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Building trust networks in the absence of trust relations*

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Abstract: User-specified trust relations are often very sparse and dynamic, making them difficult to accurately predict from online social media. In addition, trust relations are usually unavailable for most social media platforms. These issues pose a great challenge for predicting trust relations and further building trust networks. In this study, we investigate whether we can predict trust relations via a sparse learning model, and propose to build a trust network without trust relations using only pervasively available interaction data and homophily effect in an online world. In particular, we analyze the reliability of predicting trust relations by interaction behaviors, and provide a principled way to mathematically incorporate interaction behaviors and homophily effect in a novel framework, bTrust. Results of experiments on real-world datasets from Epinions and Ciao demonstrated the effectiveness of the proposed framework. Further experiments were conducted to understand the importance of interaction behaviors and homophily effect in building trust networks.

Key words: Trust network; Sparse learning; Homophily effect; Interaction behaviors
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1 Introduction

Our society increasingly relies on the digitized and aggregated opinions of others for making decisions. Trust has played more of an essential and important role in initiating social interactions, assessing quality and credibility of information, decision-making in purchasing, and so on. However, in reality, there are numerous online social networks without explicit trust relations, such as Facebook and Twitter. Even when trust relations are available (e.g., Epinions and Ciao), they are often too sparse to accurately infer between two users. Therefore, trust prediction is an important research field, especially where there is an absence of explicit social relations, which has been attracting increased attention in computer science and sociology.

Recently, a large amount of research has focused on trust prediction, yet little work has been done on predicting trust relations in the absence of trust network topology. The ratings that users have given to items or reviews provide us with rich behavior data. By using these online ratings, we can find similar preferences and interaction opinions of users for predicting trust relations and further building a trust network. Therefore, when traditional trust prediction methods fail, we take direct interactions as the principal factors from which to derive trust

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relations. When the number of interactions is small, the similarity between users becomes a secondary factor to make up for the deficiency.

In this work, we study the problem of building a trust network without any trust relations using only interaction behaviors and the homophily effect. In the absence of social or trust relations, we can use behavior data (such as rating data on reviews and emotional tendency) to investigate (1) how to capture the inducing factors of trust and (2) how to take advantage of these inducing factors for building trust networks. Our contributions are summarized as follows:

1. Demonstrate the existence of inducing factors about trust relations in terms of interaction behaviors and the homophily effect.

2. Propose an unsupervised learning framework, bTrust, to predict trust relations based on low-rank matrix factorization via introducing the reliability matrix of positive interactions and homophily regularization.

3. Evaluate the proposed framework, bTrust, on two real-world datasets, Epinions and Ciao, to understand the importance of inducing factors in building a trust network on a premise without trust relations.

In addition, this work makes it possible to turn an existing acquaintance or interaction network into a trust network, because we need social interaction feedback with a positive opinion tendency, not strictly relying on the rating data in product review communities, such as Epinions and Amazon.

2 Related work

In recent years, trust prediction has attracted more and more attention from computer science communities, and many trust prediction models have been developed, most of which can be classified as supervised learning approaches (Kuter and Golbeck, 2007; Nguyen *et al.*, 2009; Leskovec *et al.*, 2010; Zolfaghar and Aghaie, 2012) and unsupervised learning approaches (Guha *et al.*, 2004; Massa and Avesani, 2005; Zhang and Mao, 2014).

The supervised approaches first define trust relations as a set of features, and then train a classifier with a little known data and predict user pairs with the trained classifier, such as the Bayesian network (Kuter and Golbeck, 2007), support vector machine (SVM) with a radial basis function (RBF) kernel (Liu *et al.*, 2008), logistic regression (Leskovec *et al.*, 2010), and neural network (Zolfaghar and Aghaie, 2012). Based on interaction data, Kahanda and Neville (2009) proposed a supervised method to distinguish strong ties from weak ties by predicting binary relationship strength between users. Zolfaghar and Aghaie (2011) proposed a time-aware trust prediction approach, which incorporates the temporal evolution of trust networks to predict future trust relations (or links) with a supervised learning method. Liu *et al.* (2008) proposed a classification approach to predict if a user trusts another user using features derived from his/her interactions with the latter as well as from the interactions with other users. Nielsen *et al.* (2007) proposed a Bayesian model for event-based trust and defined a mathematical measure to quantitatively compare the effectiveness of probabilistic computational trust systems in various environments. Wang and Vassileva (2003) argued that trust was multi-faceted, and as Bayesian networks provide a flexible method to present differentiated trust and combine different aspects of trust, they proposed a Bayesian network based trust model in peer-to-peer networks. In the supervised model, the explicit trust value in a web of trust is necessary and critical for training the trust prediction model as an output variable; however, trust relations follow a power law distribution, making the classification problem extremely unbalanced (Wang *et al.*, 2011) and thus affecting the accuracy of classification.

