onboard deployment. To address these issues, we propose an advanced real-time capable onboard tracking controller for robotic manipulators with dynamic uncertainties, which can balance the overall control performance, computational complexity, and parameter setting. The controller is suitable for direct implementation on embedded hardware with limited computational resources, without requiring heavy dynamics computation or learning approximation. First, a model-free adaptive time-delay estimator is designed to estimate and compensate for the lumped dynamic uncertainties of the robotic manipulator, where the gain matrix is directly updated by the magnitude of the sliding mode variable of tracking error for excellent overall performance. Then, an adaptive fixed-time nonsingular terminal sliding mode controller with prescribed performance is developed to stabilize the tracking errors of the robotic manipulator in a predefined fixed time. The controller parameters determine an upper bound of the fixed convergence time through analytically derived expressions, where the actual settling time is automatically ensured regardless of the initial conditions. The fast response and accurate steady-state tracking are also guaranteed by the prescribed performance, such that the processing quality and efficiency of the robotic manipulator during real-time operation tasks can be improved. Moreover, the adaptive updating law for the tracking controller gain is designed to achieve fixed-time stability of the entire closed-loop states without the requirement of an exact upper bound of dynamic uncertainties or estimation errors. The proposed tracking control scheme for robotic manipulators is easy to implement owing to the convenient parameter setting and low computational burden, and its effectiveness is verified by simulations and experiments.

Key words: Adaptive time-delay estimation; Adaptive fixed-time control; Dynamic uncertainties; Prescribed performance; Robotic manipulator; Tracking control

1 Introduction

The tracking control performance of robotic manipulators is always subject to dynamic uncertainties caused by uncertain inertial parameters, unknown nonlinear frictions, and external disturbances

(Yilmaz et al., 2022; Saied et al., 2023). Moreover, unknown heavy payloads will cause drastic changes in robotic manipulator dynamics during packing, palletizing, and other handling tasks, which may lead to a poor tracking response or instability of manipulators (Zhu et al., 2019; Huang et al., 2023). For real-time tasks such as high-power laser welding and high-speed sorting, manipulators must simultaneously achieve a fast, smooth transient response and accurate steady-state tracking. Laser welding requires precise path following in a short time, while high-speed sorting demands fast high-precision positioning. Therefore, how to design the advanced onboard controller for robotic manipulators, which can make a reasonable tradeoff among computational complexity, parameter setting, and comprehensive control performance, is still one of the main challenges to achieve autonomous applications in industrial manufacturing fields at present.

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To address the dynamic uncertainties of robotic manipulators, many representative control methods have been proposed, such as active disturbance rejection control (Ali and Mirinejad, 2024), robust sliding mode control (SMC) (Wang B et al., 2023), and adaptive learning control with neural networks (NNs) (Xu and Wu, 2024). Tong et al. (2024) developed an adaptive SMC with chattering suppression for the trajectory tracking of a robotic manipulator under dynamic uncertainties. Zhang S et al. (2023) employed radial basis function NNs with adaptive weights to approximate unknown dynamics and then designed a cooperative trajectory tracking controller for mobile dual manipulators. In recent works, a continuous supertwisting SMC was designed to achieve bottom-layer control of the robotic manipulator mounted on an autonomous vehicle, where the extreme learning machine method was used to compensate for the inferior trajectory influences caused by unknown dynamics, base tilt, and external disturbance (Wang XY et al., 2024). These control methods can only guarantee the uniform ultimate boundedness of the tracking errors, and hardly consider the convergence speed. However, it is also superior for stabilizing the tracking error with faster convergence concurrently for the robotic manipulator, so that the time-varying dynamics and real-time operations can be effectively settled.

To accelerate the tracking response, Feng et al. (2002) proposed the nonsingular terminal SMC (TSMC) method to achieve the finite-time convergence of the trajectory tracking error for manipulators, and Yu et al. (2005) improved the TSMC algorithm by designing a continuous reaching control law. The novel NN-based TSMC with an error-shifting function and barrier function was designed for a robotic manipulator, such that the finite-time convergence for both tracking errors and NN weights can be ensured (Zhang Y et al., 2023). Recently, in Saeedi et al. (2025), a resilient event-triggered nonsingular TSMC was developed for the tracking tasks of uncertain electro-hydraulic robotic manipulators despite cyberattacks that ruin the measuring data. Moreover, fractional-order TSMC (Chaudhary and Kumar, 2023), globally fast TSMC with adaptive fuzzy learning (Soltanpour and Zaare, 2024), and adaptive backstepping fast TSMC (Van et al., 2019) were designed for the finite-time tracking control of robots. However, the settling time of these finite-time control schemes is dependent on the initial conditions of the system states (Liu et al., 2022), which limits their applications because the variable settling time makes it difficult to guarantee a timely transient response of robotic manipulators, especially for real-time operation tasks. Mostly, the initial states of the robotic manipulators are unavailable in practical engineering. In addition, the transient overshoot of the above controllers cannot be constrained, which may affect the processing quality of robotic manipulators in the manufacturing process.

To avoid the excessive control efforts caused by large control gains and to improve robust antidisturbance capabilities, many disturbance observer-based (DOB) control methods have been widely studied (Sariyildiz et al., 2020), such as DOB NN control (Wen et al., 2023) and sliding mode DOB finite-time control (Song et al., 2024). A dual-mode model predictive controller with a disturbance observer was proposed for the trajectory tracking of robotic manipulators with bounded disturbances (Yang et al., 2024). Based on two fuzzy disturbance observers for dynamic uncertainties, an adaptive fault-tolerant controller was developed to ensure the uniform ultimate boundedness of manipulator tracking errors (Ren et al., 2024). Li G

et al. (2023) integrated the fuzzy NN dynamics approximator and auxiliary disturbance observer to design a command filtered impedance controller for a robotic manipulator. Different from the above DOB controllers which always suffer from a heavy computational burden caused by the nominal dynamics model or learning approximators, the time-delay estimation (TDE) method with less computational complexity is a superior alternative to compensate for disturbances and has been extensively used in robot tracking control (Glida et al., 2023; Vo et al., 2025). Although the lumped uncertainties can be estimated and compensated properly, it should be pointed out that these DOB control methods require the exact upper bounds of system uncertainties or observer errors for closed-loop stability design; for example, the upper bounds of TDE errors are necessary for the parameter setting of TDE-based controllers (Jin et al., 2015; Van et al., 2017). This will block the practical execution of these controllers because it is very difficult to obtain the exact upper bounds of the system uncertainties of robotic manipulators, which are always nonlinear time-varying and inaccessible.

To solve the above challenges, we propose an adaptive practical fixed-time tracking controller for robotic manipulators with dynamic uncertainties, which can balance comprehensive control performance, computational complexity, and parameter setting. The main contributions are summarized as follows:

1. A model-free adaptive TDE is designed to estimate the lumped dynamic uncertainties of robotic manipulators, where the gain matrix is continuously updated by the sliding mode variable of tracking error to improve the estimation performance and facilitate parameter setting. Unlike traditional TDE with fixed gains (Jin et al., 2015; Van et al., 2017; Glida et al., 2023; Vo et al., 2025), the proposed adaptive law improves both transient estimation accuracy and robustness against time-varying disturbances without manual gain scheduling.

2. An adaptive fixed-time nonsingular TSMC scheme with prescribed performance is developed to stabilize tracking errors within a predefined fixed time, where the settling time is determined only by controller parameters without respect to the initial states of robotic manipulators. The transient-phase overshoot and steady-state tracking errors are constrained by the prescribed performance. In contrast to finite-time controllers (Zhang Y et al., 2023; Saeedi et al., 2025) whose settling time depends on the initial conditions, and existing DOB controllers (Sariyildiz et al., 2020; Wen et al., 2023; Song et al., 2024) which require the exact upper bounds of uncertainties, the proposed controller achieves fixed-time convergence without conservative requirements.

3. An adaptive updating law for the switching control gain is further designed to guarantee the fixed-time stability of the entire closed-loop system without requiring the exact upper bounds of dynamic uncertainties or TDE errors. This is superior to most existing control methods (Sariyildiz et al., 2020; Wen et al., 2023; Zhang Y et al., 2023; Song et al., 2024; Saeedi et al., 2025) that either assume known uncertainty bounds or only ensure uniform ultimate boundedness.

Note that the term onboard in this paper refers to the controller suitability for embedded real-time execution, which avoids heavy computational operations such as inverse inertia matrix or complex dynamics. The proposed controller involves basic algebraic operations and adaptive laws, making it practical for industrial robot control hardware. Regarding fixed-time convergence, the control parameters

only determine the upper bound of the convergence time through the analytical results, rather than directly specifying the value of the settling time. Although the expression of this upper bound is complex, it is used only for theoretical proof. The users can qualitatively tune the parameters following the guidelines provided in this paper. The key advantage is that the upper bound of settling time is independent of the initial conditions, which is essential for the real-time operations of robotic manipulators where the initial errors usually vary unpredictably.

Notations: The symbols $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ are defined as the minimum and maximum eigenvalues of a matrix, respectively. Let $\|\cdot\|$ denote the Euclidean norm $\|z\| = \sqrt{z^T z}$ for a vector z or its induced norm $\|\mathcal{Z}\| = \sqrt{\lambda_{\max}(\mathcal{Z}^T \mathcal{Z})}$ for a matrix $\mathcal{Z}$. $z^a = [z_1^a, z_2^a, \dots, z_n^a]^T$, $\text{diag}(z) = \text{diag}\{z_1, z_2, \dots, z_n\}$, and $\text{sgn}(z) = [\text{sign}(z_1), \text{sign}(z_2), \dots, \text{sign}(z_n)]^T$ are defined with the positive constant $a \in \mathbb{R}$. $\max\{z_1, z_2, \dots, z_n\}$ and $\min\{z_1, z_2, \dots, z_n\}$ denote the maximum and minimum values of the elements, respectively.

2 Problem formulation

2.1 System description

The standard dynamics of the n -degree-of-freedom (n -DOF) robotic manipulator can be derived as

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + \tau_d = \tau, \quad (1)$$

where $q = [\theta_1, \theta_2, \dots, \theta_n]^T$ denotes the joint coordinates of the manipulator, $M(q) \in \mathbb{R}^{n \times n}$ is the inertia tensor matrix, $C(q, \dot{q})\dot{q} \in \mathbb{R}^n$ is the Coriolis and centripetal torque, $G(q) \in \mathbb{R}^n$ is the gravitational torque, $\tau_d \in \mathbb{R}^n$ is the total unknown torque caused by system uncertainties and external disturbances, and $\tau \in \mathbb{R}^n$ is the joint input torque of the manipulator.

Define a diagonally positive definite constant matrix $\bar{M}$; then, the dynamic model (1) can be rewritten as

$$\bar{M}\ddot{q} + H(q, \dot{q}, \ddot{q}) = \tau, \quad (2)$$

where the lumped unknown dynamics are defined as

$$H(q, \dot{q}, \ddot{q}) = (M(q) - \bar{M})\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + \tau_d. \quad (3)$$

It is clear that $H(q, \dot{q}, \ddot{q})$ includes nonlinear time-varying manipulator dynamics, modeling uncertainties, and disturbances, which are always highly complex and inaccessible in practical engineering. The development of the proposed control scheme is based on the following property and assumptions:

Property 1 The inertia matrix $M(q)$ is positive definite, and $m_1 \leq \|M^{-1}(q)\| \leq m_2$ holds with constants $m_2 \geq m_1 > 0$.

Assumption 1 The nominal terms $M(q)$, $C(q, \dot{q})$, and $G(q)$ are all continuous and bounded, which are reasonable due to the mechanical and task-space limitations of the robotic manipulator.

Assumption 2 The unknown torque τ_d is bounded and can be represented as $\tau_d = \tau_{d\text{discon}} + \tau_{d\text{con}}$, where $\tau_{d\text{discon}}$ and $\tau_{d\text{con}}$ are the discontinuous and continuous terms of τ_d , respectively.

2.2 Conventional TDE control

The digital control system of the robotic manipulator is regarded as continuous since the sampling frequency is much higher than its bandwidth; thus, the lumped unknown dynamics $H(q, \dot{q}, \ddot{q})$ are treated as a continuous or piecewise continuous function. The conventional TDE uses the time-delay signals to estimate $H(q, \dot{q}, \ddot{q})$, which is defined as

$$\hat{H}'(q, \dot{q}, \ddot{q}) = \tau_{t-L} - \bar{M}\ddot{q}_{t-L}, \quad (4)$$

where $\hat{H}'(q, \dot{q}, \ddot{q})$ is the estimate of $H(q, \dot{q}, \ddot{q})$, and L is the delay time, which can be chosen as the sampling period of the control system. The tracking control objective is to make the joint position tracking error $e = q - q_d$ converge to zero as close as possible, where q_d is the desired trajectory of the robotic manipulator. Therefore, the conventional TDE-based controller can be formulated as

$$\tau = \hat{H}'(q, \dot{q}, \ddot{q}) + \bar{M}(\ddot{q}_d - K_d \dot{e} - K_p e), \quad (5)$$

where $K_d \in \mathbb{R}^{n \times n}$ and $K_p \in \mathbb{R}^{n \times n}$ are diagonally positive definite gain matrices.

