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A reversibility-gain model for integer Karhunen-Loève transform design in video coding*

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Abstract: Karhunen-Loève transform (KLT) is the optimal transform that minimizes distortion at a given bit allocation for Gaussian source. As a KLT matrix usually contains non-integers, integer-KLT design is a classical problem. In this paper, a joint reversibility-gain (R-G) model is proposed for integer-KLT design in video coding. Specifically, the 'reversibility' is modeled according to distortion analysis in using forward and inverse integer transform without quantization. It not only measures how invertible a transform is, but also bounds the distortion introduced by the non-orthonormal integer transform process. The 'gain' means transform coding gain (TCG), which is a widely used criterion for transform design in video coding. Since KLT maximizes the TCG under some assumptions, here we define the TCG loss ratio (LR) to measure how much coding gain an integer-KLT loses when compared with the original KLT. Thus, the R-G model can be explained as follows: subject to a certain TCG LR, an integer-KLT with the best reversibility is the optimal integer transform for a given non-integer-KLT. Experimental results show that the R-G model can guide the design of integer-KLTs with good performance.

Key words: Integer transform, Karhunen-Loève transform (KLT), Integer-KLT, Transform coding, Video coding
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1 Introduction

Transform plays an important role in source coding, especially for image/video coding systems and standards. It is inefficient to directly compress spatial data (i.e., image samples or inter/intra prediction residuals), because spatial data are often highly correlated and their energy tends to be evenly distributed. With a suitable transform, spatial data can be converted into a different representation, i.e., transform domain data, so that inter sample correlation of the data can be decreased or even removed. Thus, the transform coefficients are efficient to compress using scalar quantization and entropy coding.

It has been shown that Karhunen-Loève transform (KLT) is the optimal transform that minimizes distortion at a given bit allocation for a Gaussian source (Goyal, 2001). However, in the past, KLT usually played as a theoretical reference for transform design, while discrete cosine transform (DCT) (Ahmed *et al*., 1974) was widely used in video coding. This is because: (1) KLT depends highly on the characteristics of source and the characteristics of video signal are not stable but spatially or temporally changed, and (2) although video signal varies from time to time, it can be approximated as a first-order Markov process, for which DCT is very similar to KLT. Nevertheless, DCT is not as dominant as before since context-based adaptive transforms, e.g., mode-dependent directional transform (Ye and Karczewicz, 2008), rate-distortion optimized transform (Zhao *et al*., 2011), discrete sine transform (Yeo *et al*., 2011;

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Han *et al*., 2012), and secondary transform (Saxena and Fernandes, 2013), have emerged recently and showed much better rate-distortion (RD) performance than DCT in video coding. These context-based transform coding schemes have similar characteristics; i.e., they are based on KLT.

However, the basis vectors of KLTs usually consist of non-integer numbers that are not friendly for hardware implementation. In addition, encoder-decoder mismatch may occur due to different implementations of the non-integer inverse transform at the encoder and decoder, which can result in untolerated performance degradation (Hinds *et al*., 2007). To overcome the drawback of non-integer transform, integer transforms need to be designed. This leads to a problem: how to design integer-KLT from a given KLT in video coding.

Given a KLT, a large number of integer-KLTs can be obtained. To select a good integer-KLT from them, we can incorporate these integer-KLTs into a video encoder and compare compression performance. However, this wastes too much time and the results could only indicate that the chosen one has the best performance on the tested videos but not all the situations. It is necessary to develop a simple and reliable model that can be used to evaluate the performance of integer-KLTs. This is exactly the work we have done.

2 Review of KLT and the integerization

2.1 KLT

KLT is a transform that decorrelates vectors of input samples. However, it is totally dependent on the characteristics of the input signal. Let $\boldsymbol{x}$ represent a random column vector of input samples from a stationary source and $\boldsymbol{y}$ a random vector of the transform coefficients by a transform matrix $\boldsymbol{T}$. The analysis transform process can be illustrated as

$$\boldsymbol{y}=\boldsymbol{T}\boldsymbol{x}. \tag{1}$$

Let $\boldsymbol{R}_x$ be the autocorrelation matrix of the input process and $\boldsymbol{R}_{y,\boldsymbol{T}}$ the autocorrelation matrix of the transform coefficients by $\boldsymbol{T}$. Then we have

$$\boldsymbol{R}_x=E\{\boldsymbol{x}\boldsymbol{x}^{\mathrm{T}}\}, \tag{2}$$

$$\boldsymbol{R}_{y,\boldsymbol{T}}=E\{\boldsymbol{y}\boldsymbol{y}^{\mathrm{T}}\}=\boldsymbol{T}\cdot\boldsymbol{R}_x\cdot\boldsymbol{T}^{\mathrm{T}}, \tag{3}$$

where $E\{\cdot\}$ denotes the mathematical expectation. If $\boldsymbol{R}_{y,\boldsymbol{T}}$ is a diagonal matrix with eigenvalues of $\boldsymbol{R}_x$ on its main diagonal, the transform matrix $\boldsymbol{T}$ is a KLT matrix of the input signal and the row vectors of $\boldsymbol{T}$ are exactly the eigenvectors of $\boldsymbol{R}_x$.