The unsupervised approaches develop a prediction model which propagates trust values through the trust network (Guha *et al.*, 2004) or assigns a calculated score based on some criteria (Kim and Phalak, 2012; Wang *et al.*, 2015a) to each user pair so as to rank them from most likely to least likely trust relations. Agarwal and Zhou (2014) introduced an extended trust model to detect malicious activities in online social networks. The major insight was to conduct a trust propagation process over a novel heterogeneous social graph which can model different social activities. Jang *et al.* (2014) proposed a method to predict accurately the trust relationships of a target user even if he/she did not have much interaction information, considering positive, implicit, and negative information of all users in a network based on belief propagation to predict trust relationships of the target user. Au Yeung and Iwata (2011) investigated the strength of social influence in trust networks, showing that the strength of trust relation correlates with the similarity among the users. A modified matrix factorization technique was used to estimate the strengths of trust relations. Guha *et al.* (2004) developed a formal framework of trust propagation schemes. They first separated trust and distrust matrices and then performed operations on them to obtain the transitive trust between two nodes. Tang *et al.* (2013) proposed an unsupervised framework incorporating low-rank matrix factorization and homophily regularization for trust prediction, and the experimental results demonstrated the effectiveness of the proposed framework. Mishra and Bhattacharya (2011) proposed a method to model and compute the bias or truthfulness of a user in trust networks. The bias of users was their propensity to trust/distrust other users. They claimed that their model conforms well to other graph ranking algorithms and social theories such as the balance theory.

3 Motivating observations and problem statement

3.1 Motivating observations

Trust is a complex and abstract concept. It is difficult to identify the inducing factors that build trust networks. To analyze these inducing factors, the simplest and best way is to observe real-world data about trust relations from social networks. In view of user behaviors, we divide the inducing factors of trust into interaction behaviors and the homophily effect.

3.1.1 Interaction behaviors

Interaction behaviors with emotion tendency are an effective and important way to overcome the inaccuracy and sparseness problem of trust prediction. In the context of product review sites, a user can rate reviews written by another user 'helpful', which shows agreement toward the user. It is reasonable to surmise that 'helpful' ratings are more appropriate for measuring the degree to which the trustor trusts trustee.

In this study, we first compare a trustor's ratings with an average score of the trustor's ratings $\overline{r}_{\mathrm{tr}}$ and consider the interactions with scores higher than $\overline{r}_{\mathrm{tr}}$ positive. Then, we obtain a matrix $\boldsymbol{P}$ from user-review authorship relation matrix $\boldsymbol{E}$ and user-review helpfulness rating matrix $\boldsymbol{A}$, where P_{ij} is the number of positive interactions from u_i to u_j. Finally, we study the correlation between positive interactions and trust relations.

To verify the effectiveness of positive interactions on trust prediction, we try to answer the following two questions:

1. Are users with positive interactions more likely to establish trust relations than two randomly chosen users?

2. Are two users with the more positive interactions more likely to establish trust relations?

To answer the first question, we obtain two vectors **vp** and **vr**, in which each element of **vp** represents the set of pairs of users with positive interactions, while each element of **vr** denotes the set of pairs of users that are randomly chosen. In terms of each user pair $< u_i, u_j >$, if u_i trusts u_j, $\mathrm{vp}_i = 1$ or $\mathrm{vr}_i = 1$, and zero otherwise. Then, we construct a two-sample t-test on **vp** and **vr**. The null hypothesis H_0: $\mathbf{vp} \leq \mathbf{vr}$ is that the number of trust relations for those with positive interactions is smaller than that for randomly chosen users. The alternative hypothesis H_1: $\mathbf{vp} > \mathbf{vr}$ is the opposite. For both datasets, the null hypothesis is rejected at significance level $\alpha = 0.01$ with p-value of 4.27×10^{-86} and 6.13×10^{-75} for Epinions and Ciao, respectively. The evidence from the t-test suggests a positive answer to the first question: users with positive interactions are likely to have more trust relations than users chosen at random.

To answer the second question, we define the ratio of pairs with trust relations T_k in I_k when the number of positive interactions is k as $R_k = T_k/I_k$, where T_k denotes the set of pairs of users $< u_i, u_j >$ with the number of positive interactions from u_i to u_j being no less than k, and I_k represents the set of pairs with trust relations in T_k. Fig. 1 describes the distribution of the ratio of trust relations R_k with respect to k. As k increases, R_k tends to increase, which indicates the impact of the number of positive interactions on the correlation and that a large percentage of trust relations are established under interaction behaviors. It reveals that there is a strong correlation between positive interactions and trust relations, and that users with positive interactions are more likely to have trust relations.

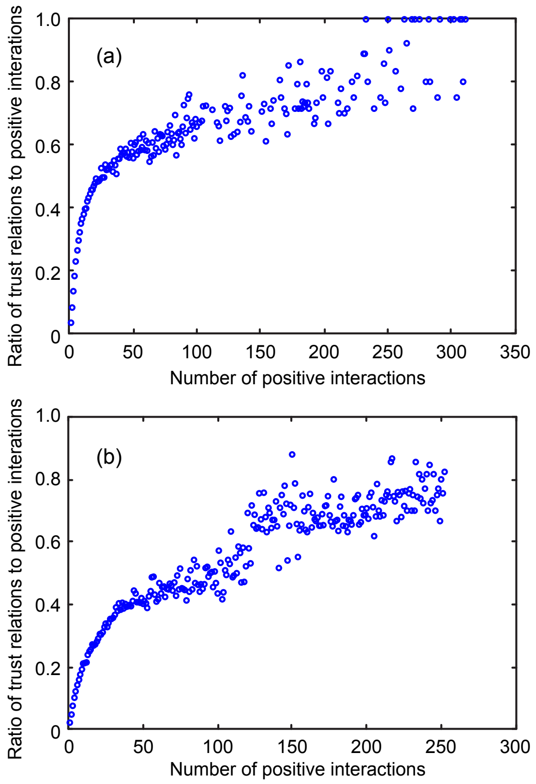

Fig. 1 The number of positive interactions vs. trust relations: (a) Epinions; (b) Ciao