Let the TDE error be $\varepsilon' = \bar{M}^{-1}(H(q, \dot{q}, \ddot{q}) - \hat{H}'(q, \dot{q}, \ddot{q}))$; then, the following closed-loop dynamics of TDE-based control can be derived by substituting Eq. (5) into Eq. (2):

$$\ddot{e} + K_d \dot{e} + K_p e = -\varepsilon'. \quad (6)$$

It is obvious that the tracking error e will be bounded if ε' is bounded, which depends on the following constraint:

$$\|I - \bar{M}^{-1}(q)\bar{M}\| < 1. \quad (7)$$

This indicates that the matrix $\bar{M}$ should be chosen properly to ensure the boundedness of the TDE error, such that the bounded convergence of the tracking error can be guaranteed. Although a smaller $\bar{M}$ can be chosen to satisfy constraint (7), the convergence speed and precision of the tracking error will also degrade. In contrast, a larger $\bar{M}$ may lead to excessive overshoot and instability of the control system while achieving a faster response. The suitable selection of $\bar{M}$ for better tracking performance of the conventional TDE-based controller is still a challenging task in practical applications, especially for robotic manipulators with dynamic uncertainties and disturbances.

2.3 Motivations and preliminaries

To improve the transient and steady-state tracking response of the robotic manipulator, the following prescribed performance of tracking error can be employed (Li YM et al., 2017):

$$-\delta_i \mu_i(t) < e_i < \delta_i \mu_i(t), \quad (8)$$

where e_i ($i = 1, 2, \dots, n$) denotes the i^{th} element of the vector e , $0 < \delta_i \leq 1$ is a constant parameter, and $\mu_i(t)$ is the prescribed performance function that is strictly monotonically decreasing and formulated as $\mu_i(t) = (\mu_{i0} - \mu_{i\infty})\exp(-\bar{\mu}_i t) + \mu_{i\infty}$ with positive design parameters

μ_{i0} , $\mu_{i\infty}$, and $\bar{\mu}_i$ satisfying $\mu_{i0} = \mu_i(0)$, $\mu_{i\infty} = \lim_{t \rightarrow \infty} \mu_i(t)$, and $\mu_{i0} > \mu_{i\infty}$. It is obvious that the convergence rate of e_i is higher than the exponential decreasing rate $\exp(-\bar{\mu}_i t)$, the maximum overshoot of e_i is also less than $\delta_i \mu_{i0}$, and the steady-state error is confined by $\delta_i \mu_{i\infty}$.

By the above analyses, it can be concluded that the challenge of practical fixed-time controller design comes from the proper selection of TDE gain matrix $\hat{M}$ to achieve satisfactory tracking response, which can be overcome by adjusting $\hat{M}$ adaptively according to the tracking control performance of the manipulator. In addition, the application of a conventional TDE-based controller is limited by the requirement of inaccessible upper bounds of system uncertainties or TDE errors. To achieve the tradeoff between tracking control performance and onboard execution efficiency for robotic manipulators, the primary motivation of this paper is to design an adaptive TDE-based fixed-time tracking controller with prescribed performance, which can rapidly drive the tracking error into a bounded set with high precision and preset settling time, eliminating the requirement for any prior knowledge of system uncertainties and TDE errors.

The following preliminaries are used for the development of the proposed control scheme.

Lemma 1 (Sun et al., 2022) Consider the following differential equation:

$$\dot{\eta}(t) = -c_1 \eta(t) - c_2 \eta^{u/v}(t) + c_3 \zeta(t), \quad (9)$$

where c_1 , c_2 , and c_3 are positive constants, u and v are positive odd integers satisfying $u > v$, and $\zeta(t) \geq 0$. If the initial state $\eta(t_0) \geq 0$, then the inequality $\eta(t) \geq 0$ holds $\forall t \geq t_0$.

Lemma 2 (Sun et al., 2022) $\forall z \geq \tilde{z} \geq 0$, the inequality

$$\tilde{z}(z - \tilde{z})^a \leq \frac{a}{a+1} (z^{a+1} - \tilde{z}^{a+1}) \quad (10)$$

holds with the constant $a > 1$.

Lemma 3 (Hu et al., 2023) The following inequalities are valid with any scalar $z_i \in \mathbb{R}$, $i = 1, 2, \dots, n$:

$$\left(\sum_{i=1}^n \|z_i\| \right)^a \leq n^{a-1} \sum_{i=1}^n \|z_i\|^a, \quad (11a)$$

$$\left(\sum_{i=1}^n \|z_i\| \right)^b \leq \sum_{i=1}^n \|z_i\|^b, \quad (11b)$$

where $a > 1$ and $0 < b < 1$.

Lemma 4 (Hu et al., 2023) For a continuous nonlinear system $\dot{x} = \mathcal{F}(x, u)$ with the state x and the control input u , if there is a Lyapunov function $V(x)$ that satisfies

$$\dot{V}(x) \leq -c_1 V^a(x) - c_2 V^b(x) + \xi, \quad (12)$$

where the constants $c_1 > 0$, $c_2 > 0$, $a > 1$, $0 < b < 1$, and $\xi > 0$, then the system is practically fixed-time stable, and the settling time t_∞ is bounded by

$$t_\infty \leq T_{\max} = \frac{1}{c_1 \xi (a-1)} + \frac{1}{c_2 \xi (1-b)} \quad (13)$$

for any initial condition, and the system state will be driven into a bounded set Ω_x that guarantees

$$\Omega_x = \left\{ x | V(x) \leq \min \left\{ \left(\frac{\xi}{c_1(1-\xi)} \right)^{\frac{1}{a}}, \left(\frac{\xi}{c_2(1-\xi)} \right)^{\frac{1}{b}} \right\} \right\} \quad (14)$$

with the constant $0 < \xi < 1$.

3 Proposed control scheme

As shown in Fig. 1, an adaptive TDE method is designed to compensate for the lumped unknown dynamics of the manipulator, where the gain matrix is directly updated by the magnitude of the sliding mode variable. Then, an adaptive fixed-time tracking controller with prescribed performance is developed to drive the tracking error to a bounded set in fixed time without respect to the initial conditions. The proposed novel control scheme is easy to implement owing to the convenient parameter setting and low computing burden, while eliminating the requirement of the exact upper bounds of system uncertainties or TDE errors, which is often unavailable in practical engineering.

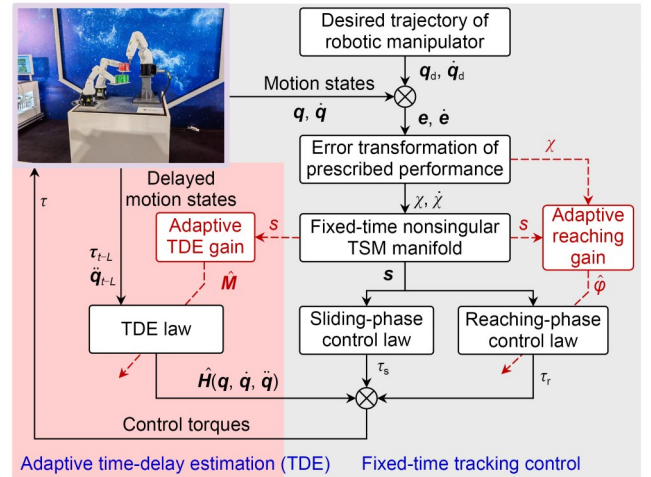


Fig. 1 Proposed tracking control scheme for robotic manipulators

3.1 Design of adaptive TDE

To improve the tracking controller performance, an adaptive TDE method for the lumped unknown dynamics of the manipulator is proposed as

$$\hat{H}(q, \dot{q}, \ddot{q}) = \tau_{t-L} - \hat{M} \ddot{q}_{t-L}, \quad (15)$$

where $\hat{H}(q, \dot{q}, \ddot{q})$ is the estimate of $H(q, \dot{q}, \ddot{q})$, and the online adaptive updating law for gain matrix $\hat{M} = \text{diag}(\hat{M}_1, \hat{M}_2, \dots, \hat{M}_n) \in \mathbb{R}^{n \times n}$ is designed as

$$\hat{M}_i = \bar{M}_i \left(1 + \mathcal{K}_i \exp \left(- \frac{\psi_i}{\|s_i\| + \varpi} \right) \right), \quad (16)$$

where $\bar{M}_i (i=1, 2, \dots, n)$ is the i^{th} element of $\bar{\mathbf{M}}$, s_i is the i^{th} element of vector $\mathbf{s}$, which is the sliding mode variable detailed later, $\mathcal{K}_i > 0$ and $\psi_i > 0$ are the design parameters, and ϖ is a small positive constant to prevent the singularity problem. It is clear that the TDE gain matrix $\hat{\mathbf{M}}$ is positive definite and upper bounded by $\bar{M}_i(1 + \mathcal{K}_i)$. Moreover, $\hat{\mathbf{M}}$ will increase along with the magnitude of the sliding mode variable $\mathbf{s}$ to accelerate the error convergence for better tracking performance, and decrease to approximately $\bar{\mathbf{M}}$ to avoid excessive control gains and transient overshoots when $\mathbf{s}$ approaches zero. Thus, the appropriate TDE gain $\hat{\mathbf{M}}$ is directly updated by the sliding mode variable $\mathbf{s}$ owing to the proposed adaptation law (16), and then the fast and accurate tracking response of the TDE controller can be achieved.

Unlike conventional TDE methods that rely on the fixed-gain matrices requiring manual calibration (Glida et al., 2023), the novel adaptive law (16) continuously updates the gain matrix $\hat{\mathbf{M}}$ according to the magnitude of the sliding variable $\mathbf{s}$. This method automatically provides larger estimation gains during transient phases to aggressively compensate for dynamic uncertainties and reduces gains at steady state to avoid noise amplification. Thus, the proposed method improves estimation accuracy and greatly simplifies parameter tuning by eliminating the need for manual gain scheduling.

Theorem 1 Let $\boldsymbol{\varepsilon} = \bar{\mathbf{M}}^{-1}(\mathbf{H}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) - \hat{\mathbf{H}}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}))$ denote the adaptive TDE error. Then, there exists an unknown constant $\bar{\varepsilon} > 0$ such that $\|\boldsymbol{\varepsilon}\| \leq \bar{\varepsilon}$ holds for a sufficiently small L .

Proof The proof of Theorem 1 is detailed as follows.

Let the adaptive TDE-based control torque be $\boldsymbol{\tau} = \boldsymbol{\tau}_{\text{TSM}} + \hat{\mathbf{H}}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})$, where $\boldsymbol{\tau}_{\text{TSM}}$ is the adaptive fixed-time control law detailed later. Then, substituting the control torque into Eq. (2), we obtain the dynamics of adaptive TDE as follows:

$$\boldsymbol{\varepsilon} = -\ddot{\mathbf{q}} + \bar{\mathbf{M}}^{-1}\boldsymbol{\tau}_{\text{TSM}}. \quad (17)$$

Multiplying both sides of Eq. (17) by $\mathbf{M}(\mathbf{q})$ and applying Eqs. (1) and (15), one can derive that

$$\begin{aligned} \mathbf{M}(\mathbf{q})\boldsymbol{\varepsilon} &= \mathbf{M}(\mathbf{q})\bar{\mathbf{M}}^{-1}\boldsymbol{\tau}_{\text{TSM}} - \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} \\ &= \mathbf{M}(\mathbf{q})\bar{\mathbf{M}}^{-1}\boldsymbol{\tau}_{\text{TSM}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) + \boldsymbol{\tau}_d - \boldsymbol{\tau} \\ &= (\mathbf{M}(\mathbf{q})\bar{\mathbf{M}}^{-1} - \mathbf{I})\boldsymbol{\tau}_{\text{TSM}} - (\mathbf{M}(\mathbf{q})_{t-L} - \hat{\mathbf{M}})\ddot{\mathbf{q}}_{t-L} + \boldsymbol{\Delta}, \end{aligned} \quad (18)$$

where $\mathbf{I}$ is the identity matrix, and $\boldsymbol{\Delta}$ denotes

$$\begin{aligned} \boldsymbol{\Delta} &= \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) + \boldsymbol{\tau}_d \\ &\quad - \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})_{t-L}\dot{\mathbf{q}}_{t-L} - \mathbf{G}(\mathbf{q})_{t-L} - \boldsymbol{\tau}_{d,t-L}. \end{aligned} \quad (19)$$

Using Assumptions 1 and 2, $\boldsymbol{\Delta}$ can be divided as

$$\boldsymbol{\Delta} = \boldsymbol{\Delta}_{\text{con}} + \boldsymbol{\Delta}_{\text{discon}}, \quad (20)$$

where

$$\begin{aligned} \boldsymbol{\Delta}_{\text{con}} &= \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) + \boldsymbol{\tau}_{\text{con}} \\ &\quad - \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})_{t-L}\dot{\mathbf{q}}_{t-L} - \mathbf{G}(\mathbf{q})_{t-L} - \boldsymbol{\tau}_{\text{con},t-L}, \\ \boldsymbol{\Delta}_{\text{discon}} &= \boldsymbol{\tau}_{\text{discon}} - \boldsymbol{\tau}_{\text{discon},t-L}. \end{aligned}$$

Thus, the approximation error $\boldsymbol{\Delta}_{\text{con}}$ of continuous terms is determined by the delay time L , i.e., $\boldsymbol{\Delta}_{\text{con}} = \mathcal{O}(L^2)$, where $\mathcal{O}(\cdot)$ denotes the approximation function. The error $\boldsymbol{\Delta}_{\text{discon}}$ is bounded by $\|\boldsymbol{\Delta}_{\text{discon}}\| \leq \bar{\tau}_d$ with the constant $\bar{\tau}_d > 0$. One can derive that $\|\boldsymbol{\Delta}\| \leq \|\mathcal{O}(L^2)\| + \bar{\tau}_d$ holds for a sufficiently small L .