2.2 Integerization of KLT

Generally, a KLT matrix, full of non-integers, is not very friendly for hardware implementation or standardization since non-integer operations may introduce untolerated performance degradation in videos due to encoder-decoder mismatch. Obviously, there is a natural way to derive integer-KLT as

$$\boldsymbol{A}=\mathrm{int}(\boldsymbol{T}\cdot\alpha), \tag{4}$$

where $\boldsymbol{T}$ is a KLT matrix with non-integers, α is a scalar factor, the int($\cdot$) operator denotes the integerization process, and $\boldsymbol{A}$ is a corresponding derived integer-KLT matrix. This derivation can be described in two steps: scaling and integerization. In the scaling process, a KLT matrix is multiplied by a scalar factor α to obtain a scaled transform matrix. In the integerization process, an integer is selected as an approximation for each element in the scaled transform matrix. The integerization process can be mathematical rounding, floor, ceiling, or other operations.

The choice of α depends on the limitation of complexity. A higher value of α could make $\boldsymbol{A}$ more approximate to $\boldsymbol{T}$, but it increases the implementation complexity of the transform process. In this study, we focus on the derivation of $\boldsymbol{A}$ for a given $\boldsymbol{T}$ and α.

2.2.1 Integer-KLTs with mathematical rounding

There is no doubt that rounding each element in the scaled transform matrix to the corresponding nearest integer is a simple and natural method, as shown in Eq. (5). The rounding method has been widely used in integer DCT design for video coding, such as the 4×4 integer DCT in the draft of H.26L (Wiegand, 2001) and the 8×8 integer DCT design proposed by Zhang *et al*. (2006):

$$A(i,j)=\mathrm{round}(T(i,j)\cdot\alpha). \tag{5}$$

2.2.2 Integer-KLTs with TCG

The TCG (Jayant and Noll, 1984) is a widely used figure of merit to measure the performance of a transform, and it has been serving as a useful guide in designing the integer transform in video coding. For a transform $\boldsymbol{A}$, TCG is defined as

$$\mathrm{TCG}(\boldsymbol{A}) = 10\lg \frac{\frac{1}{N}\cdot\sum_{i=0}^{N-1} R_{y,A}(i,i)}{\sqrt[N]{\prod_{i=0}^{N-1} R_{y,A}(i,i)}}, \tag{6}$$

where N is the transform size and $\boldsymbol{R}_{y,A}$ is the autocorrelation matrix of transform coefficients by $\boldsymbol{A}$.

Since the integerization method in Eq. (1) can be mathematical rounding, floor, ceiling, or other operations, for a given $\boldsymbol{T}$ and α, we can derive a large number of integer-KLTs as candidates and each candidate transform has its own TCG value. By using TCG as a measurement, integer-KLT with the highest TCG value will be chosen as the best one. That is,

$$\boldsymbol{A} = \arg\max_{\boldsymbol{C}\in\Phi} \{\mathrm{TCG}(\boldsymbol{C})\}, \tag{7}$$

where Φ is the candidate set of integer-KLTs and $\boldsymbol{C}$ is a sample in the set.

3 Design of integer-KLTs with the R-G model

In the previous section, we have discussed two effective methods to derive an integer-KLT: mathematical rounding and maximizing TCG. However, they could still be improved by considering more characteristics of KLT. Although depending highly on the statistics of the input signal, the KLT matrices have some common properties:

1. Orthogonality: the basis vectors of $\boldsymbol{T}$ are orthogonal, i.e., $\boldsymbol{t}_i^{\mathrm{T}}\boldsymbol{t}_j=0$ for $i\neq j$, where $\boldsymbol{t}_i$ is the ith basis vector of $\boldsymbol{T}$.

2. Normality: the basis vectors of $\boldsymbol{T}$ have equal norm, i.e., $\boldsymbol{t}_i^{\mathrm{T}}\boldsymbol{t}_i=\boldsymbol{t}_j^{\mathrm{T}}\boldsymbol{t}_j$ for $i, j=0, 1, \ldots, N-1$. It is desirable for simplifying the quantization process by eliminating the scaling matrix in quantization.