3.1.2 Homophily effect

Homophily is one of the most important sociology theories (Wang *et al.*, 2015b) that contact between similar people occurs at a higher rate than among dissimilar people (McPherson *et al.*, 2001). We investigate homophily in social networks via researching the correlation between trust relations and user similarity (Guo *et al.*, 2015). User similarity in social networks is established by similar preferences and similar behaviors (Wolff and Weyde, 2014). Next, we verify only the homophily effect as an important factor for predicting trust relations by asking the following question: are users with trust relations more similar in terms of similar behaviors than those without any relations?

For this question, Tang *et al.* (2013) have verified the answer by comparing trust similarity and random similarity without trust relations. We conduct a similar t-test between trust relations and distrust relations. It is found that user similarity is insufficient for distinguishing trust relations and distrust relations. Therefore, we conclude that user similarity is conducive to distinguish user pairs with social relations and without any relations through the above two observations.

3.2 Problem statement

Let $\mathbf{UB} = [\boldsymbol{E}, \boldsymbol{A}]$ be the set of user behaviors on product review sites, where $\boldsymbol{E} \in \mathbb{R}^{m\times n}$ is a user-review authorship relation matrix, $\boldsymbol{A} \in \mathbb{R}^{m\times n}$ is a user-review helpfulness rating matrix, m is the number of users, and n is the number of reviews. $E_{ij} = 1$ if u_i writes r_j, and $E_{ij} = 0$ otherwise. If u_i rates the helpfulness of r_j, R_{ij} is the helpfulness rating score. We assume that trust relations are unavailable, and exist only within interaction data on the social network. So, we define $\boldsymbol{X}$ as a pseudo trust matrix that is constructed based on interaction behaviors with emotion tendency. $X_{ij} = 1$ if we observe that u_i has positive interactions with u_j and $X_{ij} = 0$ otherwise. Thus, trust prediction aims to develop a predictor f to infer real trust relations $\boldsymbol{T}$ using a pseudo trust matrix $\boldsymbol{X}$:

$$f : \{\boldsymbol{X}\} \rightarrow \{\boldsymbol{T}\}. \tag{1}$$

With the above notations, we formally define trust prediction with interaction behaviors and homophily effect as follows:

Given pseudo trust matrix $\boldsymbol{X}$, user-review authorship relation matrix $\boldsymbol{E}$, and user-review helpfulness rating matrix $\boldsymbol{A}$, we aim to learn a predictor f to infer trust relations $\boldsymbol{T}$ with $\boldsymbol{X}$, $\boldsymbol{E}$, and $\boldsymbol{A}$:

$$f : \{\boldsymbol{X}, \boldsymbol{E}, \boldsymbol{A}\} \rightarrow \{\boldsymbol{T}\}. \tag{2}$$

4 Our framework—bTrust

4.1 Low-rank matrix factorization model

Previous work demonstrates that online trust has several properties such as transitivity, asymmetry, and correlation with user preferences and multiple factors (Tang *et al.*, 2012). A few factors can influence people in establishing trust relations, and a user usually establishes trust relations with a small proportion of users, resulting in trust network $\boldsymbol{X}$, which is very sparse and of low rank. Low-rank matrix tri-factorization seeks a 'factor model' for representation using a low-rank approximation.

In this study, we harness non-negative matrix tri-factorization to model trust prediction. Specifically, $\boldsymbol{X}$ is decomposed by the following three matrices:

$$\boldsymbol{X} \approx \boldsymbol{UHU}^{\mathrm{T}}, \tag{3}$$

where $\boldsymbol{U} \in \mathbb{R}^{n\times d}$ is the user preference matrix and d is the number of facets of user preferences. $\boldsymbol{H} \in \mathbb{R}^{d\times d}$ captures the correlations among $\boldsymbol{U}$. X_{ij} is modeled as $X(i,j) = \boldsymbol{U}(i,:)\boldsymbol{HU}^{\mathrm{T}}(j,:)$.

The matrix factorization model seeks a low-rank representation $\boldsymbol{U}$ and $\boldsymbol{H}$ via solving the following optimization problem:

$$\min_{\boldsymbol{U},\boldsymbol{H}} \quad \|\boldsymbol{X} - \boldsymbol{UHU}^{\mathrm{T}}\|_{\mathrm{F}}^2, \tag{4}$$

where $\|\cdot\|_{\mathrm{F}}$ is the Frobenius norm of a matrix. Specifically, if $\boldsymbol{X} \in \mathbb{R}^{m\times n}$, then $\|\boldsymbol{X}\|_{\mathrm{F}} = \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n} X_{ij}^2}$.