Substituting $\ddot{\mathbf{q}}_{t-L} = \bar{\mathbf{M}}^{-1}\boldsymbol{\tau}_{\text{TSM},t-L} - \boldsymbol{\varepsilon}_{t-L}$ into Eq. (18) and applying Eq. (16) yield

$$\begin{aligned} \mathbf{M}(\mathbf{q})\boldsymbol{\varepsilon} &= (\mathbf{M}(\mathbf{q})\bar{\mathbf{M}}^{-1} - \mathbf{I})\boldsymbol{\tau}_{\text{TSM}} + (\mathbf{M}(\mathbf{q}) - \mathbf{M}(\mathbf{q})_{t-L})\ddot{\mathbf{q}}_{t-L} \\ &\quad - (\mathbf{M}(\mathbf{q}) - \hat{\mathbf{M}})(\bar{\mathbf{M}}^{-1}\boldsymbol{\tau}_{\text{TSM},t-L} - \boldsymbol{\varepsilon}_{t-L}) + \boldsymbol{\Delta} \\ &= (\mathbf{M}(\mathbf{q}) - \hat{\mathbf{M}})\boldsymbol{\varepsilon}_{t-L} \\ &\quad + (\mathbf{M}(\mathbf{q})\bar{\mathbf{M}}^{-1} - \mathbf{I})(\boldsymbol{\tau}_{\text{TSM}} - \boldsymbol{\tau}_{\text{TSM},t-L}) \\ &\quad + \bar{\mathcal{K}}\boldsymbol{\tau}_{\text{TSM},t-L} + (\mathbf{M}(\mathbf{q}) - \mathbf{M}(\mathbf{q})_{t-L})\ddot{\mathbf{q}}_{t-L} + \boldsymbol{\Delta}, \end{aligned} \quad (21)$$

where $\bar{\mathcal{K}}$ is a diagonal matrix whose elements are bounded and given as $\bar{\mathcal{K}}_i = \mathcal{K}_i \exp(-\psi_i/(\|s_i\| + \varpi_i))$, $i=1, 2, \dots, n$.

Let $\boldsymbol{\omega}_1 = (\mathbf{M}(\mathbf{q})\bar{\mathbf{M}}^{-1} - \mathbf{I})(\boldsymbol{\tau}_{\text{TSM}} - \boldsymbol{\tau}_{\text{TSM},t-L}) + \bar{\mathcal{K}}\boldsymbol{\tau}_{\text{TSM},t-L}$ and $\boldsymbol{\omega}_2 = (\mathbf{M}(\mathbf{q}) - \mathbf{M}(\mathbf{q})_{t-L})\ddot{\mathbf{q}}_{t-L} + \boldsymbol{\Delta}$. Then, one has

$$\boldsymbol{\varepsilon} = (\mathbf{I} - \mathbf{M}^{-1}(\mathbf{q})\hat{\mathbf{M}})\boldsymbol{\varepsilon}_{t-L} + \mathbf{M}^{-1}(\mathbf{q})(\boldsymbol{\omega}_1 + \boldsymbol{\omega}_2). \quad (22)$$

According to Assumption 1 and bounded $\bar{\mathcal{K}}$, $\boldsymbol{\omega}_1$ and $\boldsymbol{\omega}_2$ are bounded for a sufficiently small L , and then $\mathbf{M}^{-1}(\mathbf{q})(\boldsymbol{\omega}_1 + \boldsymbol{\omega}_2)$ is also bounded.

Thus, one can conclude that the solution to the equivalent differential equation of Eq. (22) is asymptotically bounded if

$$\|\mathbf{I} - \mathbf{M}^{-1}(\mathbf{q})\hat{\mathbf{M}}\| < 1. \quad (23)$$

As a result, the existence of $\bar{\varepsilon} > 0$ such that $\|\boldsymbol{\varepsilon}\| \leq \bar{\varepsilon}$ is guaranteed. Theorem 1 is proved.

Remark 1 The delay time L and adaptive gain matrix $\hat{\mathbf{M}}$ are essential for achieving good performance for TDE. The constant L is usually chosen as the sufficiently small sampling period of the control system. Considering that $\|\mathbf{I} - \mathbf{M}^{-1}(\mathbf{q})\bar{\mathbf{M}}\| \leq \frac{m_2 - m_1}{m_2 + m_1} < 1$ when $\bar{\mathbf{M}} = \frac{m_1 + m_2}{2}\mathbf{I}$ (Spong and Vidyasagar, 1987), inequality (23) can also be easily satisfied with the appropriate selection of $\hat{\mathbf{M}}$, such as letting $\bar{M}_i = (m_1 + m_2)/2$ and $\mathcal{K}_i = m_1/((m_2 - m_1)m_2)$; then, one can derive that $\|\mathbf{I} - \mathbf{M}^{-1}(\mathbf{q})\hat{\mathbf{M}}\| \leq m_2/(m_2 + m_1) < 1$.

Remark 2 Superior to the existing adaptive TDE methods (Baek et al., 2018; Wang YY et al., 2019) employing the discontinuous updating laws of gain parameters, which may lead to serious chattering and obvious deterioration of tracking response, the proposed adaptive TDE gain $\hat{\mathbf{M}}$ is continuously updated using the magnitude of sliding variable $\mathbf{s}$ directly to ensure excellent overall performance. The conventional adaptation laws of TDE gain (Baek et al., 2018; Wang YY et al., 2019) are also limited by the high computational burden due to the complex auxiliary variables, and the appropriate selection of the design parameters for bounded-input bounded-output (BIBO) stability requires further study. Fortunately, the proposed adaptive TDE gain $\hat{\mathbf{M}}$, as given by Eq. (16), is computationally

efficient and convenient for parameter setting. Moreover, compared with the adaptive TDE in Baek et al. (2018) and Wang YY et al. (2019), which relies on a tedious trial-and-error tuning process for the gain adaptation law, the proposed method directly links the gain update to the magnitude of the sliding variable of tracking error, greatly reducing the number of tuning parameters.

3.2 Design of the tracking controller

To satisfy the prescribed performance on the tracking error, the inequality constraint (8) can be transformed into the equivalent equality constraint as

$$e_i(t) = \mu_i(t) \mathcal{G}_i(\chi_i(t)), \quad \forall t \geq 0, \quad (24)$$

where $\chi_i(t)$ denotes the transformed error, and $\mathcal{G}_i(\chi_i(t))$ is a monotonically increasing smooth function, defined as

$$\mathcal{G}_i(\chi_i(t)) = \frac{\delta_i(\exp(\chi_i(t)) - \exp(-\chi_i(t)))}{\exp(\chi_i(t)) + \exp(-\chi_i(t))}.$$

Then, the transformed error can be solved as

$$\chi_i(t) = \mathcal{G}_i^{-1}\left(\frac{e_i(t)}{\mu_i(t)}\right) = \frac{1}{2} \ln \frac{\mathcal{G}_i\left(\frac{e_i(t)}{\mu_i(t)}\right) + \delta_i}{\delta_i - \mathcal{G}_i\left(\frac{e_i(t)}{\mu_i(t)}\right)}. \quad (25)$$

Let the transformed error vector be $\chi = [\chi_1, \chi_2, \dots, \chi_n]^T$. Taking the time derivative of Eq. (25) and substituting Eq. (2) into it yield

$$\dot{\chi} = \Phi(\Lambda \dot{e} - \dot{\Lambda} e), \quad (26)$$

$$\ddot{\chi} = \dot{\Phi}(\Lambda \dot{e} - \dot{\Lambda} e) - \Phi \ddot{\Lambda} e + \Phi \Lambda (\bar{M}^{-1}(\tau - H(q, \dot{q}, \ddot{q})) - \ddot{q}_d), \quad (27)$$

where $\Phi = \text{diag}\{\Phi_1, \Phi_2, \dots, \Phi_n\}$ is a diagonally positive definite matrix with the element $\Phi_i = \frac{1}{2} \left(\frac{1}{\mathcal{G}_i\left(\frac{e_i(t)}{\mu_i(t)}\right) + \delta_i} - \frac{1}{\mathcal{G}_i\left(\frac{e_i(t)}{\mu_i(t)}\right) - \delta_i} \right)$ and $\Lambda = \text{diag}\{\mu_1(t), \mu_2(t), \dots, \mu_n(t)\}$ is also diagonally positive definite. It can be concluded that the prescribed performance constraint (8) of e can be satisfied when χ is bounded (Li YM et al., 2017).

The fixed-time nonsingular terminal sliding mode (NTSM) manifold of transformed error χ is designed as

$$s = \chi + \alpha_1 \chi^{u_1/v_1} + \beta_1 \dot{\chi}^{p_1/q_1}, \quad (28)$$

where $\alpha_1 \in \mathbb{R}^{n \times n}$ and $\beta_1 \in \mathbb{R}^{n \times n}$ are both diagonally positive definite gain matrices, and u_1, v_1, p_1 , and q_1 are positive odd integers satisfying $1 < p_1/q_1 < 2, v_1 < u_1, u_1/v_1 > p_1/q_1$.

Differentiating Eq. (28) with respect to time and applying Eqs. (26) and (27), the dynamics of the terminal sliding mode (TSM) manifold can be derived as

$$\begin{aligned} \dot{s} = & \dot{\chi} + \frac{u_1}{v_1} \alpha_1 \text{diag}\left(\chi^{\frac{u_1}{v_1}-1}\right) \dot{\chi} \\ & + \frac{p_1}{q_1} \beta_1 \text{diag}\left(\dot{\chi}^{\frac{p_1}{q_1}-1}\right) \left(\dot{\Phi}(\Lambda \dot{e} - \dot{\Lambda} e) - \Phi \ddot{\Lambda} e\right. \\ & \left. + \Phi \Lambda (\bar{M}^{-1}(\tau - H(q, \dot{q}, \ddot{q})) - \ddot{q}_d)\right). \end{aligned} \quad (29)$$

Assumption 3 Let an unknown positive constant φ satisfy

$$\frac{1}{\lambda_{\min}(\Pi(\dot{\chi}))} \|\Pi(\dot{\chi}) \Phi \Lambda\| \|\varepsilon\| \leq \varphi, \quad (30)$$

where $\Pi(\dot{\chi}) = \beta_1 \text{diag}\left(\dot{\chi}^{\frac{p_1}{q_1}-1}\right)$ denotes a diagonally positive definite matrix, and $\dot{\chi}_i \neq 0$ ($i = 1, 2, \dots, n$).

Theorem 2 For the robotic manipulator system (2) with a fixed-time NTSM manifold (28), if the adaptive TDE is developed as Eqs. (15) and (16), the practical fixed-time tracking control with prescribed performance is designed as

$$\tau = \tau_{\text{TSM}} + \hat{H}(q, \dot{q}, \ddot{q}), \quad (31a)$$

$$\tau_{\text{TSM}} = \tau_s + \tau_r, \quad (31b)$$

$$\begin{aligned} \tau_s = & -\frac{q_1 u_1}{p_1 v_1} \bar{M} \Lambda^{-1} \Phi^{-1} \beta_1^{-1} \alpha_1 \text{diag}\left(\chi^{\frac{u_1}{v_1}-1}\right) \dot{\chi}^{2-\frac{p_1}{q_1}} \\ & - \frac{q_1}{p_1} \bar{M} \Lambda^{-1} \Phi^{-1} \beta_1^{-1} \dot{\chi}^{2-\frac{p_1}{q_1}} \end{aligned} \quad (31c)$$

$$\begin{aligned} & + \bar{M} \Lambda^{-1} \Phi^{-1} (-\dot{\Phi}(\Lambda \dot{e} - \dot{\Lambda} e) + \Phi \ddot{\Lambda} e) + \bar{M} \ddot{q}_d, \\ \tau_r = & -\bar{M} \Lambda^{-1} \Phi^{-1} \left(\alpha_2 s^{\frac{u_2}{v_2}} + \beta_2 s^{\frac{q_2}{p_2}} + (\hat{\varphi} + \sigma_0) \text{sgn}(s) \right), \end{aligned} \quad (31d)$$

with the adaptation law

$$\dot{\hat{\varphi}} = -\sigma_1 \hat{\varphi} - \sigma_2 \hat{\varphi}^{\frac{u_2}{v_2}} + \frac{p_1}{q_1} \lambda_{\min}(\Pi(\dot{\chi})) \|s\|, \quad (32)$$

where τ_s and τ_r are the sliding- and reaching-phase control laws, respectively, $\alpha_2 \in \mathbb{R}^{n \times n}$ and $\beta_2 \in \mathbb{R}^{n \times n}$ are diagonally positive definite gain matrices, u_2, v_2, p_2 , and q_2 are positive odd integers satisfying $u_2 > v_2$ and $p_2 > q_2$, $\hat{\varphi}$ is the estimate of φ , and $\sigma_0 > 0, \sigma_1 > 0$, and $\sigma_2 > 0$ are design parameters. Then, one can conclude that the transformed error χ will be driven into a bounded set within a fixed time and that the tracking error e of the robotic manipulator will converge to a bounded set within a fixed time and satisfy the prescribed performance (8).