3. Maximizing TCG: the basis vectors of $\boldsymbol{T}$ have been shown to provide the best energy compaction or the best TCG, which is desirable for compression efficiency.

The first two properties can also be named orthonormality, which makes KLT an orthonormal transform that satisfies $\boldsymbol{T}^{-1}=\boldsymbol{T}^{\mathrm{T}}$, which is desirable for a simple synthesis transform. The synthesis transform is

$$\hat{\boldsymbol{x}} = \boldsymbol{T}^{-1}\cdot\hat{\boldsymbol{y}} = \boldsymbol{T}^{\mathrm{T}}\cdot\hat{\boldsymbol{y}}, \tag{8}$$

where $\hat{\boldsymbol{x}}$ is the reconstructed signal and $\hat{\boldsymbol{y}}$ the quantization of $\boldsymbol{y}$. In the following, we would discuss how to embed these good properties into the design of integer-KLTs.

3.1 Reversibility of a transform

From the analysis transform in Eq. (1) and synthesis transform in Eq. (8), we can see that, if no quantization error is introduced (meaning $\hat{\boldsymbol{y}}=\boldsymbol{y}$), a signal could be perfectly reconstructed by KLTs (meaning $\hat{\boldsymbol{x}}=\boldsymbol{x}$), since KLT is orthonormal. We can say that KLT is reversible. However, this lossless reconstruction could not be held by integer-KLTs, if basis vectors of integer-KLTs are not orthogonal or have different norms. It indicates that the reconstruction error is highly associated with the orthogonality and normality of a transform. Unfortunately, we have no idea whether orthogonality is more important than normality or not. To solve this, we should first study the relationship between the reconstruction error and the integer-KLT matrix.

In a practical transform coding system, the analysis and synthesis transforms with an integer-KLT can be described as follows:

$$\boldsymbol{y}=\boldsymbol{A}\boldsymbol{x}/\alpha, \tag{9}$$

$$\hat{\boldsymbol{x}} = \boldsymbol{A}^{\mathrm{T}}\hat{\boldsymbol{y}}/\alpha. \tag{10}$$

Assuming $\hat{\boldsymbol{y}}=\boldsymbol{y}$ and ignoring the round-off error of the division, the reconstruction error vector $\boldsymbol{e}$ is

$$\boldsymbol{e} = \boldsymbol{x}-\hat{\boldsymbol{x}} = \boldsymbol{x}-\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A}\boldsymbol{x}/\alpha^2 = \boldsymbol{U}\boldsymbol{x}, \tag{11}$$

where

$$\boldsymbol{U}=(\alpha^2\boldsymbol{I}-\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A})/\alpha^2, \tag{12}$$

in which $\boldsymbol{I}$ is an identity matrix.

That is, without quantization, the reconstruction error can be analytically described by the transform matrix and the input signal vector. A similar conclusion was derived in the integer DCT design (Dong *et al.*, 2009). Using the Euclidean norm (denoted as $\|\cdot\|$) to measure the distortion D, statistically we have

$$D=E\{\|\boldsymbol{e}\|\}=E\{\|\boldsymbol{U}\boldsymbol{x}\|\}. \tag{13}$$

Using the property of induced norm, we can obtain the upper bound of the distortion:

$$D\leq\|\boldsymbol{U}\|\cdot E\{\|\boldsymbol{x}\|\}=c\cdot\|\boldsymbol{U}\|. \tag{14}$$

Since the input signal $\boldsymbol{x}$ is a random vector that obeys a certain distribution, its Euclidean norm can be treated as a random variable and the corresponding expectation is a constant, denoted as c.

By now we have demonstrated how the integer-KLT matrix affects the reconstruction distortion for the case without quantization. As given in Eq. (14), the reconstruction distortion is bounded by matrix $\boldsymbol{U}$. Accordingly, we define the reversibility (denoted as symbol R) of an integer-KLT $\boldsymbol{A}$ as the norm of $\boldsymbol{U}$, which is shown as follows:

$$R(\boldsymbol{A})=\|\boldsymbol{U}\|. \tag{15}$$

Obviously, in the design of integer-KLTs, we would like the value of R to be as small as possible, which can make the integer-KLTs more orthonormal. Therefore, the normality and orthogonality of KLTs can be well retained.

3.2 TCG loss ratio of a transform

KLT maximizes the rate distortion performance for stationary Gaussian sources if scalar quantizers for transform coefficients are optimized (Goyal, 2000). In other words, TCG can be maximized by KLT. Thus, TCG has been working as a criterion in integer cosine transform (ICT) design in video coding. Note that the definition of TCG comes from high rate coding for Gaussian sources. However, in practical video coding systems, the prediction residue to be transformed is not a Gaussian source but obeys the Laplace distribution (Chen, 2012), and the coding rate ranges from low to high. On one side, the TCG of a transform should be high; on the other side, a transform with the highest TCG may not be the best transform that maximizes the rate distortion performance for a practical video coding system.