To avoid over-fitting, we add a smoothness regularization on $\boldsymbol{U}$ and $\boldsymbol{H}$, and non-negative constraints are always applied to $\boldsymbol{U}$ and $\boldsymbol{H}$. We have

$$\begin{aligned} &\min_{\boldsymbol{U},\boldsymbol{H}} \quad \|\boldsymbol{X} - \boldsymbol{UHU}^{\mathrm{T}}\|_{\mathrm{F}}^2 + \alpha\|\boldsymbol{H}\|_{\mathrm{F}}^2 + \beta\|\boldsymbol{U}\|_{\mathrm{F}}^2 \\ &\text{s.t.,} \quad \boldsymbol{U} \geq 0,\ \boldsymbol{H} \geq 0, \end{aligned} \tag{5}$$

where $\alpha\|\boldsymbol{H}\|_{\mathrm{F}}^2 + \beta\|\boldsymbol{U}\|_{\mathrm{F}}^2$ are introduced to avoid over-fitting and non-negative parameters α and β are used to control the capability of $\boldsymbol{H}$ and $\boldsymbol{U}$, respectively. It is easy to verify that Eq. (5) can model the properties of trust mentioned above, and performance improvement was reported by Huang *et al.* (2012; 2013) and Tang *et al.* (2013) in terms of trust prediction. Next we will introduce an approach to model interaction beahvior and the homophily effect based on the low-rank matrix tri-factorization method.

4.2 Modeling interaction behaviors

In our study, since we assume that a trust network is not available, interaction behaviors are modeled in trust prediction by capturing the correlation between positive interactions and trust relations. We can leverage users' positive interactions to build a pseudo trust network. Let $\boldsymbol{X} \in \mathbb{R}^{n\times n}$ be the pseudo trust matrix, defined as

$$X_{ij} = \begin{cases} 1, & \text{if } I_{ij} > 0, \\ 0, & \text{otherwise}, \end{cases} \tag{6}$$

where I_{ij} denotes the number of positive interactions from u_i to u_j.

The matrix of pseudo trust relations $\boldsymbol{X}$ is constructed based on positive interactions, and their values may not be equally reliable. X_{ij} is very reliable due to the existence of trust relation from u_i to u_j, while values in the pseudo matrix with more positive interactions are more reliable based on our previous findings on the impact of the number of positive interactions. Therefore, we incorporate a weight matrix $\boldsymbol{W}$ to indicate the reliability of values in X, and the contribution of X_{ij} to the learning process is controlled by W_{ij} during the matrix factorization process:

$$W_{ij} = \begin{cases} g(I_{ij}), & \text{if } u_i \to u_j \text{ has a} \\ & \quad \text{positive interaction}, \\ c, & \text{otherwise}, \end{cases} \tag{7}$$

where $g(x) = 1 - [1 + \ln(x+1)]^{-1}$.

We may not directly apply Eq. (5) to our problem since the values in $\boldsymbol{X}$ may not be available. We modify Eq. (5) with the pseudo trust relation matrix $\boldsymbol{X}$ and weight matrix $\boldsymbol{W}$ by solving the following optimization problem:

$$\begin{aligned} &\min_{\boldsymbol{U},\boldsymbol{H}} \quad \|\boldsymbol{W} \odot (\boldsymbol{X} - \boldsymbol{UHU}^{\mathrm{T}})\|_{\mathrm{F}}^2 + \alpha\|\boldsymbol{H}\|_{\mathrm{F}}^2 + \beta\|\boldsymbol{U}\|_{\mathrm{F}}^2 \\ &\text{s.t.} \quad \boldsymbol{U} \geq 0,\ \boldsymbol{H} \geq 0. \end{aligned} \tag{8}$$

4.3 Modeling homophily effect

Homophily in social networks supports that users with higher similarity are more likely to establish trust relations than those with lower similarity. We define $\zeta(i,j)$ as the 'homophily coefficient' between u_i and u_j, satisfying (1) $\zeta(i,j) \in [0,1]$, (2) $\zeta(i,j) = \zeta(j,i)$, and that (3) the larger $\zeta(i,j)$ is, the more likely a trust relation is established between u_i and u_j (Tang *et al.*, 2013; Wang *et al.*, 2015b). With the homophily coefficient, homophily regularization is to minimize the following term:

$$\min \quad \sum_{i=1}^{n}\sum_{j=1}^{n} \zeta(i,j)\|\boldsymbol{U}(i,:) - \boldsymbol{U}(j,:)\|_{\mathrm{F}}^2. \tag{9}$$

Users close to each other in the low-rank space are more likely to establish trust relations (Zheng *et al.*, 2014), and their distances in the latent space are controlled by their homophily coefficients. For example, $\zeta(i,j)$ controls the latent distance between u_i and u_j. A larger value of $\zeta(i,j)$

indicates that u_i and u_j are more likely to establish trust relations according to property (3) of the homophily coefficient. Thus, their latent representations should be as close as possible, while a smaller value of $\zeta(i,j)$ tells that the distance of their latent representations should be larger.