Remark 3 According to Lemma 1, $\hat{\varphi} \geq 0$ can be guaranteed by choosing a positive initial value $\hat{\varphi}(t_0) \geq 0$. As a result, the inequality $\varphi \geq \hat{\varphi}$ holds with the estimation error $\tilde{\varphi} = \varphi - \hat{\varphi}$.

Remark 4 Since the real velocity $\dot{q}$ and its desired value $\dot{q}_d$ of the robotic manipulator are bounded in practical engineering, both $\dot{e}$ and $\dot{\chi}$ can be assumed to be bounded. Therefore, Assumption 3 is reasonable due to the bounded TDE error ε .

Proof The stability analysis of the proposed control scheme is divided into two steps. The practical fixed-time stability of the entire closed-loop control system is proved in step 1. The fixed-time convergence

of the transformed error χ and tracking error e is further detailed in step 2.

Step 1: Applying the control law (31a)–(31d) to Eq. (29), one has

$$\dot{s} = \frac{p_1}{q_1} \Pi(\dot{\chi}) \begin{pmatrix} -\alpha_2 s^{\frac{u_2}{v_2}} - \beta_2 s^{\frac{q_2}{p_2}} \\ -(\hat{\phi} + \sigma_0) \operatorname{sgn}(s) - \Phi \Lambda \varepsilon \end{pmatrix}. \quad (33)$$

Define the Lyapunov candidate function as

$$V = s^T s / 2 + \tilde{\varphi}^2 / 2. \quad (34)$$

In the case of $\dot{\chi} \neq 0$, taking the time derivative of Eq. (34) and substituting Eq. (33) into it yield

$$\begin{aligned} \dot{V} &= s^T \frac{p_1}{q_1} \Pi(\dot{\chi}) \begin{pmatrix} -\alpha_2 s^{\frac{u_2}{v_2}} - \beta_2 s^{\frac{q_2}{p_2}} \\ -(\hat{\phi} + \sigma_0) \operatorname{sgn}(s) - \Phi \Lambda \varepsilon \end{pmatrix} - \tilde{\varphi} \dot{\hat{\varphi}} \\ &\leq s^T \frac{p_1}{q_1} \Pi(\dot{\chi}) (-\alpha_2 s^{\frac{u_2}{v_2}} - \beta_2 s^{\frac{q_2}{p_2}}) - \tilde{\varphi} \dot{\hat{\varphi}} \\ &\quad - \frac{p_1}{q_1} \lambda_{\min}(\Pi(\dot{\chi})) \hat{\phi} \|s\| + \frac{p_1}{q_1} \|\Pi(\dot{\chi}) \Phi \Lambda\| \|\varepsilon\| \|s\| \quad (35) \\ &\leq s^T \frac{p_1}{q_1} \Pi(\dot{\chi}) (-\alpha_2 s^{\frac{u_2}{v_2}} - \beta_2 s^{\frac{q_2}{p_2}}) - \tilde{\varphi} \dot{\hat{\varphi}} \\ &\quad + \frac{p_1}{q_1} \lambda_{\min}(\Pi(\dot{\chi})) \hat{\phi} \|s\|. \end{aligned}$$

Applying the adaptation law (32) yields

$$\begin{aligned} \dot{V} &\leq s^T \frac{p_1}{q_1} \Pi(\dot{\chi}) \begin{pmatrix} -\alpha_2 s^{\frac{u_2}{v_2}} - \beta_2 s^{\frac{q_2}{p_2}} \\ \sigma_1 \hat{\phi} + \sigma_2 \hat{\phi}^{\frac{u_2}{v_2}} \end{pmatrix} \\ &\leq -\frac{p_1}{q_1} \lambda_{\min}(\Pi(\dot{\chi}) \alpha_2) s^{\frac{u_2 + v_2}{v_2}} - \frac{p_1}{q_1} \lambda_{\min}(\Pi(\dot{\chi}) \beta_2) s^{\frac{q_2 + p_2}{p_2}} \quad (36) \\ &\quad + \sigma_1 \tilde{\varphi} (\phi - \hat{\phi}) + \sigma_2 \tilde{\varphi} (\phi - \hat{\phi})^{\frac{u_2}{v_2}}. \end{aligned}$$

According to Lemmas 2 and 3, one can derive that

$$\begin{aligned} \dot{V} &\leq -\frac{p_1}{q_1} \lambda_{\min}(\Pi(\dot{\chi}) \alpha_2) s^{\frac{u_2 + v_2}{v_2}} - \frac{p_1}{q_1} \lambda_{\min}(\Pi(\dot{\chi}) \beta_2) s^{\frac{q_2 + p_2}{p_2}} \\ &\quad - \frac{\sigma_1}{2} \tilde{\varphi}^{\frac{q_2 + p_2}{p_2}} + \frac{\sigma_1}{2} \tilde{\varphi}^{\frac{q_2 + p_2}{p_2}} + \frac{\sigma_1}{2} (\varphi^2 - \tilde{\varphi}^2) \quad (37) \\ &\quad + \frac{\sigma_2 v_2}{u_2 + v_2} \left(\varphi^{\frac{u_2 + v_2}{v_2}} - \tilde{\varphi}^{\frac{u_2 + v_2}{v_2}} \right) \\ &\leq -\rho_1 V^{\frac{u_2 + v_2}{2v_2}} - \rho_2 V^{\frac{q_2 + p_2}{2p_2}} + \bar{\varphi}, \end{aligned}$$

where

$$\begin{aligned} \rho_1 &= \frac{\sqrt{2}}{2^{(u_2 + v_2)/(2v_2)}} \min \left\{ \frac{p_1}{q_1} \lambda_{\min}(\Pi(\dot{\chi}) \alpha_2), \frac{\sigma_2 v_2}{u_2 + v_2} \right\}, \\ \rho_2 &= \sqrt{2} \min \left\{ \frac{p_1}{q_1} \lambda_{\min}(\Pi(\dot{\chi}) \beta_2), \frac{\sigma_1}{2} \right\}, \end{aligned}$$

$$\begin{aligned} \bar{\varphi} &= \frac{\sigma_2 v_2}{u_2 + v_2} \varphi^{\frac{u_2 + v_2}{v_2}} + \frac{\sigma_1}{2} \tilde{\varphi}^{\frac{q_2 + p_2}{p_2}} - \frac{\sigma_1}{2} \tilde{\varphi}^2 + \frac{\sigma_1}{2} \varphi^2 \\ &\leq \frac{\sigma_2 v_2}{u_2 + v_2} \varphi^{\frac{u_2 + v_2}{v_2}} + \frac{\sigma_1}{2} \varphi^2 \\ &\quad + \frac{\sigma_1}{2} \left(\left(\frac{p_2 + q_2}{2p_2} \right)^{\frac{p_2 + q_2}{p_2 - q_2}} - \left(\frac{p_2 + q_2}{2p_2} \right)^{\frac{2p_2}{p_2 - q_2}} \right) = \bar{\varphi}_{\max}. \end{aligned}$$

Then, inequality (37) can be further derived as

$$\dot{V} \leq -\rho_1 V^{\frac{u_2 + v_2}{2v_2}} - \rho_2 V^{\frac{q_2 + p_2}{2p_2}} + \bar{\varphi}_{\max}. \quad (38)$$

As the positive odd integers satisfy $u_2 > v_2$ and $p_2 > q_2$, the terms ρ_1 , ρ_2 , and $\bar{\varphi}_{\max}$ are all positive and bounded. According to Lemma 4, the NTSM manifold s will be driven into a bounded set

$$\Omega_s = \left\{ s \mid V \leq \min \left\{ \left(\frac{\bar{\varphi}_{\max}}{\rho_1(1-\varsigma)} \right)^{\frac{2v_2}{u_2 + v_2}}, \left(\frac{\bar{\varphi}_{\max}}{\rho_2(1-\varsigma)} \right)^{\frac{2p_2}{q_2 + p_2}} \right\} \right\}, \quad (39)$$

and the settling time is bounded by

$$t_1 \leq T_{\max 1} = \frac{1}{\rho_1 \varsigma \left(\frac{u_2 + v_2}{2v_2} - 1 \right)} + \frac{1}{\rho_2 \varsigma \left(1 - \frac{q_2 + p_2}{2p_2} \right)}, \quad (40)$$

where $0 < \varsigma < 1$, and $T_{\max 1}$ is determined only by the design parameters without respect to the initial conditions.

It has already been proven that $\dot{\chi} = 0$ is not an attractor for the NTSM manifold (Yu et al., 2005); thus, the reachability of the bounded set $s \in \Omega_s$ is guaranteed all the time. Therefore, the fixed-time convergence of the NTSM manifold s in Eq. (28) is achieved as Eq. (39) and inequality (40). The practical fixed-time stability of the entire closed-loop control system is ensured.

Step 2: Combining Eqs. (34) and (39), the fixed-time convergence region of s can be further formulated as

$$\|s\| \leq \bar{\Omega} = \sqrt{2} \min \left\{ \left(\frac{\bar{\varphi}_{\max}}{\rho_1(1-\varsigma)} \right)^{\frac{v_2}{u_2 + v_2}}, \left(\frac{\bar{\varphi}_{\max}}{\rho_2(1-\varsigma)} \right)^{\frac{p_2}{q_2 + p_2}} \right\}. \quad (41)$$

In the case of $\dot{\chi}_i \neq 0$, substituting inequality (41) into Eq. (28) yields

$$\chi_i + \alpha_{1_i} \chi_i^{\frac{u_1}{v_1}} + \left(\beta_{1_i} - s_i \dot{\chi}_i^{-p_1/q_1} \right) \dot{\chi}_i^{p_1/q_1} = 0, \quad (42)$$

with $i = 1, 2, \dots, n$. If the convergence region formulated by Eq. (42) is reached and the inequality $\tilde{\beta}_{1_i} = \beta_{1_i} - s_i \dot{\chi}_i^{-p_1/q_1} > 0$ holds, then the transformed error χ_i will persistently converge along the fixed-time NTSM surface (28) until $\tilde{\beta}_{1_i} \leq 0$; thus,

$$\|\dot{\chi}_i\| \leq \left(\beta_{1_i}^{-1} \bar{\Omega} \right)^{q_1/p_1}, \quad (43)$$

where the convergence time is bounded by

$$t_{2_i} \leq T_{\max 2_i} = \left(\frac{\beta_{1_u}}{\alpha_{1_u}} \right)^{q_1/p_1} \frac{p_1/q_1}{u_1/v_1 - p_1/q_1} + \beta_{1_u}^{q_1/p_1} \frac{1}{1 - q_1/p_1}. \quad (44)$$

Considering Eq. (28), the following inequality can be obtained:

$$\left\| \chi_i + \alpha_{1_u} \chi_i^{u_1/v_1} \right\| \leq \left\| s_i \right\| + \left\| \beta_{1_u} \dot{\chi}_i^{p_1/q_1} \right\|. \quad (45)$$

Substituting inequality (43) into inequality (45), one can derive that

$$\left\| \chi_i \right\| \leq \max \left\{ 2\bar{\Omega}, \left(2\alpha_{1_u}^{-1} \bar{\Omega} \right)^{v_1/u_1} \right\}. \quad (46)$$

For the case of $\dot{\chi}_i = 0$, it can be easily derived that

$$\left\| \chi_i \right\| \leq \max \left\{ \bar{\Omega}, \left(\alpha_{1_u}^{-1} \bar{\Omega} \right)^{v_1/u_1} \right\}. \quad (47)$$

The total convergence time is determined as follows:

$$\begin{aligned} t_{\text{total}} &= t_1 + t_2 \leq T_{\max 1} + T_{\max 2} \\ &= T_{\max 1} + \max \left\{ T_{\max 2_1}, T_{\max 2_2}, \dots, T_{\max 2_n} \right\}, \end{aligned} \quad (48)$$

which depends only on the design parameters.