Therefore, we define the TCG loss ratio (LR) to indicate how much coding gain is lost from the integerization process when compared with the original KLT:

$$\mathrm{LR}(\boldsymbol{A})=\frac{\mathrm{TCG}(\boldsymbol{T})-\mathrm{TCG}(\boldsymbol{A})}{\mathrm{TCG}(\boldsymbol{T})}\times 100\%. \tag{16}$$

Based on the definition of LR, we regard the integer-KLTs that satisfy inequality (17) as the ones that have excellent rate distortion performance close to the original KLT:

$$0\leq\mathrm{LR}(\boldsymbol{A})\leq\eta, \tag{17}$$

where η is a threshold to control TCG($\boldsymbol{A}$) within a left neighborhood of TCG($\boldsymbol{T}$). Different from Eq. (7), which selects the integer-KLT with the closest TCG to that of the original KLT, inequality (17) defines a group of integer-KLTs with TCG close enough to that of the original KLT.

It is difficult to derive a proper value for η mathematically. Fortunately, previous studies on ICT design could be used as a reference to find a suitable value. Table 1 illustrates the LR of the 4×4 ICT in H.264/AVC, for a stationary Gauss-Markov input with correlation coefficient ρ ranging from 0.5 to 0.9, which simulates the characteristic of prediction residue in video coding. We can find that the value of LR ranges from 0.62% to 3.57%. As a result, a value less than 3.57% for η can ensure high enough TCG for a 4×4 integer-KLT.

Table 1 Loss ratio of the 4×4 ICT in H.264/AVC

ρ	TCG (dB)		LR (ICT)
	KLT	ICT	
0.5	0.937	0.904	3.57%
0.6	1.454	1.415	2.67%
0.7	2.193	2.152	1.88%
0.8	3.328	3.288	1.19%
0.9	5.409	5.376	0.62%

ICT: integer cosine transform; KLT: Karhunen-Loève transform

3.3 R-G model for designing integer-KLT

So far, we have used reversibility to measure the orthonormality of a transform, and have a constraint on the LR to keep a high TCG for the transform.

Naturally, we combine the two items and obtain the R-G model for designing integer-KLT as follows:

$$\boldsymbol{A}=\arg\min_{\boldsymbol{C}\in\Phi}\{R(\boldsymbol{C})\}\ \text{ s.t. } 0\le \mathrm{LR}(\boldsymbol{C})\le\eta. \quad (18)$$

Eq. (18) seems to be an optimization problem. However, the relationship between the LR and reversibility acts randomly. According to our observation, in a candidate set of integer-KLTs for a given KLT, with the decrease of LR, the value of R is not monotonous, which means the problem is not a convex optimization problem. Therefore, we use an enumeration method to solve Eq. (18). Given a KLT matrix $\boldsymbol{T}$ and a scalar factor α, the derivation method for an integer-KLT according to the R-G model has the following four steps:

Step 1: Generate the rounding integer-KLT matrix $\boldsymbol{A}_\mathrm{r}$ using Eq. (5).

Step 2: For each element $A_\mathrm{r}(i, j)$ in $\boldsymbol{A}_\mathrm{r}$, an integer set $S_{ij}=\{b_{ij}|A_\mathrm{r}(i, j)-k\le b_{ij}\le A_\mathrm{r}(i, j)+k\}$ is built, where k is an integer that specifies the search range, $i, j=0, 1, \ldots, N-1$. There are $N\times N$ integer sets, and we can obtain an integer-KLT candidate by picking up $N\times N$ elements from these integer sets (one element from one set). Totally $(2k+1)^{N\times N}$ integer-KLT candidates can be obtained which form a candidate set Φ.

Step 3: For each sample in Φ, the value of LR is calculated. All the integer-KLTs with $0\le\mathrm{LR}\le\eta$ are reserved and form another set, denoted as Ψ.

Step 4: For each integer-KLT in Ψ, the value of R is calculated, and the one with the minimal value is the target integer-KLT.

During the derivation process, the value of k is usually small to reduce the complexity of the derivation method, and different values are set to η for different transform sizes, e.g., 0.62% for the 4×4 transform. In step 4, if there is more than one target integer-KLT with the minimal value of R, we select the one with the highest value of TCG among these target ones as the final derived integer-KLT.