For a particular user u_i, the terms in homophily regularization related to the latent representation $\boldsymbol{U}(i,:)$ are

$$\sum_{j=1}^{n} \zeta(i,j) \|\boldsymbol{U}(i,:) - \boldsymbol{U}(j,:)\|_{\mathrm{F}}^2. \tag{10}$$

We can see that the latent representation for u_i is smoothed with other users, controlled by the homophily coefficient. Hence, even for long-tail users with a few or even no trust relations, we can still obtain an approximate estimate of their latent representations via homophily regularization, addressing the sparsity problem with traditional unsupervised methods.

After some derivations, we can obtain the homophily regularization as follows:

$$\begin{aligned}
&\sum_{i=1}^{n}\sum_{j=1}^{n} \zeta(i,j) \|\boldsymbol{U}(i,:) - \boldsymbol{U}(j,:)\|_{\mathrm{F}}^2 \\
&= \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{d} \zeta(i,j)[U(i,k) - U(j,k)]^2 \\
&= 2\sum_{k=1}^{d} \boldsymbol{U}^{\mathrm{T}}(:,k)(\boldsymbol{\mathcal{Q}} - \boldsymbol{\mathcal{Z}})\boldsymbol{U}(:,k) = 2\mathrm{tr}(\boldsymbol{U}^{\mathrm{T}}\boldsymbol{\mathcal{L}}\boldsymbol{U}),
\end{aligned} \tag{11}$$

where $\boldsymbol{\mathcal{L}} = \boldsymbol{\mathcal{Q}} - \boldsymbol{\mathcal{Z}}$ is the Laplacian matrix, $\boldsymbol{\mathcal{Q}}$ is a diagonal matrix with the ith diagonal element $\mathcal{Q}(i,i) = \sum_{j=1}^{n} \zeta(i,j)$, and $\boldsymbol{\mathcal{Z}}$ is the homophily coefficient matrix. $\zeta(i,j)$ is measured as users' similarity by 'ratings similarity'.

4.4 Optimization algorithm for bTrust

When considering both interaction behaviors and homophily effect, X_{ij} can be modeled by combining Eqs. (8) and (9). The proposed framework, bTrust, is to solve the following optimization problem:

$$\begin{aligned}
\min_{\boldsymbol{U},\boldsymbol{H}} \quad & \|\boldsymbol{W} \odot (\boldsymbol{X} - \boldsymbol{U}\boldsymbol{H}\boldsymbol{U}^{\mathrm{T}})\|_{\mathrm{F}}^2 + 2\lambda \mathrm{tr}(\boldsymbol{U}^{\mathrm{T}}\boldsymbol{\mathcal{L}}\boldsymbol{U}) \\
& + \alpha\|\boldsymbol{H}\|_{\mathrm{F}}^2 + \beta\|\boldsymbol{U}\|_{\mathrm{F}}^2 \\
\text{s.t.} \quad & \boldsymbol{U} \geq 0,\ \boldsymbol{H} \geq 0.
\end{aligned} \tag{12}$$

Next, we use $\boldsymbol{\mathcal{L}}_k$ to denote the Lagrangian function of Eq. (12) in the kth iteration:

$$\begin{aligned}
\boldsymbol{\mathcal{L}}_k =& \|\boldsymbol{W} \odot (\boldsymbol{X} - \boldsymbol{U}\boldsymbol{H}\boldsymbol{U}^{\mathrm{T}})\|_{\mathrm{F}}^2 + \alpha\|\boldsymbol{H}\|_{\mathrm{F}}^2 + \beta\|\boldsymbol{U}\|_{\mathrm{F}}^2 \\
& + 2\lambda\mathrm{tr}(\boldsymbol{U}^{\mathrm{T}}\boldsymbol{\mathcal{L}}\boldsymbol{U}) - \mathrm{tr}(\boldsymbol{\Lambda}^{(1)}\boldsymbol{U}) - \mathrm{tr}(\boldsymbol{\Lambda}^{(2)}\boldsymbol{H}),
\end{aligned} \tag{13}$$

where $\boldsymbol{\Lambda}^{(1)}$ and $\boldsymbol{\Lambda}^{(2)}$ are Lagrangian multipliers for non-negativity of $\boldsymbol{U}$ and $\boldsymbol{H}$, respectively.