Therefore, the fixed-time bounded stability of the transformed error χ is achieved with the convergence region formulated by inequalities (43), (46), and (47); thus, the prescribed performance (8) of tracking error e can be satisfied. Let $\left\| \chi_i(t) \right\| \leq \bar{\chi}_i, t \geq t_{\text{total}}$ with a positive constant $\bar{\chi}_i$; then, one can derive that $\left\| e_i(t) \right\| \leq \mu_i(t_{\text{total}}) \mathcal{G}_i(\bar{\chi}_i), t \geq t_{\text{total}}$ according to Eq. (24). As a result, the tracking error e will also converge to a bounded set within the fixed time t_{total} . Theorem 2 is proved.

Remark 5 Superior to conventional adaptive TDE-based controllers (Baek et al., 2018; Wang YY et al., 2019), which require the exact upper bound of TDE errors for the tuning of controller parameters, the proposed controller successfully eliminates the prior information of the system uncertainties or TDE errors, which is almost unavailable in practical engineering. Different from the controllers only ensuring uniform ultimate boundedness of the tracking error (Baek et al., 2018; Wang YY et al., 2019), the designed controller can drive the tracking error into a bounded set within a fixed time where the maximum settling time can be predefined without respect to the initial conditions, such that faster and robust tracking can be achieved. Moreover, the prescribed tracking error performance is designed to constrain the transient response and the steady-state errors, and better comprehensive control performance for the robotic manipulator can be achieved. It is worth emphasizing that most existing fixed-time controllers for robot manipulators (Hu et al., 2023) assume the lumped uncertainty or its derivative to be bounded by a known constant. In sharp contrast, the proposed adaptive switching gain law removes this assumption, making the controller truly practical for industrial applications where such bounds are rarely available a priori.

Remark 6 Despite the presence of multiple adaptive laws, the proposed controller is computationally lightweight. At each sampling step, the required operations include 6 scalar multiplications and 6 additions of diagonal matrix for TDE gain updating, 24 floating-point operations (power, sign, multiplication) for fixed-time NTSM

control law, and 12 scalar operations for switching gain adaptation. The total operations are approximately 60 FLOPs per joint, and 360 FLOPs for a 6-DOF manipulator. In contrast, a typical NN-based fixed-time controller (Hu et al., 2023) requires at least 30 hidden neurons, implying 2000 FLOPs per step, including activation functions and weight updates. Thus, the proposed controller reduces the computational cost by approximately 80%, making it suitable for onboard real-time execution on standard industrial hardware at a 1000 Hz sampling rate. No matrix inversion, regression matrix, or iterative optimization is involved.

Remark 7 To avoid the chattering caused by the discontinuous signum function $\text{sign}(s_i)$ in Eq. (31d), the saturation function

$$\text{sat}(\text{sign}(s_i), \Xi) = \begin{cases} \text{sign}(s_i), & \|s_i\| > \Xi, \\ \frac{s_i}{\Xi}, & \|s_i\| \leq \Xi, \end{cases} \quad (49)$$

with a small positive constant Ξ can be used as a replacement.

3.3 Parameter tuning guidelines

To facilitate the practical implementation of the proposed control method without loss of generality, this subsection provides qualitative yet actionable guidelines for selecting the controller parameters. Users can adjust the control parameters iteratively according to their specific manipulator and task requirements.

1. Prescribed performance function: Choose the maximum allowable overshoot δ_p , the initial error bound μ_{i0} , the steady-state error bound $\mu_{i\infty}$, and the convergence rate $\bar{\mu}_i$. A smaller $\mu_{i\infty}$ and a larger $\bar{\mu}_i$ demand faster and more precise tracking but may require higher control effort. The prescribed bound must be initially larger than the expected initial error to avoid violation.

2. Fixed-time NTSM exponents far from equilibrium: The exponents u_1/v_1 and p_1/q_1 dominate the convergence behavior when the sliding variable is far from zero. A larger ratio u_1/v_1 accelerates convergence but may induce chattering; a smaller ratio slows convergence but smooths the response. The ratio p_1/q_1 should be chosen to be greater than 1 and close to 2 for a balance between speed and smoothness.

3. Fixed-time NTSM exponents near equilibrium: The exponents u_2/v_2 and p_2/q_2 govern the convergence near the sliding manifold, and are selected to be smaller than their far-from-equilibrium counterparts to avoid excessive control input near the steady state. A value slightly above 1 ensures fixed-time convergence without singularity.

4. Adaptive TDE gain matrix: The diagonal gain matrix $\hat{M}$ is updated online based on the sliding variable magnitude. The lower and upper bounds should be set according to the expected range of the lumped uncertainty. A wider interval allows faster adaptation but may increase sensitivity to measurement noise, and a narrower interval improves robustness but may limit estimation accuracy. The adaptation rate is governed by the design parameters ψ_i and ϖ , where larger values yield faster adjustment.

5. Fixed-time NTSM gains: The diagonal matrices $\alpha_1, \beta_1, \alpha_2$, and β_2 directly affect the convergence rate of the sliding variable. Increasing any of these gains will increase the convergence rate but also increase the control torque magnitude, and may excite unmodeled dynamics. Thus, a practical rule is to start with moderate

values and gradually increase them until the desired transient response is achieved while monitoring the control input for saturation or chattering.

6. Switching-gain adaptation: Parameters σ_0 , σ_1 , and σ_2 determine the adaptation law of $\hat{\phi}$, which compensates for the residual TDE error. A higher adaptation rate with larger σ_1 , σ_2 enables faster compensation of sudden disturbances but may cause temporary overestimation. The threshold σ_0 defines the activation condition; it should be set slightly above the expected steady-state sliding variable level to avoid unnecessary adaptation.

7. Validation and iterative refinement: After initial selection, run a simulation or a low-risk experiment. If the tracking error violates the prescribed-performance bound, increase the value of each element of α_1 , β_1 or tighten the prescribed-performance parameters. If chattering is excessive, reduce the far-from-equilibrium exponents or increase the saturation boundary Ξ . If the steady-state error is too large, reduce μ_{∞} or increase the near-equilibrium gains. The process is intuitive and typically converges within a few iterations.

4 Simulation and experimental studies

The proposed control scheme is validated by simulation and experimental studies on a 6-DOF robotic manipulator. The Denavit-Hartenberg (D-H) convention and nominal inertia parameters are listed in Table 1.

4.1 Simulation results

During the simulation, the desired trajectory of the manipulator is chosen as $q_d = [0.2 \sin(0.1\pi t), 0.2 \sin(0.2\pi t), 0.1 \sin(0.4\pi t)]$,

$0.3 \sin(0.5\pi t), 0.2 \sin(0.2\pi t), 0.6 \sin(0.1\pi t)]^T$ rad, where the amplitude of the sinusoidal trajectory for each joint is selected according to its mechanical limitations. The tracking control performance is verified by conducting three scenarios: (1) the different initial conditions are applied to validate the fixed-time convergence property, where the settling time is dependent only on controller parameters regardless of the initial states; (2) the main tunable parameters of the tracking controller, including L , $\mathcal{K}_p$, u_1/v_1 , p_1/q_1 , u_2/v_2 , and q_2/p_2 , and their influence on the tracking performance, are demonstrated under time-varying disturbances; (3) comparative studies are carried out with state-of-the-art tracking controllers for robots (Jin et al., 2009; He et al., 2017; Baek et al., 2018; Hu et al., 2023) to verify the superiority of the proposed control method.

4.1.1 Tracking performance under different initial conditions

With the different initial conditions of the robotic manipulator and controller parameters set as given in Table 2, Fig. 2 shows that the tracking errors can converge to the steady state within 1.9 s for four different initial conditions. It is clear that fixed-time convergence of the proposed controller is achieved, where the settling time is determined only by the controller parameters without respect to the initial conditions of the manipulator. Note that the fastest transient response will be upper bounded by the mechanical limitations of the robotic manipulator, such as the constraints of joint velocity and driving torque.

Owing to the prescribed performance for tracking error given in inequality (8), Fig. 2 demonstrates that the tracking response is also constrained by the prescribed performance function $\delta_i \mu_i(t)$ in both the transient phase and steady-state phase. As a result, the convergence rate of the tracking error is higher than the exponential

Table 1 D-H convention and inertia parameters of the robotic manipulator

Link index	Link length (m)	Link twist (rad)	Joint offset (m)	Joint angle (rad)	Mass (kg)	Inertia (kg·m ²)
1	0.040	0	0.342	θ_1	5.167	1.167
2	0.275	$-\pi/2$	0	θ_2	4.301	2.957
3	0.025	0	0	θ_3	3.500	1.056
4	0	0	0.280	θ_4	2.333	0.245
5	0	0	0	θ_5	1.750	0.042
6	0	0	0	θ_6	0.500	0.005

Table 2 Parameters of the proposed controller during simulations

Section	Parameters
Initial case 1	$q(0) = [-\pi/15, \pi/15, -\pi/10, \pi/10, -\pi/9, \pi/9]^T$
Initial case 2	$q(0) = [-\pi/8, \pi/8, -\pi/20, \pi/20, -\pi/17, \pi/17]^T$
Initial case 3	$q(0) = [\pi/20, -\pi/20, \pi/15, -\pi/15, \pi/11, -\pi/11]^T$
Initial case 4	$q(0) = [\pi/10, -\pi/10, \pi/8, -\pi/8, \pi/7, -\pi/7]^T$
Tuned case 1	$L = 0.001$ s, $u_1/v_1 = 7/3$, $p_1/q_1 = 17/9$, $u_2/v_2 = 9/5$, $q_2/p_2 = 3/5$, $\mathcal{K} = [0.8, 0.6, 0.2, 0.1, 0.01, 0.001]^T$
Tuned case 2	$L = 0.004$ s, $u_1/v_1 = 7/3$, $p_1/q_1 = 17/9$, $u_2/v_2 = 9/5$, $q_2/p_2 = 3/5$, $\mathcal{K} = [0.8, 0.6, 0.2, 0.1, 0.01, 0.001]^T$
Tuned case 3	$L = 0.001$ s, $u_1/v_1 = 11/3$, $p_1/q_1 = 13/11$, $u_2/v_2 = 9/5$, $q_2/p_2 = 3/5$, $\mathcal{K} = [0.8, 0.6, 0.2, 0.1, 0.01, 0.001]^T$
Tuned case 4	$L = 0.001$ s, $u_1/v_1 = 7/3$, $p_1/q_1 = 17/9$, $u_2/v_2 = 31/29$, $q_2/p_2 = 27/29$, $\mathcal{K} = [0.8, 0.6, 0.2, 0.1, 0.01, 0.001]^T$
Tuned case 5	$L = 0.001$ s, $u_1/v_1 = 7/3$, $p_1/q_1 = 17/9$, $u_2/v_2 = 9/5$, $q_2/p_2 = 3/5$, $\mathcal{K} = [10.0, 8.0, 3.0, 1.0, 0.1, 0.01]^T$
Other control parameters	$\bar{M} = \text{diag}\{1, 0.8, 0.3, 0.1, 0.01, 0.001\}$, $\psi_i = 1.2$, $\varpi = 0.001$, $\sigma_0 = 0.1$, $\sigma_1 = 0.6$, $\sigma_2 = 0.8$, $\hat{\phi}(0) = 0.5$, $\delta_i = 0.5$, $\mu_{\infty} = 1$, $\mu_{\infty} = 0.02$, $\bar{\mu}_i = 0.5$, $\alpha_1 = \text{diag}\{2.5, 2.3, 2.1, 2.0, 2.0, 1.8\}$, $\beta_1 = \text{diag}\{2.3, 2.2, 2.0, 1.8, 1.8, 1.6\}$, $\alpha_2 = \text{diag}\{1.5, 1.5, 1.5, 1.4, 1.4, 1.4\}$, $\beta_2 = \text{diag}\{1.3, 1.3, 1.3, 1.2, 1.2, 1.2\}$, $\Xi = 0.01$

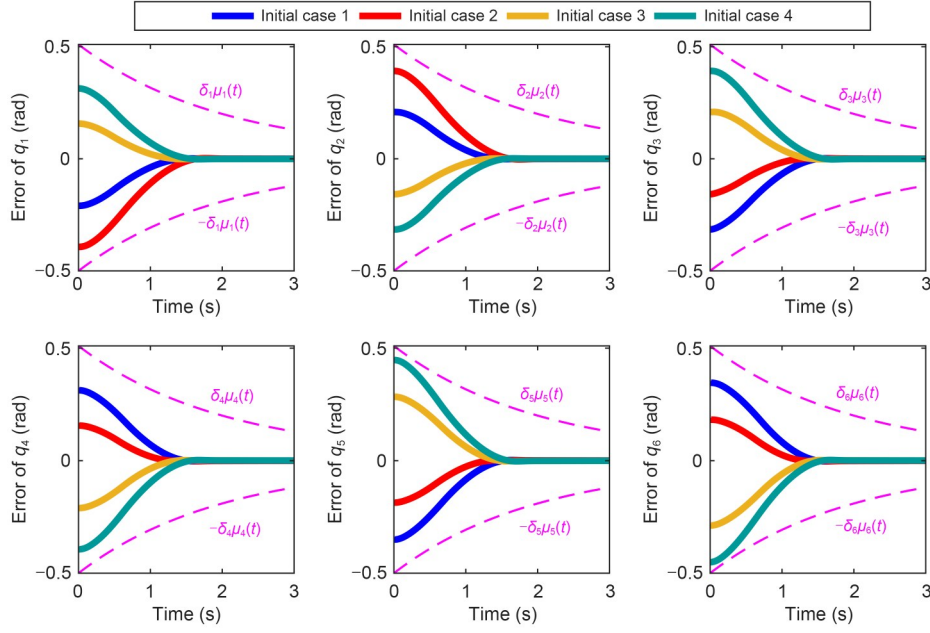


Fig. 2 Fixed-time convergence of tracking errors for the robotic manipulator with different initial conditions as given in Table 2

one, $-0.245\exp(-0.5t)$ rad/s, and the steady-state error is upper bounded within 0.01 rad. Moreover, the transient-phase maximum overshoot of the tracking error is less than 0.5 rad despite the large initial deviation of the joint configuration.