4 Performance and analysis

In this section, we design integer-KLTs to verify the effectiveness of the R-G model using the mode-dependent directional transform (MDDT), since the transform matrices used in MDDT are KLTs. Before starting the experiments, we briefly describe intra coding in the H.264/AVC standard and MDDT.

4.1 Mode-dependent directional transform

In intra coding of H.264/AVC, an image block is first predicted by spatially neighboring pixels through a certain intra prediction mode and then the resulting residual block is transform coded by a 2D transform with column and row ICTs. Fig. 1a illustrates the eight directional intra modes for 4×4 blocks. Figs. 1b and 1c give the prediction processes of mode 0 (vertical mode: using top neighbors to predict the current block) and mode 1 (horizontal mode: using left neighbors to predict the current block), respectively.

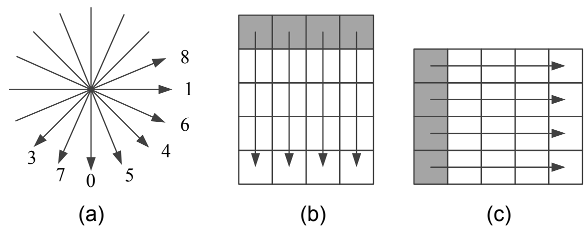

Fig. 1 Intra prediction modes for a 4×4 block in H.264/AVC
(a) Eight directional modes; (b) Vertical mode; (c) Horizontal mode

Later, some researchers found that residual blocks still contain significant directional information (Ye and Karczewicz, 2008); i.e., the residual amplitudes tend to increase along the prediction direction as the distance from the reference samples increases. MDDT exactly uses this property to maximally compact residual signal energy. Generally, MDDT replaces ICT by a pair of KLTs for residue blocks according to the prediction modes. All KLTs come from singular value decomposition on a training set of residuals and then are integerized. Although the characteristics differ from sequences, the property of directional information in residual blocks is similar, which makes KLTs in MDDT be offline trained. Many publications show that MDDT further removes correlation in residues and outperforms the ICT-based transform coding scheme in video compression (Ye and Karczewicz, 2008; Yeo *et al.*, 2011).

4.2 Comparison of integer-KLTs with different integerization methods

In this part, we follow the MDDT design to generate 16 float KLT matrices for the eight directional modes (two for each mode) of 4×4 blocks. In the integerization process, α is set to be 128. Besides the group of integer-KLTs generated with the R-G model (denoted as $\boldsymbol{A}_{\text{R-G}}$), integer-KLTs are generated with the following two methods:

1. Integerization with rounding: in this group, 16 integer-KLTs are derived according to Eq. (5), denoted as $\boldsymbol{A}_{\text{r}}$.

2. Integerization with maximal TCG: in this group, 16 integer-KLTs are derived according to Eq. (7), denoted as $\boldsymbol{A}_{\text{g}}$. The integer-KLTs' candidate set Φ is the same as the one used in integerization with the R-G model.

Figs. 2 and 3 list the column and row integer-KLTs, respectively, with different integerization methods for mode 0. $\boldsymbol{A}_{\text{g}}$ or $\boldsymbol{A}_{\text{R-G}}$ can be seen as optimization based on $\boldsymbol{A}_{\text{r}}$ with some element modification according to the corresponding criterion.

The LR and reversibility of column and row integer-KLTs for all directional modes are shown in Tables 2 and 3, respectively, from which we obtain:

For the LR, LR($\boldsymbol{A}_{\text{g}}$)<LR($\boldsymbol{A}_{\text{r}}$)<LR($\boldsymbol{A}_{\text{R-G}}$)<0.2%. This indicates that all the integer-KLTs have high TCG that is very close to that of KLTs.

For reversibility, $R(\boldsymbol{A}_{\text{R-G}})<R(\boldsymbol{A}_{\text{r}})<R(\boldsymbol{A}_{\text{g}})$ holds for all matrices. This means that $\boldsymbol{A}_{\text{R-G}}$ results in a lower bound of distortion than the other two.

$$\begin{bmatrix} 0.450 & 0.534 & 0.535 & 0.475 \\ -0.682 & -0.279 & 0.326 & 0.593 \\ -0.513 & 0.535 & 0.385 & 0.550 \\ 0.264 & -0.592 & 0.678 & -0.348 \end{bmatrix} \quad \begin{bmatrix} 57.64 & 68.40 & 69.50 & 60.76 \\ -87.25 & -35.68 & 41.72 & 75.89 \\ -65.67 & 68.52 & 49.25 & -70.36 \\ 33.73 & -75.75 & 86.75 & -44.53 \end{bmatrix}$$