By moving constants, $\boldsymbol{\mathcal{L}}_k$ can be rewritten as

$$\begin{aligned}
\boldsymbol{\mathcal{L}}_k =& -2\mathrm{tr}[(\boldsymbol{W}^{\mathrm{T}} \odot \boldsymbol{W}^{\mathrm{T}} \odot \boldsymbol{X}^{\mathrm{T}})\boldsymbol{U}\boldsymbol{H}\boldsymbol{U}^{\mathrm{T}}] \\
& + \alpha\mathrm{tr}(\boldsymbol{H}^{\mathrm{T}}\boldsymbol{H}) + \beta\mathrm{tr}(\boldsymbol{U}^{\mathrm{T}}\boldsymbol{U}) \\
& + \mathrm{tr}\{[\boldsymbol{W}^{\mathrm{T}} \odot \boldsymbol{W}^{\mathrm{T}} \odot (\boldsymbol{U}\boldsymbol{H}^{\mathrm{T}}\boldsymbol{U}^{\mathrm{T}})]\boldsymbol{U}\boldsymbol{H}\boldsymbol{U}^{\mathrm{T}}\} \\
& + 2\lambda\mathrm{tr}(\boldsymbol{U}^{\mathrm{T}}\boldsymbol{\mathcal{L}}\boldsymbol{U}) - \mathrm{tr}(\boldsymbol{\Lambda}^{(1)}\boldsymbol{U}) - \mathrm{tr}(\boldsymbol{\Lambda}^{(2)}\boldsymbol{H}).
\end{aligned} \tag{14}$$

The KKT complementary condition (Ye, 2006) is

$$U_{ik}\Lambda_{ik}^{(1)} = 0, H_{ik}\Lambda_{ik}^{(2)} = 0, \forall\ i \in [1,n], k \in [1,d]. \tag{15}$$

Setting $\frac{\partial \boldsymbol{\mathcal{L}}_k}{\partial \boldsymbol{U}} = 0$ and $\frac{\partial \boldsymbol{\mathcal{L}}_k}{\partial \boldsymbol{H}} = 0$ and adopting an alternative optimization schema (Ding *et al.*, 2008), we update $\boldsymbol{U}$ and $\boldsymbol{H}$ alternately with the following updating rules:

$$\begin{cases}
U_{ik} \leftarrow U_{ik}\sqrt{\dfrac{A_{ik}}{B_{ik}}}, \\
H_{ik} \leftarrow H_{ik}\sqrt{\dfrac{C_{ik}}{D_{ik}}},
\end{cases} \tag{16}$$

where $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, and $\boldsymbol{D}$ are defined as

$$\begin{aligned}
\boldsymbol{A} =& 2(\boldsymbol{W}^{\mathrm{T}} \odot \boldsymbol{W}^{\mathrm{T}} \odot \boldsymbol{X}^{\mathrm{T}})\boldsymbol{U}\boldsymbol{H} \\
& + 2(\boldsymbol{W} \odot \boldsymbol{W} \odot \boldsymbol{X})\boldsymbol{U}\boldsymbol{H}^{\mathrm{T}} + 4\lambda\boldsymbol{\mathcal{Z}}\boldsymbol{U}, \\
\boldsymbol{B} =& [\boldsymbol{W}^{\mathrm{T}} \odot \boldsymbol{W}^{\mathrm{T}} \odot (\boldsymbol{U}\boldsymbol{H}^{\mathrm{T}}\boldsymbol{U}^{\mathrm{T}})]\boldsymbol{U}\boldsymbol{H} \\
& + [\boldsymbol{W} \odot \boldsymbol{W} \odot (\boldsymbol{U}\boldsymbol{H}\boldsymbol{U}^{\mathrm{T}})]\boldsymbol{U}\boldsymbol{H}^{\mathrm{T}} + 2\alpha\boldsymbol{U} \\
& + [\boldsymbol{W}^{\mathrm{T}} \odot \boldsymbol{W}^{\mathrm{T}} \odot (\boldsymbol{U}\boldsymbol{H}\boldsymbol{U}^{\mathrm{T}})]\boldsymbol{U}\boldsymbol{H}^{\mathrm{T}} \\
& + [\boldsymbol{W} \odot \boldsymbol{W} \odot (\boldsymbol{U}\boldsymbol{H}^{\mathrm{T}}\boldsymbol{U}^{\mathrm{T}})]\boldsymbol{U}\boldsymbol{H} + 4\lambda\boldsymbol{\mathcal{Q}}\boldsymbol{U}, \\
\boldsymbol{C} =& 2\boldsymbol{U}^{\mathrm{T}}(\boldsymbol{W} \odot \boldsymbol{W} \odot \boldsymbol{X})\boldsymbol{U}, \\
\boldsymbol{D} =& \boldsymbol{U}^{\mathrm{T}}[\boldsymbol{W} \odot \boldsymbol{W} \odot (\boldsymbol{U}\boldsymbol{H}\boldsymbol{U}^{\mathrm{T}})]\boldsymbol{U} \\
& + \boldsymbol{U}^{\mathrm{T}}[\boldsymbol{W}^{\mathrm{T}} \odot \boldsymbol{W}^{\mathrm{T}} \odot (\boldsymbol{U}\boldsymbol{H}\boldsymbol{U}^{\mathrm{T}})]\boldsymbol{U} + 2\alpha\boldsymbol{H}.
\end{aligned} \tag{17}$$

After learning $\boldsymbol{U}$ and $\boldsymbol{H}$, $\boldsymbol{U}_i\boldsymbol{H}\boldsymbol{U}_j^{\mathrm{T}}$ suggests the likelihood of a trust relation established from u_i to u_j; namely, $\tilde{\boldsymbol{X}} = \boldsymbol{U}\boldsymbol{H}\boldsymbol{U}^{\mathrm{T}}$ is the new low-rank representation of $\boldsymbol{X}$.