4.1.2 Tracking performance under time-varying disturbances

Typical time-varying disturbances, including periodic disturbances $[\tau_{d1}, \tau_{d2}]^T = [50 \sin(0.4\pi t), 60 \sin(0.5\pi t)]^T$ N·m, viscous frictions $[\tau_{d3}, \tau_{d4}]^T = [48\dot{q}_3, 40\dot{q}_4]^T$ N·m, and Coulomb frictions $[\tau_{d5}, \tau_{d6}]^T = [3\text{sign}(\dot{q}_5), 1.5\text{sign}(\dot{q}_6)]^T$ N·m, are added to the joints of the manipulator at 12.0 s to verify the robustness of the proposed controller. The root-mean-square error (RMSE) of tracking errors, defined as $\text{RMSE} = \sqrt{\sum_{i=1}^n (q_i - q_{di})^2 / n}$, is calculated and used to demonstrate the tracking performance of the manipulator.

As depicted in Fig. 3, a fast transient response, precise steady-state tracking, and superior antidisturbance capabilities can be achieved by the proposed controller. The main tunable parameters and their effects on the tracking performance are also shown in Fig. 3. From tuned cases 1 and 2, the delay time selected as the controller sampling period $L=0.001$ s ensures better transient and steady-state responses, compared with $L=0.004$ s. The proposed controller with large time delays cannot settle the time-varying disturbances added at 12.0 s and causes greater tracking errors. Larger u_1/v_1 , u_2/v_2 and smaller p_1/q_1 , q_2/p_2 will achieve faster convergence when far away from and close to the equilibrium point, respectively. As verified by tuned case 3, inappropriate u_1/v_1 and p_1/q_1 cause serious chattering and unstable responses. The excessive value of the gain matrix parameter $\mathcal{K}_i$ will also lead to inferior transient convergence.

Moreover, the adaptive TDE gain $\hat{\mathbf{M}}$ and switching control gain $\hat{\phi}$ of tuned case 1 are shown in Fig. 4. It is clear that the rapidly updated $\hat{\mathbf{M}}$ and $\hat{\phi}$ in the transient phase result from the excessive

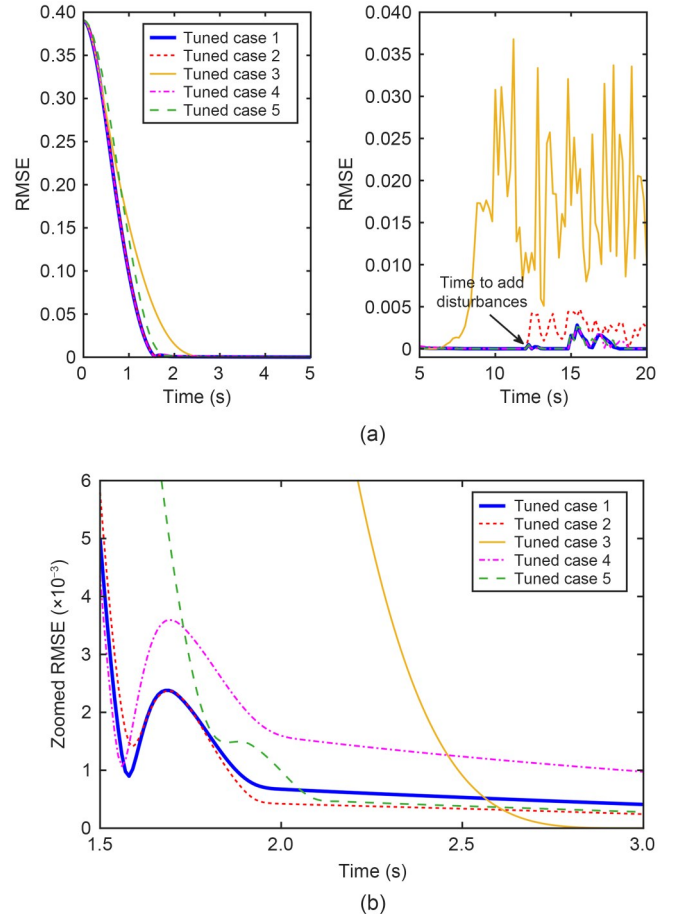


Fig. 3 RMSE of joint-space tracking errors of the robotic manipulator under different controller parameters as given in Table 2: (a) transient and steady-state RMSE; (b) zoomed RMSE

initial errors and can stabilize the tracking controller quickly, whose steady-state values under the time-varying disturbances change very little owing to robustly stabilized tracking errors. The above analyses

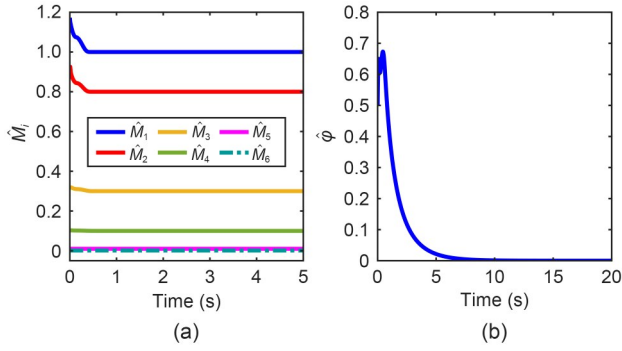


Fig. 4 Adaptive gains of tuned case 1 of the proposed controller: (a) adaptive TDE gain; (b) switching control gain

can provide a guideline for the parameter tuning and practical execution of the proposed controller.

4.1.3 Comparative studies

To validate the superiorities of the proposed controller, the NN-based adaptive fixed-time SMC (NN-SMC) (Hu et al., 2023), the representative adaptive TDE controller (ATDEC) (Baek et al., 2018), the traditional TDE-based terminal SMC (TDE-SMC) (Jin et al., 2009), and the disturbance-observer-based adaptive neural network controller (DOB-NNC) (He et al., 2017) are employed in comparative studies. The disturbances with rapidly changing amplitudes $\tau_d = [30, 30, 24, 15, 2, 0.2]^T \text{ N}\cdot\text{m}$, $10 \text{ s} \leq t < 15 \text{ s}$ and $\tau_d = [60, 60, 48, 30, 5, 0.4]^T \text{ N}\cdot\text{m}$, $t \geq 15 \text{ s}$ are also applied to all joints of the robotic manipulator.

As shown in Figs. 5 and 6, the settling time of 0.9 s of NN-SMC is slightly smaller than the 1.5 s of the proposed controller, but its steady-state tracking error is sharply greater than that of the proposed controller when the disturbances are introduced and visibly exceeds the prescribed performance. The antidisturbance capability of traditional NN-SMC is also inferior to that of the proposed controller, whose adaptive switching control gain can address time-varying disturbances effectively. Moreover, the designed controller is superior to the conventional ATDEC, in both the transient response and the steady-state tracking performance. Specifically, as shown in Fig. 5, under the same time-varying disturbances, ATDEC exhibits visibly larger steady-state tracking errors that even exceed the prescribed performance bound, whereas the proposed controller maintains tracking errors well within the prescribed bounds. This directly confirms the improved estimation performance of our adaptive TDE with a continuously updated gain matrix. As shown in Fig. 4, this improvement is achieved because the proposed adaptation law automatically increases the TDE gain during large tracking errors to compensate for uncertainties more aggressively and reduces the gain at steady state to avoid noise injection, which is a novel mechanism absent in the conventional ATDEC. Furthermore, the proposed adaptation law significantly facilitates parameter setting compared to ATDEC, which requires laborious trial-and-error tuning of many gain parameters. The proposed method needs only a small initial value, and then the gain adapts online, making the controller much more practical for real-world deployment. Conventional TDE-SMC without adaptive gain relies on a fixed large gain to maintain stability, which leads to conservative control effort and

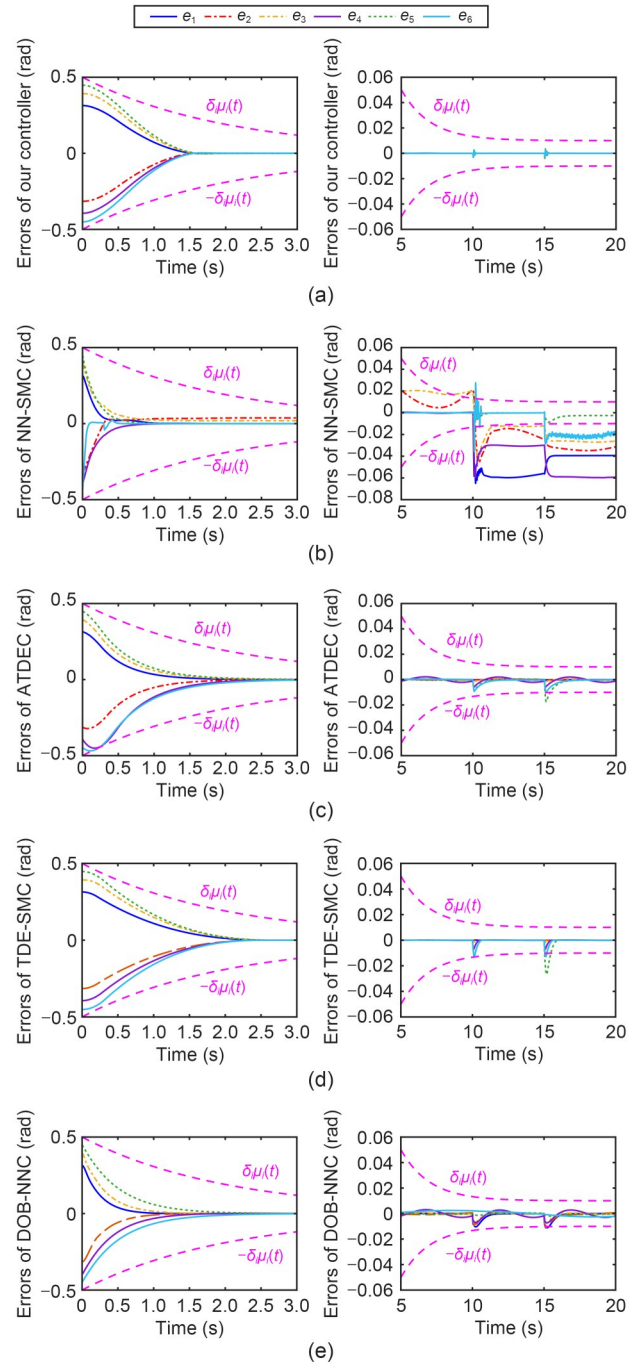


Fig. 5 Transient and steady-state tracking errors of the comparative controllers for the robotic manipulator under disturbances: (a) proposed controller; (b) NN-SMC (Hu et al., 2023); (c) ATDEC (Baek et al., 2018); (d) TDE-SMC (Jin et al., 2009); (e) DOB-NNC (He et al., 2017)

steady-state errors that exceed the prescribed performance bound. DOB-NNC exhibits the slowest convergence among all compared methods and suffers from noticeable steady-state errors.