(a) (b)

$$\begin{bmatrix} 58 & 68 & 69 & 61 \\ -87 & -36 & 42 & 76 \\ -66 & 69 & 49 & -70 \\ 34 & -76 & 87 & -45 \end{bmatrix} \quad \begin{bmatrix} 58 & 68 & 69 & 61 \\ -87 & -35 & 41 & 76 \\ -67 & 68 & 49 & -71 \\ 35 & -76 & 86 & -45 \end{bmatrix} \quad \begin{bmatrix} 57 & 68 & 70 & 60 \\ -87 & -37 & 41 & 76 \\ -67 & 68 & 48 & -70 \\ 33 & -76 & 86 & -46 \end{bmatrix}$$

(c) (d) (e)

Fig. 2 Column integer-KLTs for vertical prediction mode
(a) Float KLT matrix; (b) Scaled KLT matrix with α=128; (c) $\boldsymbol{A}_{\text{r}}$; (d) $\boldsymbol{A}_{\text{g}}$; (e) $\boldsymbol{A}_{\text{R-G}}$

The LR and reversibility cannot directly reflect the RD performance of transform in video coding. To show this, we implement MDDT with the three groups of integer-KLTs into KTA-1.9. KTA-1.9 is a widely used study platform for video coding. Since MDDT is used for intra coding, we test only all intra coding cases, and the test condition is as follows:

8×8 transform: off, since here we compare only the 4×4 transform.

Quantization parameter (QP) setting: high bitrate, QP=4, 7, 10, 13; median bitrate, QP=22, 27, 32, 37.

$$\begin{bmatrix} 0.322 & 0.538 & 0.592 & 0.506 \\ -0.741 & -0.351 & 0.311 & 0.481 \\ -0.557 & 0.623 & 0.180 & -0.519 \\ 0.194 & -0.446 & 0.721 & -0.493 \end{bmatrix} \quad \begin{bmatrix} 41.26 & 68.90 & 75.77 & 64.75 \\ -94.83 & -44.88 & 39.82 & 61.58 \\ -71.24 & 79.78 & 23.02 & -66.44 \\ 24.80 & -57.08 & 92.34 & -63.12 \end{bmatrix}$$

(a) (b)

$$\begin{bmatrix} 41 & 69 & 76 & 65 \\ -95 & -45 & 40 & 62 \\ -71 & 80 & 23 & -66 \\ 25 & -57 & 92 & -63 \end{bmatrix} \quad \begin{bmatrix} 41 & 69 & 76 & 65 \\ -96 & -44 & 39 & 63 \\ -72 & 79 & 24 & -67 \\ 25 & -57 & 92 & -62 \end{bmatrix} \quad \begin{bmatrix} 42 & 70 & 75 & 64 \\ -94 & -45 & 41 & 62 \\ -72 & 79 & 23 & -67 \\ 26 & -57 & 92 & -63 \end{bmatrix}$$

(c) (d) (e)

Fig. 3 Row integer-KLTs for vertical prediction mode
(a) Float KLT matrix; (b) Scaled KLT matrix with α=128; (c) $\boldsymbol{A}_{\text{r}}$; (d) $\boldsymbol{A}_{\text{g}}$; (e) $\boldsymbol{A}_{\text{R-G}}$

Table 2 Loss ratio and reversibility of column integer-KLTs for all directional modes

Mode	LR (%)			R		
	$\boldsymbol{A}_{\text{r}}$	$\boldsymbol{A}_{\text{g}}$	$\boldsymbol{A}_{\text{R-G}}$	$\boldsymbol{A}_{\text{r}}$	$\boldsymbol{A}_{\text{g}}$	$\boldsymbol{A}_{\text{R-G}}$
0	0.00	0.00	0.14	0.0134	0.0253	0.0079
1	0.00	0.00	0.02	0.0121	0.0251	0.0058
3	0.00	0.00	0.07	0.0118	0.0323	0.0089
4	0.01	0.00	0.06	0.0124	0.0329	0.0056
5	0.00	0.00	0.09	0.0120	0.0274	0.0092
6	0.01	0.00	0.03	0.0118	0.0266	0.0060
7	0.01	0.00	0.03	0.0157	0.0312	0.0074
8	0.01	0.00	0.03	0.0110	0.0207	0.0076

Table 3 Loss ratio and reversibility of row integer-KLTs for all directional modes

Mode	LR (%)			R		
	$\boldsymbol{A}_{\text{r}}$	$\boldsymbol{A}_{\text{g}}$	$\boldsymbol{A}_{\text{R-G}}$	$\boldsymbol{A}_{\text{r}}$	$\boldsymbol{A}_{\text{g}}$	$\boldsymbol{A}_{\text{R-G}}$
0	0.00	0.00	0.07	0.0102	0.0244	0.0065
1	0.01	0.00	0.05	0.0110	0.0307	0.0053
3	0.00	0.00	0.08	0.0075	0.0278	0.0065
4	0.02	0.00	0.11	0.0115	0.0228	0.0062
5	0.01	0.00	0.07	0.0081	0.0307	0.0065
6	0.00	0.00	0.02	0.0105	0.0261	0.0057
7	0.01	0.00	0.18	0.0107	0.0287	0.0056
8	0.01	0.00	0.13	0.0140	0.0238	0.0050

Others: default as in the configuration file.