5 Performance evaluation

We have conducted experiments to evaluate the effectiveness of the proposed framework, bTrust. Through the experiments, we aim to answer the following two questions:

1. How effective is bTrust compared with other baseline methods?

2. How do interaction behaviors and the homophily effect affect bTrust?

5.1 Experimental settings

We took Epinions (http://epinions.com) and Ciao (http://www.ciao.co.uk) as experimental datasets. Both sites are famous product review sites where users not only write a review by rating items with 1–5 star(s), but also rate other reviews using some tags which range from 'not helpful' to 'most helpful'. Therefore, we first collected the available datasets for this research and paid more attention to information with interaction behaviors. Then, we filtered out users with less than two in-degrees and less than two reviews and ratings, aiming to obtain data that are large enough and that have sufficient information for the purpose of evaluation. Finally, we ranked all ratings in descending order of the time when they were established, and filtered out the portions of ratings that were after the establishment time of trust relations. Table 1 shows some statistics of the collected data.

Table 1 Statistics of the datasets

Parameter	Value	
	Epinions	Ciao
Number of users	10 137	9176
Number of trust relations	388 076	149 784
Number of items	309 641	264 438
Number of ratings	7 326 961	7 062 974
Mean value of ratings	4.35	3.95

We collected user review helpfulness ratings and sorted them in chronological order in terms of the time when helpfulness information was rated, which is denoted as $\mathcal{T}$. For time t_x, we ranked user pairs according to the weight of positive interactions and took the first $x\%$ of user pairs in $\mathcal{T}$ to form a pseudo matrix of trust relations, namely an evaluation dataset $\mathcal{D}$. We collected all trust relations until time t_x as the testing dataset, which is denoted as $\mathcal{R}$. In this study, we varied x as $\{5, 10, 20, 30, 50\}$ and correspondingly constructed several evaluation datasets introduced in Section 3.

We followed the common metric for trust evaluation in Liben-Nowell and Kleinberg (2007) to assess the prediction performance. For each dataset above, $\mathcal{R}$ was the set of pairs with trust relations. Detailedly, each predictor ranked user pairs in $\mathcal{D}$ in descending order of confidence and we took the first $|\mathcal{R}|$ pairs as the set of predicted trust relations, denoted as $\mathcal{P}$. Then we have the prediction quality (PQ):

$$\mathrm{PQ} = \frac{|\mathcal{P} \cap \mathcal{R}|}{|\mathcal{R}|}, \tag{18}$$

where $|\cdot|$ denotes the size of a set. As noticed in Liben-Nowell and Kleinberg (2007), all users may be inferred trust relations so that the PQ score is usually low. Therefore, a random predictor is usually used as a baseline method. We executed the experiments 10 times for each dataset, and averaged the results on the evaluation metric to obtain the performance.

5.2 Comparison of different methods

To answer the first question, we compared bTrust with the following baseline methods:

1. HighInter: HighInter is based on the strong correlation between positive interactions and trust relations. It ranks pairs of users based on the number of positive interactions.

2. HighSimi: HighSimi is based on the strong correlation between high similarity and trust relations. It ranks pairs of users based on the similarity of user behaviors.

3. MF: MF conducts matrix factorization on the matrix representation of trust relations (Zhu *et al.*, 2007). It uses matrix factorization to compute the trust scores of pairs of users.

4. Random: Random ranks pairs of users randomly. Liben-Nowell and Kleinberg (2007) suggested that a random predictor should be used as a baseline method to meaningfully demonstrate the predictor quality since the PQ value is usually low.

In our experiments, we tuned α and λ via a common cross-validation for bTrust. For general experiment purposes, we set $\alpha = 0.5$, $\lambda = 0.5$ for Epinions and $\lambda = 1$ for Ciao, respectively. For all baseline methods, we reported the best performance. The experiment results are shown in Fig. 2.

By comparing the results of different methods, we have the following observations:

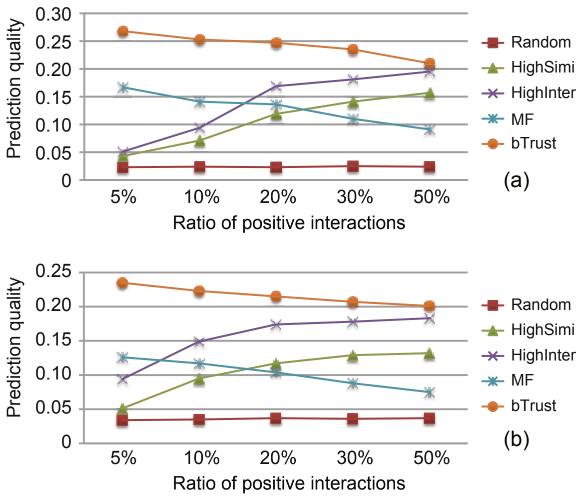

Fig. 2 Performance comparison on Epinions (a) and Ciao (b)

1. Our proposed framework, bTrust, consistently outperforms other baseline methods, especially when using a small dataset. There are two potential contributions for this improvement: (1) incorporating a reliable weight matrix $\boldsymbol{W}$ based on interaction behaviors with pseudo trust relations $\boldsymbol{X}$ and (2) modeling the homophily effect based on behavior data of user rating.