In addition, Fig. 7 illustrates the RMSE of tracking errors for the five compared controllers under time-varying disturbances. Among all methods, NN-SMC achieves the fastest initial convergence; however, its steady-state error remains the largest throughout the operation. DOB-NNC and ATDEC exhibit similar steady-state RMSE levels, with ATDEC showing a slightly faster transient response. The conventional TDE-SMC suffers from the slowest convergence

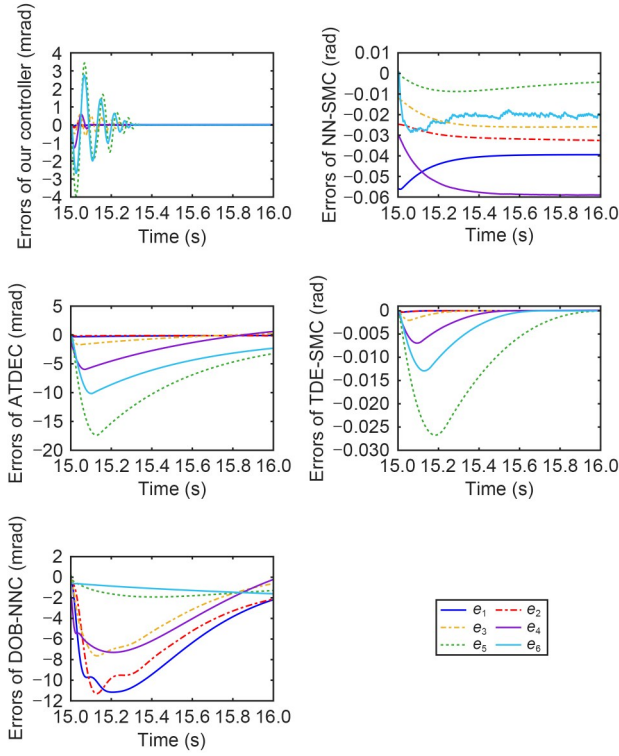


Fig. 6 Zoomed steady-state tracking errors of the comparative controllers for the robotic manipulator under disturbances

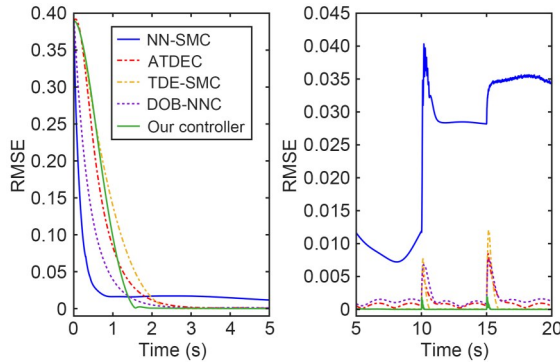


Fig. 7 RMSE of tracking errors for comparative controllers under time-varying disturbances

and pronounced error fluctuations when disturbances are injected. In contrast, the proposed controller consistently achieves both a rapid transient response and the smallest steady-state tracking error. These results confirm that only the proposed method successfully balances the tracking accuracy and response speed under dynamic uncertainties.

Table 3 shows that the proposed controller achieves the smallest steady-state RMSE 3.99×10^{-5} rad averaged over the interval $[5\text{ s}, 9\text{ s}]$ and the lowest control energy 4.38×10^7 (N·m)²s calculated by $E = \sum_{i=1}^n \int_0^T \|\tau_i\|^2 dt$ among all the controllers while maintaining a moderate settling time of 1.5 s. NN-SMC has the least settling time of 0.9 s, but its steady-state error 8.60 mrad is explicitly larger than that of the proposed method, and it consumes a significantly higher control energy of 1.19×10^8 (N·m)²s. ATDEC exhibits a larger convergence time of 2.1 s and moderate errors. TDE-SMC relies on a fixed

large gain, which results in a high control energy of 6.44×10^7 (N·m)²s and a steady-state error of 4.90×10^{-5} rad. DOB-NNC shows the largest convergence time of 2.4 s and a relatively large steady-state error of 1.20 mrad. These results confirm that the proposed controller offers the best balance among convergence speed, steady-state accuracy, and energy efficiency owing to the combination of adaptive TDE, fixed-time NTSM, and prescribed performance.

4.2 Experimental results

As illustrated in Fig. 8, the experimental platform consists of a SIASUN SN4A-4 robotic manipulator, six Minas-A5B servo drivers of Panasonic Inc., a self-developed real-time controller, and a laptop. The 6-DOF manipulator is driven by brushless direct current (DC) motors with Panasonic Inc.'s servo drivers, where the joint position is measured through a 21-bit rotary electric encoder. The servo driver at each joint is connected to the real-time controller using the EtherCAT communication protocol and the master Acontis developed by Beckhoff Inc. During the experiments, the control algorithms are executed on the real-time controller with the sampling frequency of 1000 Hz, and then the servo drivers regulate the motors to generate the controller input torques and transmit–receive joint motion states. The regulated controller parameters in the experiments are illustrated in Table 4.

4.2.1 Joint-space exciting trajectory tracking control

To validate the tracking performance of the proposed controller for a robotic manipulator under various motion states where the joint frictions and dynamic uncertainties are time-varying, the periodic exciting trajectory calculated by the fifth-order Fourier series (Huang et al., 2023) is adopted as the reference trajectory for each joint. Different initial joint-space deviations of the robotic

Table 3 Quantitative comparison of tracking performance in simulations

Controller	Settling time T_s (s)	Steady-state RMSE (rad)	Control energy E ((N·m) ² s)
Proposed	1.5	3.99×10^{-5}	4.38×10^7
NN-SMC	0.9	8.60×10^{-3}	1.19×10^8
ATDEC	2.1	5.86×10^{-4}	5.24×10^7
TDE-SMC	2.3	4.90×10^{-5}	6.44×10^7
DOB-NNC	2.4	1.20×10^{-3}	5.87×10^7

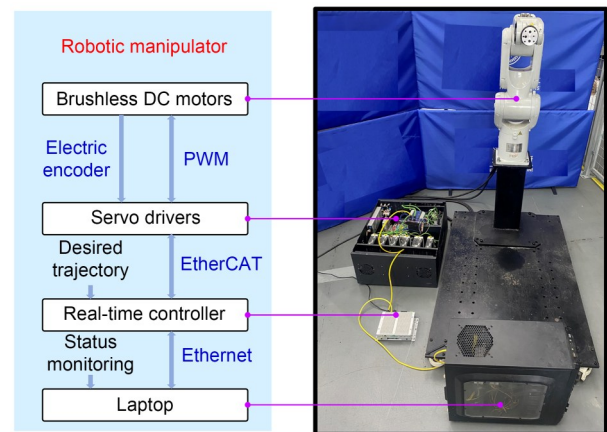


Fig. 8 Experimental platform of the robotic manipulator

Table 4 Parameters of the proposed controller in experiments

Section	Parameters
Prescribed performance	$\delta_i=0.5, \mu_{i0}=1, \mu_{i\infty}=0.04, \bar{\mu}_i=0.4$
Adaptive TDE	$\bar{M}=\text{diag}\{30, 20, 16, 10, 10, 2\},$
	$\mathcal{K}=[1, 1, 0.8, 0.8, 0.5, 0.5]^T,$
	$L=0.001 \text{ s}, \psi_i=1.2, \varpi=0.001$
Adaptive fixed-time NTSMC	$\alpha_1=\text{diag}\{50, 70, 40, 40, 36, 30\}, u_1/v_1=7/3,$
	$p_1/q_1=17/9, \beta_1=\text{diag}\{25, 25, 20, 20, 15, 12\},$
	$u_2/v_2=9/5, q_2/p_2=3/5, \hat{\phi}(0)=0.5,$
	$\alpha_2=\text{diag}\{30, 30, 30, 28, 28, 28\},$
	$\beta_2=\text{diag}\{26, 26, 26, 24, 24, 24\}, \sigma_0=0.5,$
	$\sigma_1=0.6, \sigma_2=0.8, \Xi=0.01$

manipulator are also used to verify the fixed-time convergence of the tracking error for the proposed controller in the experiment.

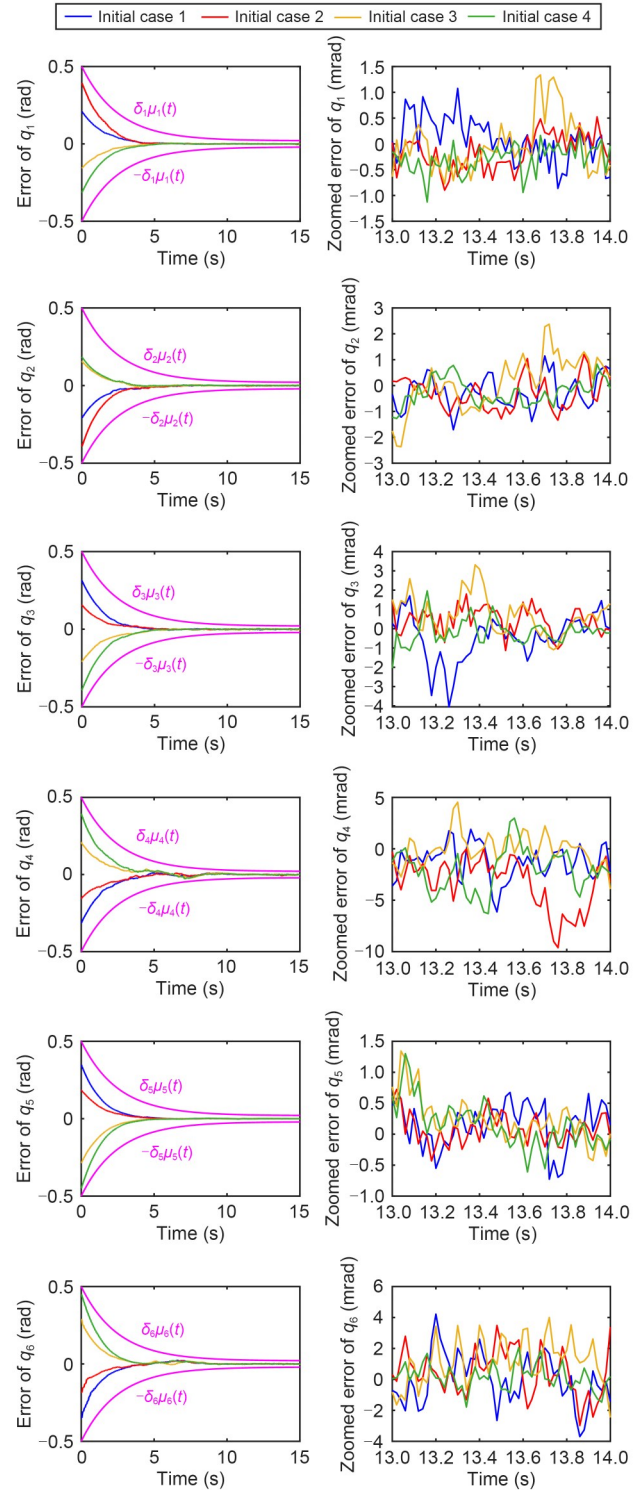
According to the tracking control performance depicted in Fig. 9, the 6-DOF joint-space tracking errors can be stabilized within 6 s, and the steady-state tracking errors are all less than 0.007 rad. The joint-space trajectory can converge to the neighborhood of the real trajectory around the same time despite the different initial conditions, which is achieved by the fixed-time convergence property of the proposed controller. Note that the convergence time of tracking error will also be bounded by the physical limitations of the robotic manipulator, such as actuator saturation. Moreover, both the transient response and steady-state performance of the tracking error are constrained by the prescribed performance in inequality (8); namely, the convergence rate is higher than that of $-0.192\exp(-0.4t)$ rad/s, and the steady-state error is less than 0.02 rad. The above experimental results verify the effectiveness of the proposed fixed-time tracking controller with prescribed performance for robotic manipulators.

Moreover, an unknown payload is added to the end-effector of the manipulator to verify the robustness of the proposed controller against payload variations. As shown in Fig. 10, the tracking errors remain within the prescribed performance bounds under both conditions, with only a slight increase in the steady-state error under the heavier load. Namely, the steady-state tracking error with an unknown payload increases slightly but remains within the prescribed bound of 0.02 rad, and the convergence time remains below 6 s. This confirms that the proposed controller is able to effectively compensate for uncertain payload dynamics with no need to retune parameters.

4.2.2 Operation task trajectory tracking control

To demonstrate the feasibility of the proposed control method in practical operation tasks of robotic manipulators, task-space end-effector trajectory tracking is carried out in the experiment. The regular circular trajectory and the complex flower-shaped trajectory are designed as the tracking references for the end-effector. Then, the joint-space tracking reference is easily computed by applying the inverse kinematics and calibrated D-H convention parameters of the robotic manipulator, which is omitted here as it is beyond the scope of this paper.