To evaluate the RD performance, we use the Bjontegaard delta (BD) bitrate (Bjontegaard, 2001), which means that the bitrate increases compared with anchor at the same peak signal-to-noise ratio (PSNR). Since mathematical rounding is the simplest integerization method, we set MDDT with $\boldsymbol{A}_{\mathrm{r}}$ as the anchor. The test video sequences are listed in Table 4, and RD performance in Table 5.

Table 4 Description of test video sequences

Sequence	Sequence number	Resolution	Frame rate (frame/s)	Duration (s)
Carphone	0	176×144	30	10
Foreman	1	176×144	30	10
Football	2	352×288	30	10
News	3	352×288	30	10
Mobile	4	352×288	30	10
BigShips	5	1280×720	60	10
Crew	6	1280×720	60	10
Night	7	1280×720	60	10
City	8	1280×720	60	10
ShuttleStart	9	1280×720	60	10

Table 5 BD-rates of $\boldsymbol{A}_{\mathrm{g}}$ and $\boldsymbol{A}_{\mathrm{R\text{-}G}}$ when compared with $\boldsymbol{A}_{\mathrm{r}}$

Sequence number	High bitrate (%)		Median bitrate (%)	
	$\boldsymbol{A}_{\mathrm{g}}$	$\boldsymbol{A}_{\mathrm{R\text{-}G}}$	$\boldsymbol{A}_{\mathrm{g}}$	$\boldsymbol{A}_{\mathrm{R\text{-}G}}$
0	2.74	−0.78	−0.06	0.03
1	3.60	−0.92	0.02	−0.11
2	1.98	−0.35	0.03	−0.08
3	1.80	−0.77	0.08	−0.05
4	2.30	−1.03	0.29	−0.11
5	2.19	−0.32	0.11	0.06
6	1.00	−0.25	0.00	0.02
7	2.23	−0.72	0.05	0.04
8	3.05	−0.91	0.05	0.01
9	0.83	−0.25	−0.06	−0.07
Average	2.17	−0.63	0.05	−0.02

When comparing $\boldsymbol{A}_{\mathrm{g}}$ with $\boldsymbol{A}_{\mathrm{r}}$ at the same PSNR, $\boldsymbol{A}_{\mathrm{g}}$ will introduce an increase of about 2.17% bitrate on average, while $\boldsymbol{A}_{\mathrm{R\text{-}G}}$ can save 0.63% bitrate in the high bitrate condition. High bitrate corresponds to low QP or low quantization distortion, and for this case, distortion due to the non-orthonormal integer transform becomes important. Thus, the transform with the best reversibility, i.e., $\boldsymbol{A}_{\mathrm{R\text{-}G}}$, shows the best RD performance.

At the median bitrate, simulation results indicate that $\boldsymbol{A}_{\mathrm{R\text{-}G}}$ has about 0.02% bitrate saving and $\boldsymbol{A}_{\mathrm{g}}$ has about 0.05% bitrate increase on average when compared with $\boldsymbol{A}_{\mathrm{r}}$. In fact, it is fair to say that the three groups show similar RD performance. On one side, the BD-bitrate of $\boldsymbol{A}_{\mathrm{R\text{-}G}}$ (or $\boldsymbol{A}_{\mathrm{g}}$) is negative on some sequences and positive on others, meaning that $\boldsymbol{A}_{\mathrm{R\text{-}G}}$ (or $\boldsymbol{A}_{\mathrm{g}}$) does not outperform $\boldsymbol{A}_{\mathrm{r}}$ on all the test videos. On the other side, the BD-bitrate fluctuations are almost within 0.1%, which can be ignored in RD performance comparison. In this case, since quantization distortion dominates the total distortion and the distortion due to the non-orthonormal integer transform becomes less important, the advantage of $\boldsymbol{A}_{\mathrm{R\text{-}G}}$ with the best reversibility is not as remarkable as that in the high bitrate case.

4.3 Discussion

Based on 16 KLTs, we have derived three groups of integer-KLTs by different integerization methods and then compared the three groups through TCG, reversibility, and RD performance.

The TCGs of integer-KLTs from the three groups are all very close to the corresponding KLTs, which indicates that all the integerization methods well retain the third property mentioned in Section 3.