2. The performance of HighInter is much better than that of Random, which further demonstrates the existence of a strong correlation between positive interactions and trust relations. Similarly, the performance of HighSimi demonstrates a strong correlation between high similarity and trust relations.

3. The sparse learning methods, such as MF and bTrust, achieve better performance than other methods when using a small dataset. However, with the increase of x, the performances of all methods degrade. It demonstrates that sparse learning methods have more advantages over other machine learning methods.

In summary, we have performed a t-test on all comparisons and the t-test results suggested that all improvement is significant. With the help of regularization based on interaction behaviors and the homophily effect, bTrust gains significant improvement over representative baseline methods, which answers the first question asked at the beginning of this section.

5.3 Impact of two components of bTrust

bTrust has two important components: (1) incorporation of interaction behaviors and (2) modeling of the homophily effect. We have conducted experiments by separating these two components to further understand their impact on the performance of trust prediction and accordingly answer the second question asked at the beginning of this section. In detail, we took bTrust without the two components as the baseline method and systematically eliminated their impacts from bTrust by defining variants as follows:

1. bTrust-PIHE: disabling the impact of interaction behaviors and homophily effect by setting $\lambda = 0$ and $\boldsymbol{W} = 1$;

2. bTrust-PI: disabling the impact of interaction behaviors by setting $\boldsymbol{W} = 1$;

3. bTrust-HE: disabling the impact of homophily effect by setting $\lambda = 0$.

The experiment results are shown in Fig. 3. We have the following observations:

1. Intuitively, for all datasets, the performances of bTrust, bTrust-PI, and bTrust-HE were better than that of bTrust-PIHE. The performance of bTrust-HE was better than that of bTrust-PI. This demonstrates that interaction behaviors are more effective than the homophily effect in predicting trust relations.

2. When disabling the impact of homophily effect, the performance of bTrust-HE degraded. Compared to bTrust, the performance of bTrust-HE averagely reduced by 0.016 for Epinions and by 0.0126 for Ciao. Hence, bTrust-HE is close to bTrust. These results demonstrated that incorporating interaction behaviors can improve performance.

3. When disabling the impact of interaction behaviors, the performance of bTrust-PI averagely degraded by 0.0428 for Epinions and by 0.0452 for Ciao. These results demonstrated that modeling the homophily effect can improve performance.

In summary, the results of these experiments further demonstrated the importance of incorporating interaction behaviors and modeling the homophily effect in trust prediction, which correspondingly answers the second question. In addition, an appropriate combination of the sparse learning model and regularization based on social theories can greatly improve the performance of bTrust.

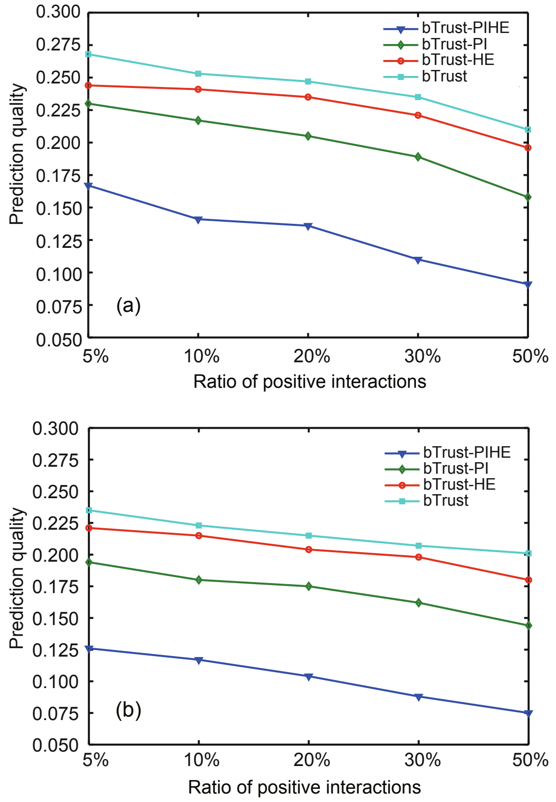

Fig. 3 Performance comparison on Epinions (a) and Ciao (b)

6 Conclusions and future work

In this study, we investigated whether we can build a trust network indirectly by studying the problem of predicting trust relations from interaction behaviors and the homophily effect. We first verified that there are strong correlations between positive interactions and trust relations and between users' similarity and trust relations. Then, we modeled interaction behaviors and the homophily effect mathematically. Finally, we proposed a novel framework, bTrust, to predict trust relations and build a trust network by incorporating interaction behaviors and modeling the homophily effect. Results of experiments on real-world datasets showed that bTrust can accurately predict trust relations. Further experiments were conducted to understand the importance of interaction behaviors and homophily effect in trust prediction.

There are several interesting directions for future work: (1) applying the proposed framework to predict positive links in the context of no explicit trust relation data and recommending some trustworthy users and items (Wang *et al.*, 2013; Forsati *et al.*, 2014); (2) exploring other social theories to predict negative links or distrust relations and further investigating some applications of the signs in social networks.

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