According to the operation task trajectory tracking results of the robotic manipulator given in Fig. 11, the position error of the

**Fig. 9** Joint-space exciting trajectory tracking control with different initial conditions for the robotic manipulator in the experiment

regular circular trajectory is less than 0.53 mm, and that of the complex flower-shaped trajectory is no more than 1.24 mm. In addition, the RMSEs of the joint-space tracking errors for circular and flower-shaped trajectories are upper bounded by 0.0001 and 0.0012, respectively. It is clear that the tracking errors of operation task trajectories will be driven into a bounded set at the level of 0.001 m, which can further verify the tracking accuracy and superiority of the proposed controller. The tracking control performance of the complex

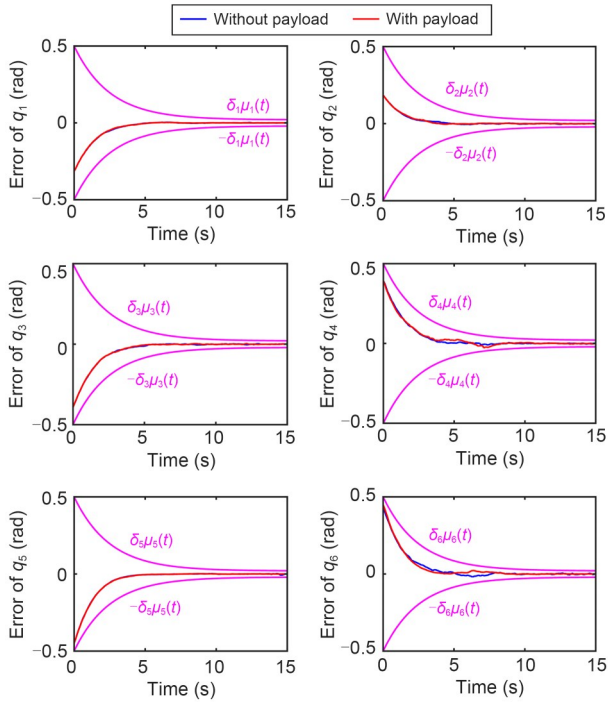


Fig. 10 Joint-space exciting trajectory tracking control with unknown payload for the robotic manipulator in the experiment

flower-shaped trajectory is slightly inferior to that of the regular circular trajectory because the sharply changed velocity of the complex trajectory and the corresponding dynamic uncertainty will affect the bounded convergence region of the proposed controller.

Fig. 12 depicts the control input torques of the robotic manipulator during the operation task trajectory tracking, which are smooth and beneficial for actuator execution. The larger variation range of the second joint τ_2 of the robotic manipulator is attributed to its role as the shoulder joint, which directly sustains the dominant gravitational and inertial loads, especially under the rapidly changing velocities of the flower trajectory. This behavior is physically expected, and the torque remains smooth and free of chattering, rather than indicating chattering or instability. Thus, the practicality and convenient execution of the proposed tracking controller are further verified.

4.2.3 Robustness of the proposed controller

The robustness of the proposed controller against varying motion speeds is tested using the circular trajectory. The baseline velocity v_0 of the manipulator end-effector is reduced by 25% and 50% and increased by 100% and 300%, resulting in four additional speed levels relative to the baseline velocity: $0.5v_0$, $0.75v_0$, $2v_0$, and $4v_0$. As the velocity changes, the joint frictions, inertial forces, and other dynamic uncertainties vary significantly, which can effectively confirm the robustness of the proposed controller.

The tracking errors for all velocity levels are depicted in Fig. 13 and summarized in Table 5. The results show that the position error remains below 1.73 mm even at 400% baseline velocity. Fig. 14 shows the maximum tracking error in each of the X , Y , and Z axes of the end-effector workspace as a function of the relative velocity. Interestingly, the tracking error does not increase monotonically with motion speed. The largest error over 1.73 mm along the X -axis

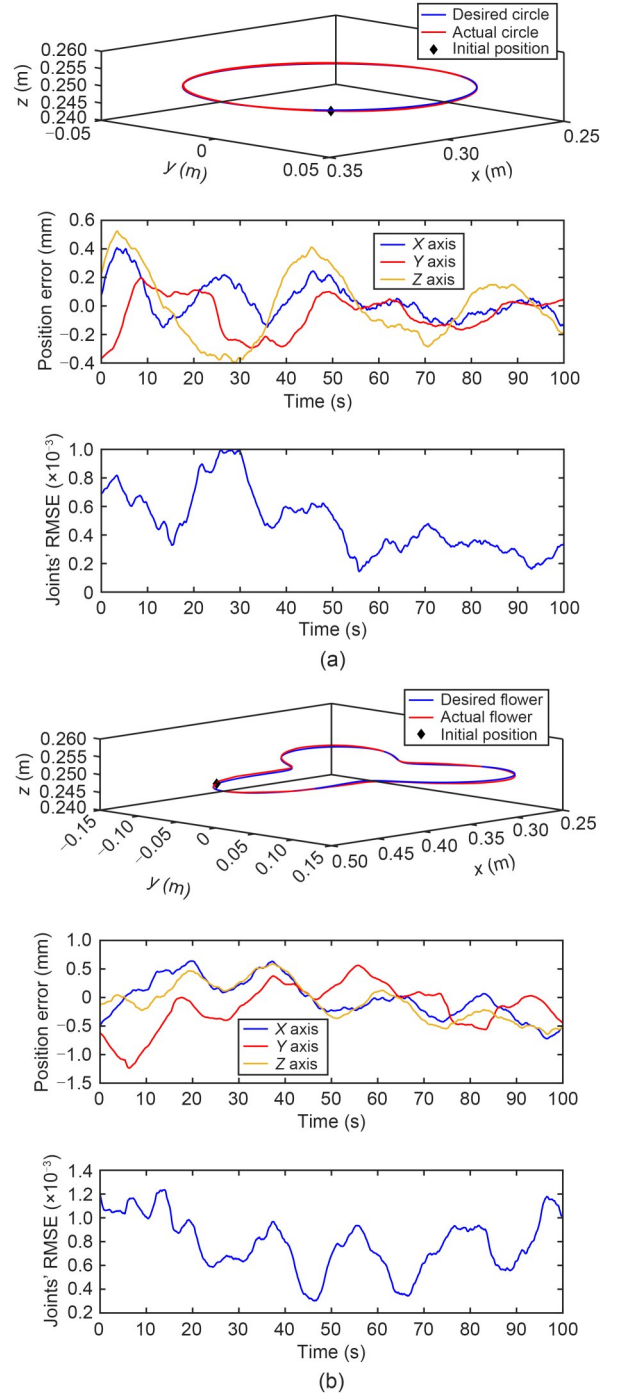


Fig. 11 Operation task trajectory tracking control of the robotic manipulator in the experiment: (a) regular circular trajectory tracking control; (b) complex flower-shaped trajectory tracking control

is observed at the middle-level speed $2v_0$. This can be explained by the fact that the frictions exhibit strong nonlinearity and become the dominant uncertain terms at low velocities, which makes it more challenging for the time-delay estimator to predict the uncertain terms accurately. As the speed increases to $4v_0$, the friction enters a more regular viscous-dominant regime, and the overall dynamic variations become more consistent, allowing the adaptive TDE and fixed-time NTSM to achieve better compensation. Consequently, the tracking error decreases at $2v_0$ and remains low at $4v_0$. At the highest speed $4v_0$, the error rises slightly compared to

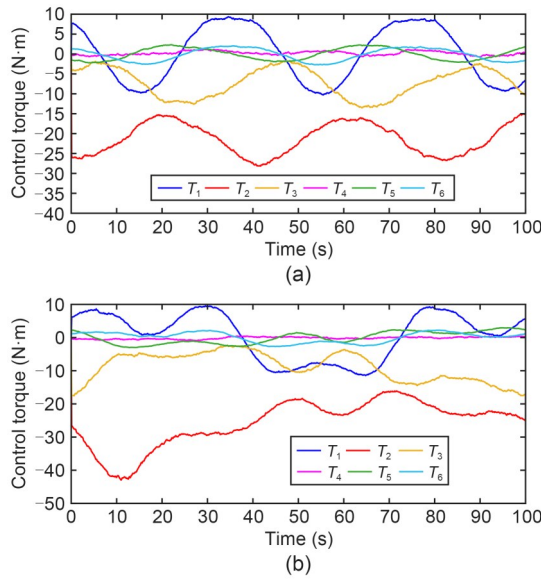


Fig. 12 Control torque of the robotic manipulator during operation task trajectory tracking in the experiment: (a) circular trajectory tracking control; (b) flower trajectory tracking control

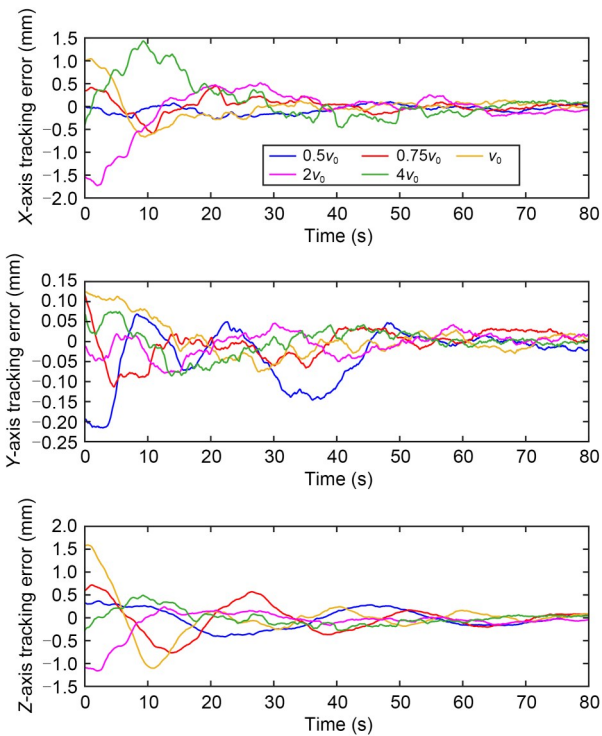


Fig. 13 Operation task circular trajectory tracking control with different motion speeds of the robotic manipulator in the experiment

Table 5 Tracking performance of the proposed controller for the circular trajectory in the X, Y, and Z axes under different motion speeds

Speed level percentage	Absolute maximum position error (mm)	Absolute mean position error (mm)
50%	[0.28, 0.22, 0.42]	[0.09, 0.05, 0.19]
75%	[0.57, 0.12, 0.77]	[0.12, 0.03, 0.24]
100%	[1.05, 0.11, 1.59]	[0.17, 0.03, 0.24]
200%	[1.73, 0.07, 1.16]	[0.30, 0.02, 0.15]
400%	[1.43, 0.08, 0.49]	[0.31, 0.02, 0.12]

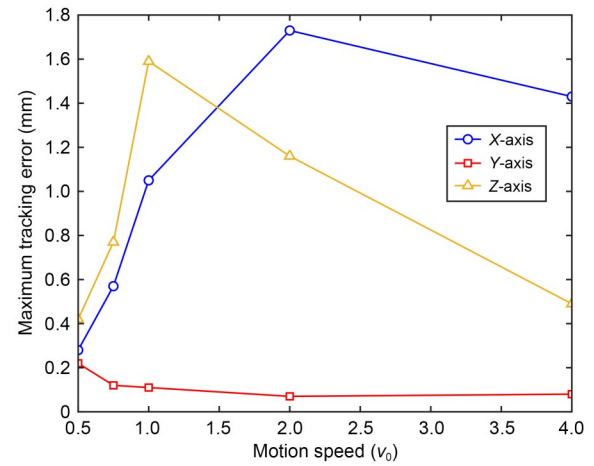


Fig. 14 Maximum tracking errors of the end-effector circular trajectory in the X, Y, and Z axes under different motion speeds

that at the baseline velocity v_0 but remains below 1.43 mm well within the prescribed performance bound.

These results highlight an important insight that the proposed controller is not only effective at high speeds but also robust against the difficult low-speed regime where uncertain frictions are most severe. The non-monotonic tracking errors confirm that the controller performance is limited more by friction modeling at low speeds than by inertial variations at high speeds. Overall, the experimental results demonstrate that the proposed controller can handle wide variations in operating speed and the associated dynamic changes with no need to retune parameters, which further validates the engineering applicability of the proposed control method across a wide range of motion speeds.

5 Conclusions

We propose an adaptive fixed-time motion controller based on adaptive TDE and prescribed performance for a robotic manipulator with dynamic uncertainties, which can effectively balance the overall control performance, computational complexity, and parameter setting. First, the model-free adaptive TDE method with a continuously updated gain matrix is devised to estimate and compensate for the lumped dynamic uncertainties of the robotic manipulator, which is computationally efficient and more applicable for onboard execution. In addition, an adaptive fixed-time tracking controller with an NTSM manifold is developed to achieve the fixed-time convergence of the tracking error for the robotic manipulator, where both transient-phase and steady-state-phase responses are constrained by the prescribed performance. Moreover, the novel adaptation law for tracking controller gain ensures the fixed-time stability of closed-loop states while eliminating the requirement for prior knowledge of dynamic uncertainties and TDE errors, which is beneficial for the practical implementation of the proposed controller. The extensive simulation and experimental results show that the proposed controller can achieve superior tracking performance of robotic manipulators despite uncertain system dynamics, various initial conditions, and complex time-varying disturbances.

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Author contributions

Xiaofei LI designed the research, processed the data, and drafted the paper. Yifei CHEN, Jiachun WU, and Chi ZHANG assisted with the experiments. Guodong LU helped organize the paper. Jin WANG and Haiyun ZHANG revised and finalized the paper.

Conflict of interest

All the authors declare that they have no conflict of interest.

Data availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Declaration on the use of generative AI tools

The authors declare that they have not used any generative AI tool during the preparation of this paper.

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