Since only integerization with the R-G model considers the reversibility of a transform, $\boldsymbol{A}_{\mathrm{R\text{-}G}}$ undoubtedly owns the best value of R. As we have mentioned in Section 3.1, better reversibility leads to better orthonormality. This is proved by Table 6, where normality (norm) and orthogonality (orth) are defined as

$$\mathrm{norm}(\boldsymbol{A})=\sum_{i=0}^{N-1}|Q(i,i)-1|, \tag{19}$$

$$\mathrm{orth}(\boldsymbol{A})=\sum_{i=0}^{N-1}\sum_{j=0,\,j\neq i}^{N-1}|Q(i,j)|, \tag{20}$$

where

$$\boldsymbol{Q}=\boldsymbol{A}\boldsymbol{A}^{\mathrm{T}}/\alpha^2. \tag{21}$$

Almost all the integer-KLTs in the $\boldsymbol{A}_{\mathrm{R\text{-}G}}$ group have the best normality and orthogonality, with few

Table 6 Normality and orthogonality of A_r, A_g, and $A_{R\text{-}G}$ for all directional modes

Mode*	norm			orth			Mode**	norm			orth		
	A_r	A_g	$A_{R\text{-}G}$	A_r	A_g	$A_{R\text{-}G}$		A_r	A_g	$A_{R\text{-}G}$	A_r	A_g	$A_{R\text{-}G}$
0	0.0173	0.0261	0.0059	0.0266	0.0912	0.0251	0	0.0186	0.0381	0.0054	0.0332	0.0724	0.0238
1	0.0217	0.0153	0.0092	0.0322	0.1100	0.0137	1	0.0191	0.0386	0.0038	0.0205	0.0758	0.0172
3	0.0226	0.0553	0.0058	**0.0277**	0.0907	**0.0374**	3	0.0173	0.0335	0.0051	**0.0164**	0.0679	**0.0270**
4	0.0278	0.0621	0.0040	0.0409	0.0859	0.0220	4	0.0172	0.0245	0.0081	0.0337	0.0831	0.0200
5	0.0132	0.0463	0.0032	**0.0333**	0.0833	**0.0338**	5	0.0138	0.0276	0.0087	0.0228	0.1050	0.0177
6	**0.0066**	0.0187	**0.0078**	0.0461	0.1263	0.0153	6	0.0119	0.0281	0.0081	0.0345	0.0743	0.0134
7	0.0119	0.0291	0.0080	0.0208	0.0978	0.0126	7	0.0134	0.0397	0.0062	0.0352	0.0856	0.0187
8	0.0117	0.0273	0.0050	0.0394	0.0677	0.0193	8	0.0192	0.0381	0.0056	0.0374	0.0640	0.0167

* Column integer-KLTs; ** row integer-KLTs

exceptions (in bold in Table 6). In these few cases, A_r owns the best normality (or orthogonality), but its orthogonality (or normality) is much larger than that of $A_{R\text{-}G}$. Take the column transform of mode 6 as an example. The normality of A_r is 0.0066, which is a little better than that of $A_{R\text{-}G}$. However, $A_{R\text{-}G}$ has much better orthogonality, i.e., 0.0153 vs. 0.0461.

In summary, $A_{R\text{-}G}$ has the best orthonormality followed by A_r and then A_g. This is consistent with the order of reversibility of the three groups.

Generally, mathematical rounding is a simple and effective integerization method for integer-KLTs design, since it keeps high TCG and good reversibility. It can be further improved. Integerization with minimal TCG does not take reversibility into account, which may lead to a poor RD performance in the high bitrate (low quantization distortion) case. The R-G model ensures that the derived integer-KTLs have the best reversibility and retain high and adequate TCG. It tries best to keep the good properties of KLT in integerization.

Although we extend the discussion based on MDDT, it is evident that the proposed approach is applicable for any KLT, such as discrete sine transform (Yeo *et al.*, 2011; Han *et al.*, 2012) and KLTs used in secondary transform (Saxena and Fernandes, 2013).

5 Conclusions

In this paper, we discuss integerization of KLT and focus on how to maintain useful properties (orthonormality and high TCG) of KLT for video compression. An R-G model that jointly considers the TCG and reversibility of a transform is proposed for integer-KLT design. The experimental results show the effectiveness of the R-G model.

In the discussion, we always assume a scalar factor in integerization. In fact, this factor could be a diagonal matrix, and a scaling process is needed after transform for normalization of transform coefficients. This is similar to the ICT design in H.264/AVC. For this case, the mind of the R-G model still works, but the modeling of reversibility is much more complex, which would be our future research.

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